Liquid Crystals

Proceedings of an International Conference

December 3-8, 1979

Edited by

S. CHANDRASEKHAR

Raman Research Institute
BANGALORE 550 080. India

LIQUID CRYSTALS

Proceedings of an International Conference held at the Raman Research Institute, Bangalore, December 3-8, 1979.

© Heydon & Son Ltd., 1980 166N Ø 85501 163 7

ARMINIED IN INDIA

Edited by

Heyden Heyden Heyden a son unu

S CHANDRASEKHAR



LONDON · PHILADELPHIA · RHEINE

Editor's address Professor S. Chandrasekhar Raman Research Institute BANGALORE 560 080, India

CIAICYMU URBULI

Proceedings of an International Conference held at the Raman Research Institute. Bandalore, December 3 — 8, 1979.

© Heyden & Son Ltd., 1980 ISBN 0 85501 163 7

PRINTED IN INDIA

Heyden & Son Ltd., Spectrum House, Hillview Gardens, London NW4 2JQ, U.K. Heyden & Son Inc., 247 South 41st Street, Philadelphia, Pennsylvania 19104, U.S.A. Heyden & Son GmbH, Munsterstrasse 22, 4440 Rheine, West Germany.

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of Heyden & Son Ltd.

F-1 COVER CONTROL OF THE CONTROL OF

LONDON PHILADELPHIA RHEINE

PREFACE

This volume forms the Proceedings of a Conference on Liquid Crystals organized at the Raman Research Institute, Bangalore, from December 3-8, 1979.

About a hundred scientists from sixteen countries, including the USSR and China, participated in the Conference and 84 papers were presented.

The opening paper in the proceedings is by Professor Sir Charles Frank on What is the Director of a Liquid Crystal? - a consideration of the definitional status of some of the basic concepts of liquid crystal theory. This is followed by a paper by Professor W. Helfrich describing the three classes of thermotropic liquid crystals - columnar (or canonic, discovered in 1977), smectic and nematic, representing singly, doubly and triply melted phases - in terms of fluidity and defects. Probably the two most thoroughly discussed topics at the meeting were mesophases of disc-like molecules and the re-entrant phenomenon, and the proceedings contain a number of important and up-to-date contributions on both of them. In addition, the volume includes papers on practically all other aspects of the physics and chemistry of liquid crystalline materials - their preparation, their structures, their thermodynamic, optical and mechanical properties, and their response to external fields. The variety of topics covered emphasizes once again the remarkable growth of activity in this area of research and the impact that it is having on so many disciplines.

I should like to take this opportunity to place on record our sense of gratitude to a number of organizations for their very generous support: the Department of Science and Technology, the Indian Academy of Sciences, the Raman Research Institute, the Department of Atomic Energy, the University Grants Commission, the Council of Scientific and Industrial Research, Bharat Electronics Ltd., Indian Petrochemicals Corporation Ltd., Spectrospin AG, Hindustan Lever Ltd., Larsen & Toubro Ltd., Dadha Pharma Private Ltd., and the Immu-Kimia Laboratory. Finally, it is a pleasure to express my thanks to all my friends at the Institute but for whose encouragement and help this conference would certainly not have taken place.

Raman Research Institute December 1979

CONTENTS

F	C FRANK: What is the director of a liquid crystal?	1
W	HELFRICH: Three-dimensional classification of liquid crystals in terms of fluidity and defects	ृ -7
A	M LEVELUT: X-ray diffraction by mesophases of some hexa- alkanoates of terphenylene	21
С	DESTRADE, M C BERNAUD, H GASPAROUX, A M LEVELUT and NGUYEN HUU TINH: Disc-like mesogens with columnar and	
	B L MOOTITUAXA and two MADRUSUDANA: The reserve and their	29
S	CHANDRASEKHAR, B K SADASHIVA and K A SURESH: New disc- like mesogens - an explanatory note	33
A	QUEGUINER, A ZANN, J C DUBOIS and J BILLARD: Mesogenic	
	disc-like molecules with twofold symmetry axis	35
M	SORAI, K TSUJI, H SUGA and S SEKI: Heat capacities of a disc-like mesogen and its precursor having a	
	structure of C ₆ (OCO-n-C _m H _{2m+1}) ₆	41
J	PROST and N A CLARK: Hydrodynamic properties of two dimensionally ordered liquid crystals	53
E :	P MINEEV: Smectic B liquid crystals	59
	J LEADBETTER, M A MAZID and R M RICHARDSON: Structures	9
^	of the smectic B, F and H phases of the N-(4-n-	
	alkoxybenzylidene) - 4'-alkylanilines and the transitions between them	65
A	DE VRIES: On the structure of the uniaxial smectic B	
	ecompound of abhasis pressure	81
G	ALBERTINI, B DUBINI, S MELONE, M G PONZI-BOSSI and F RUSTICHELLI: Two dimensional disorder as a pre-	
	transitional effect in smectic H spinord Ly sassions	89

D	DEMUS, H J DEUTSCHER, D MARZOTKO, H KRESSE and	
	A WIEGELEBEN: Calorimetric investigation of	
	4-n-alkyloxyphenyl 4'-n-alkyloxybenzoates	97
P	E CLADIS: The re-entrant nematic phase	105
R	SHASHIDHAR and K V RAO: A high pressure X-ray cell	
	for liquid crystals: pressure dependence of the	
	smectic A layer spacing of 8 OCB in relation to its	
	re-entrant behaviour A CASTAROUM H . GUAMMES O M . EGARTE	115
K	P L MOODITHAYA and N V MADHUSUDANA: The re-entrant and m	
	nematic phase in some pure compounds and their	
	mixtures at atmospheric pressure AAGAAA A 8 AAGAAAAAAAAAAAAAAAAAAAAAAAA	121
S	CHANDRASEKHAR, K A SURESH and K V RAO: Temperature	
	dependence of the smectic A layer spacing in two	
	pure re-entrant nematogens	131
В	R RATNA, R SHASHIDHAR and K V RAO: Dielectric studies	
	of some re-entrant nematogens	135
	samearre as C ₆ (QCO-r-C ₇ Zp+1) i	133
A	M LEVELUT, F HARDOUIN and G'SIGAUD: X-ray study of a	
	pure material exhibiting re-entrant nematic and	
	smectic A phases	143
G	HEPPKE and F SCHNEIDER: Thermodynamic theory of re-	
	entrant nematic mixtures	147
. 1	BILLARD: Phase diagrams of mesogens exhibiting adversaria	
0.	re-entrant mesophases	100
	Te entrare mesophases	155
В	K SADASHIVA: Re-entrant nematic phase in a pure	
	compound at atmospheric pressure	165
G	V VANI, KALYANI VIJAYAN and S CHANDRASEKHAR: Absolute configuration of cholesteryl chloride and	
	cholesteryl bromide	167

PZ	ATRICIA SAWZIK and B M CRAVEN: Cholesteryl esters:	
25	crystal and mesophase structures	171
	HERVET, & RONDELEZ and W URBETH: Transport propertion	
A	DE VRIES and S B QADRI: X-ray studies of liquid	
	crystals VII. The temperature dependence of the	
	parameters for a skewed cybotactic nematic phase	179
2.6	to slonel shear flow	
K	USHA DENIZ, A S PARANJPE, V AMIRTHALINGAM and	
	K V MURALIDHARAN: Structure of the nematic phases	185
	of nBABA (n = 1 to 6) siretesions itemen notice high	100
A	M LEVELUT, M LAMBERT and N ALBON: X-ray diffraction	
	on oriented samples of LB' phase of the dipalmitoyl	
	lecithin (DPL)	191
Н	M CONRAD, K SALM and W STOFFEL: Determination of the	
	bilayer thickness of lecithin-(DLPC)-liposomes by	
	neutron small-angle scattering has a post-body to the le	193
R	NITYANANDA and G S RANGANATH: Disclinations and their	
	radial force of interaction in elastically aniso-	
	tropic nematic liquid crystals	205
	PRIXULAN, CHU GUININ and INAME ZHIGUO: Four-wave	
G	S RANGANATH: Angular forces between disclinations in	
	elastically anisotropic nematic liquid crystals assault places of the control of	213
G	S RANGANATH: Attraction between two like half singu-	
	larities in nematic liquid crystals oldgromosem 1938	219
S	T LAGERWALL and B STEBLER: Orientation defects in the	
	smectic C phase studied by the oblique contrast	
	method	223
E	DUBOIS-VIOLETTE and F ROTHEN: First bifurcation in	
	a planar nematic sample heated from below	239
	transition	
S	CHANDRASEKHAR, U D KINI and G S RANGANATH: Radial	
	flow in a cholesteric subjected to a rotary	
	(torsional) shear about the helical axis	247

o b kini: Cholesteric shear flow normal to the helical	
axis: the effect of a magnetic field	255
H HERVET, F RONDELEZ and W URBACH: Transport properties	
in liquid crystals	263
J WAHL: Cholesteric inversion structures generated by	
torsional shear flow	267
S C JAIN, S CHANDRA and V G BHIDE: Bubble domains in	
high pitch nematic-cholesteric mixtures	281
E F CARR and R W H KOZLOWSKI: Molecular alignment and	
material flow due to electric fields in nematic	
el liquid crystals (1901 middicel	287
K P L MOODITHAYA and N V MADHUSUDANA: Induced smectic mesomorphism in some binary mixtures: a new type	
of electrohydrodynamic pattern in the nematic phase	
of the mixtures	297
S A PIKIN: Polarized states in liquid crystals	303
YE PEIXUAN, CHU GUIYIN and ZHANG ZHIGUO: Four-wave	
mixing and its relaxation effect in liquid crystals	311
ALFRED SAUPE: Amphiphilic nematic and cholesteric phases	317
F CSER: Mesomorphic polymers: hopes and facts servings	329
E DUBOIS-VIOLETTE, P PIERANSKI, F ROTHEN and Law JIAWA SOAT	T S
L STRZELECKI: Static and dynamic properties of	
SS latex bodden	339
M ANISIMOV, V MAMNITSKY and E SORKIN: Tricritical beha-	
viour of heat capacity near nematic-isotropic	
transition	347

flow in a chalesteric subjected to a rotary

T.TN	LEI: Critical properties of nematic-isotropic	
	transition in liquid crystals	355
	Rematic phase transition in CBOOA	
RG	PRIEST: New results from the Landau-de Gennes model of the nematic-isotropic phase transition	361
G VI	ENKATESH, R SHASHIDHAR and D S PARMAR: High pres-	
	sure studies on 4,4'-di-n-alkoxyazoxybenzenes	373
SON	PRASAD and S VENUGOPALAN: Orientational statistics	
	in 8 OCB - Raman and infrared study	381
V K	KELKAR, J V YAKHMI and C MANOHAR: Nematic to iso-	
465	tropic transition in molecules forming dimers:	
	effect of monomer orientation	387
JA	JANIK, J M JANIK and K OTNES: End-chain and body	
469	reorientational motions in liquid cyrstalline	
	di-methoxyazoxybenzene and di-heptyloxyazoxybenzene	391
GK	GUPTA, V K AGARWAL and B BAHADUR: Dielectric	
	studies of a nematic liquid crystal	401
P C	JAIN and S R S KAFLE: Phase transition studies in	
	MBBA and EBBA using positron as a probe	405
B D	MALHOTRA, P C JAIN and V G BHIDE: Positron anni-	
	hilation in liquid crystals: role of dielectric	
	anisotropy THE RUBAYASUR STURWING DAS ALAM THE	411
MC	KANDAPAL and V G BHIDE: Mössbauer studies of	
	anisotropic diffusion and glass transition in some liquid crystals	421
H.M	CONRAD, W KRASSER, H H STILLER and A WERGIN:	
	Collective modes in liquid crystals	429

H M CONRAD and F MEZEI: Neutron spin-echo measurement	
of the critical dynamics of the smectic A to	Villi
nematic phase transition in CBOOA	441
S BHATTACHARYA, I D CALDER, B Y CHENG, J B KETTERSON	8 G
and B K SARMA: Ultrasonic studies of the NA and	
AB transitions : RAMMAR and D s PARMAR: RECEIVED S HESTAXN	449
C L KHETRAPAL, A C KUNWAR and K P SINHA: A new appli-	
cation of NMR spectroscopy of oriented molecules ARASA	461
C L KHETRAPAL, ANIL KUMAR and A C KUNWAR: Proton NMR	
study including 13C-1H satellites in acetone	X V
oriented in a nematic solvent and noither and point	465
C L KHETRAPAL, ANIL KUMAR, A C KUNWAR, P C MATHIAS and	
K V RAMANATHAN: 2-Dimensional NMR spectroscopy	A T
of molecules oriented in liquid crystals	469
S R SINGHAL, S K GUPTA and V G BHIDE: EPR study of	1
molecular order in smectic liquid crystals A V ATQUO	473
C A CROXTON: A density gradient theory of the inter-	
facial properties of smectic, nematic and isotropic	0 9
liquid crystals dong a sa nontison paleu ASSE bas ASSM	479
S KRISHNASWAMY: Experimental determination of the ASTOLIAM	d. 8
surface tension of two liquid crystals plant noiseling	487
ITOSHI MADA and SHUNSUKE KOBAYASHI: Order parameter	
near the surface of nematic liquid crystals as JAGACHAR	491
M HARENG, R HEHLEN and S LE BERRE: Two applications	
of the thermo-optic effect in smectic A liquid	
CONRAD, W KRASSER, H H STILLER and A WERGIN:	497
N V MADHUSUDANA and T N RUCKMONGATHAN: A convenient	
multiplexing scheme for addressing small liquid	
crystal matrix displays	499

G GOTTARELLI and B SAMORI: Applications of liquid bas RAHO.	IML
crystals_in_chemistry_mannioyxoxLs-n-1q)-q :: I-sneposem	505
lidene-p"-toluidines	
R EIDENSCHINK, J KRAUSE, L POHL and J EICHLER: New	
nematic compounds derived from the 1.4-trans-	
282 cyclohexane system qqqriosins piribelelb	515
G R VAN HECKE, T H SMITH and R D PROTTAS: Excess volumes	V. A. S
as functions of composition and temperature for	
ese binary mixtures of asymmetric, homologous nemato-	
genic azobenzenes	525 808 V
J M LOHAR and G H PATEL: Study of mixed mesophases:	-
typical binary mixtures comprising Schiff's bases	533
JeM LOHAR and URVASHI MASHRU: Enantiotropic mixed webal	autho:
mesophases in binary mixtures of a monotropic	
mesomorph and certain Schiff's bases	543
J S DAVE, GEORGE KURIAN and B C JOSHI: Influence of	
naphthelene moiety on liquid crystalline proper-	549
ties of some esters	343
R A VORA, M T CHHANGAWALA and R S GUPTA: Effect of	
middle flexible gorups on mesomorphic state	555
middle ileniale goodfo in man in	
C B UPASANI, J S DAVE, RENU GUPTA and NILESH DIXIT:	
Mesomorphism in azo dyes	559
N H SHAH, R A VORA and N D JADAV: Azomesogens-(II):	
(A) Methyl 4-(4'-n-alkoxyphenylazo)benzoates,	565
(B) n-propyl 4-(4'-n-alkoxyphenylazo) benzoates	505
J M LOHAR and G H PATEL: A new homologous series of	
mesogens and its characteristics: p-(p'-n-alkoxy-	
cinnamoyloxy) benzaldehydes	571

and best and best baves are personal mesogens-I: p-(p'	-n-alkoxycinnamoyl	tudy of new LLISAT loxy) benzy-	TOD
lidene-p"-toluidir	nes		F 7 0
HIER: New	DIS I bas IHOY I ,	ENSCHINK, J KRAUSE	579
R A VORA and NILESH DIX	XIT: New mesogens	with positive	
anisotr		cyclohexane system	
R A VORA and RENU GUPTA	: New mesogenic h	lomolgous EXPER MA	v s s
series having a la	teral and terminal	as functionxoid	
groups tamen auopo	asymmetric, homol	binary mixtures of	589
V SURENDRANATH and M SU	BRAMANYA RAJ URS:	Mesomorphic	
properties of some	phenyl benzoate d	erivatives as AABO	
with lateral hydro	xyl substituent	kim vancio (spievo	505
		OHAR and UKVASHI M	599
Subject Index	my mixtures of a m	mesophases in bina	
Judgett Index	ein Schiff's bases	mesomorph and cert	601
	I SINGOL D & BLO	SAVE, GEORGE KURTAN	
5.49		ties of some ester	
	LA and R S GUPTA:	VOEA, M T CHHANGAWA	
ic state 555	muns on mesomorphi		
	HENU GUPTA BAG NII	UPASANI, J S DAVE.	
		Mesomorphism in a	
:(II)-ananoa	emonA :VAGAS e w	SHAHA E A VORA and	
	l(ozslvne ngy x di		
	-n-alkoxyphenylax		
	solver Transmit I was pro- 15.		
	opolomod wen A		MT
-vxox(s=n=1n)-n			

F.C. Frank 100 al II. Lodowill a flat ew Paille aw beilineu au emos ai jeer

H.H. Wills Physics Laboratory, University of Bristol, Bristol BS8 1TL, UK

The director should not be called the direction of the molecules, nor the average direction of the molecules. The former is a term incapable of exact definition, except when the molecules possess symmetry axes, which is rarely the case: and there are uniaxially ordered sets of directions for which the average direction is indeterminate.

The director, whenever exactly definable, is a symmetry element: the symmetry axis for the orientational distribution of <u>any</u> direction which can be associated with the molecule: or the normal to a mirror plane for these distribution functions.

Where there is a complex curvature, definitions of the director by symmetry fail, and it is only definable by a limiting process: with the consequence that the elastic theory of liquid crystals is strictly speaking limited to being a small strain theory.

For smectic-C, there is no director with any correspondence to "the direction of the molecules".

Several of the concepts commonly used in liquid crystal theory, including order-parameters, are ill-defined unless the molecules possess symmetry axes, which usually they do not. Since there are indications that molecular asymmetry promotes occurrence of mesomorphs, this carries a danger of introducing essential error.

INTRODUCTION

Most publications about liquid crystals will be found to make reference, somewhere or another, to "the direction of the molecules". Sometimes the phrase used will be "the average direction of the molecules". The latter sounds the more careful way of putting it, but in fact raises a second question: "How do you define an average direction?", as well as "How do you define the direction of a molecule?".

Sometimes the term used will be "the director". That is clearly a technical term, and its use carries the implication that it has a precise definition, on which the author and the reader, if properly educated in the subject, would be agreed. But is i't so? Often enough, I think, if you asked either the author or the reader, he would tell you it is a synonym for the average direction of the molecules, and take you no further than that.

CONTINUUM THEORY

There is one context in which I suppose the director is well defined. That is in mathematical continuum theories of anisotropic fluids, where it is

assumed, as an axiom of the theory, that in every small neighbourhood of the fluid there is an associated unique direction, determined by molecular arrangement in some unspecified way, which we call a director. It is conveniently represented by a vector, n, let us say, with the admission that its sign is ambiguous. We assume that the components of n vary continuously in space and time, that the free energy depends on their gradients, which are measures of the curvature of the director field, and that the rate of creation of entropy depends on their time derivatives, together with those of other vectors which are lines connecting neighbouring fluid elements. That way, we obtain a selfconsistent system, conforming, at least to a very good approximation, with the real behaviour of, let us say, a nematic fluid. We cannot, however, remain content with this, as our persistent reference to the direction of the molecules shows. We can elaborate the continuum theory, for example by introducing coupling coefficients to electric fields: in doing so we inevitably interpret these by reference to molecular properties and orientations: usually through some simplified assumption about what they might be rather than what they really are . It is easy enough to suppose you know what you are talking about when discussing pictures in which each molecule is represented by a straight line, or a rod, or perhaps a banana: but real molecules are none of these things. We easily get ourselves into sophisticated theories the concepts of which belong to models simulating the liquid crystal, rather than the real thing.

SYMMETRIC MOLECULES

There are some molecules with a unique and precisely definable direction, in which all the atoms lie in a straight line. Such are carbon disulphide, SCS, acetylene, HCCH, diacetylene, HCCCH: unfortunately, I think none of these form mesomorphs. Quinquephenyl, five benzene rings in a row, is a nematogen in which the molecule has three diad axes of symmetry, one of which may be singled out as the long axis, to be called the direction of the molecule. Para-cyano-diphenyl has one diad, which is also its long axis, but to produce the many valuable mesomorphs based on this molecule we add a symmetry-breaking aliphatic or etheric tail in the para position. This is rather general. I suppose that rigid molecules of high symmetry too readily form true lattice-crystals, and these symmetry breaking rather flexible tails are needed to lower the stability of the true crystal, to permit the mesomorphs to appear.

MOLECULAR AXES

It is certainly the case that the vast majority of mesomorphogenic molecules have no symmetry axes whatever. How then shall we define their direction? For many years "the long axis" has given us an imperfect but useable working concept for this purpose. One always kept in mind the possibility that there might be liquid crystals in which it would be the shortest rather than the longest molecular axes which would be strongly correlated to one direction.

This very likely occurs in the tars of coking-coal, but chemically well-defined examples of it have now been produced here in Bangalore. When the molecule is planar, the shortest axis is well-defined as the normal to the plane: but they won't always be. The long axis might be more precisely defined as the line joining the remotest pair of atoms in the molecule: or, more usefully I think, as the line joining the remotest pair of atoms in the rigid portion of the molecule, without regard to flexible appendages. Moments of inertia are mathematically convenient, but not particularly relevant, I think, to the liquid crystal. The axes of the ellipsoid of diamagnetic susceptibility are well-defined, and truly very relevant to actual experimental observations. The axes of electric polarizability of the molecule are even more directly related to the experimental observations of birefringence etc., which we can make on liquid crystals. But here we have the internal field complications arising from the fact that these electric polarizabilities are large, and each molecule feels not only the applied field but also the induced field from all surrounding molecules. The apparent polarizability of a molecule in the condensed phase is not that of the bare molecule but of a pseudo-molecule, dressed with its environment, and variability in that environment is something we can be interested in. Refractive indices vary with frequency, and so, unless the molecule is symmetric, some variation in the axes of molecular polarizability with frequency must be anticipated. Other optical ways of observing molecular orientation, such as dichroism or fluorescence, are subject to similar complications. Nuclear magnetic resonance experiments give information about the vectors joining pairs of atoms. In a molecule of N atoms there are N(N-1) of these, though only some of them can be readily singled out for NMR study. Diffraction of X-rays, or other short-wave radiation, yields a complex overlay of information about positional correlation of atoms, from which it is hard to extract information about molecular orientation in uncontaminated form: though with neutron scattering intermolecular effects can be eliminated by using a dilute solution of deuterated molecules in a protonated host.

To summarize then, there is an infinite number of ways of defining a molecular axis \underline{M}_i , a direction determined by the orientation of the molecule, and a fairly large subset of these are open to observation of some statistical information about their distribution functions. A number of these \underline{M}_i may merge together into one direction if the molecule is symmetric, but in general they are all distinct. Most mesomorphogenic molecules are asymmetric, and that is probably no accident. Any theory of liquid crystals which eliminates this molecular asymmetry is liable to exclude an essential element of the truth.

What, then, <u>is</u> the director? It is <u>not</u> the average direction of the molecules. Direction of the molecules does not in general have any exact meaning, and we have not yet considered how to define the average of a direction. The director <u>is</u>, whenever it can be precisely defined, a crystallographic symmetry element, <u>the axis of symmetry for the orientational distribution function for any chosen molecular axis M_i.</u>

THE SYMMETRY ELEMENTS OF LIQUID CRYSTALS

For a nematic in its ground state that is very straightforward. By every observational test, and therefore we may assume for every statistical property of its internal structure, it is uniaxial, with infinite rotational symmetry about its unique axis, n, and every plane normal to that axis is a mirror plane. So is every plane parallel to that axis. The orientational distribution function for any molecular axis M, must therefore be expressible by a series of even zonal spherical harmonics. However, in the elastic theory we consider non-uniformity of n, and that implies that the symmetry is spoilt. For curvatures uniformly of one kind enough symmetry is preserved for the purpose of the definition. Uniform bend preserves mirror planes normal and parallel to m. Uniform but biaxial splay preserves mirror planes in two orthogonal orientations parallel to n. Uniform twist corresponds to the cholesteric case which I shall consider presently: n remains well defined, but all mirror planes are necessarily destroyed. Hence with mixed curvatures we are left with no symmetry test strictly applicable for identification of n. We can only say, I think, that as we shrink the volume element under consideration it becomes less and less distinguishable from a volume element from a sample with uniform n, see the particularly if the curvatures are not too strong. We have similar subtle difficulties in the elastic theory of solids, where the simpler definitions of strain are only valid in the limit in which strains are small. I think we have to conclude that the elastic theory of nematics is only strictly valid as a small-strain theory. I wish I knew how to improve on that, but I don't.

For the cholesteric in ground state, or the uniformly twisted nematic, the symmetry elements are a continuous screw axis, of pitch p, and a periodic translation of p in the same direction, together with diad axes at intervals of p/4 in every plane parallel to the screw axis, alternately of two kinds, respectively parallel and orthogonal to n, which is everywhere orthogonal to the screw axis. The p translations, or alternatively the diads orthogonal to n, perform the function of ensuring that the sign of n is ambiguous, like the diads, or mirror plane orthogonal to n in the ground state nematic. The diads parallel to n tell us that we are not entitled to expect purely zonal spherical harmonics in the orientational distribution functions for molecular axes, but more general distribution functions of two-fold symmetry about n, requiring even-even tesseral spherical harmonics for their description. This applies particularly for cholesterics of very short pitch.

What about smectics? There is no problem about defining \underline{n} for smectic-A: it is both the normal to the layering, and an axis of infinite rotational symmetry for orientational distribution functions as in a nematic. But smectic-C is quite another case. In the conventional models in which molecules are represented as rods, they are presented as parallel and inclined by an angle θ to the layer-normal, and this θ may be employed as a symmetry-breaking variable parameter in theories of the phase-change smectic-A to smectic-C. Here I believe we are introducing a "direction of the molecules" which has no exact definable meaning. The symmetry elements are the periodic translation $\underline{\ell}$, normal to the layers, two discrete sets of diads orthogonal to $\underline{\ell}$, in a single plane parallel to $\underline{\ell}$, one set centring the layers and the other

set between them, and a mirror plane orthogonal to these diads. For smectic-C*, the cholesteric-related optically active equivalent of smectic-C, this mirror plane disappears: but the diads of both kinds are still present, and now recur with a translation period equal to the pitch of the screw rotation, when this is commensurate with the density-layering. The direction of these diad axes is now the only direction we can have as a symmetry axis for orientational distribution functions. These should be of two-fold non-polar symmetry, comprising even-even spherical harmonics, among which the zonal harmonics are present, but cannot be expected to predominate. The symmetry axis of these orientational distribution functions is now orthogonal to the imagined molecular directions in the model with rod-molecules. Whether it is now useful to call it a director I very much doubt. I think it is better to say that for smectic-C neither the director nor the direction of the molecules is an even approximately useful concept, and quite other descriptive parameters should be sought.

AVERAGE DIRECTIONS

I raised the question of defining an average direction. Clearly, when we have equal numbers of vectors pointing opposite ways, we do not do it by adding the vectors and dividing by their number, since that must give zero. We can define an average direction as that from which the sum of the squares of the deviations, θ , is minimum, with the convention that of the two alternative angles between a pair of directions, we always choose $\theta < \pi/2$. A neater avoidance of the 90° discontinuity, with essentially the same result in simple cases, is obtained by choosing that direction which minimises $\Sigma \sin^2 \theta$, instead of $\Sigma \theta^2$. That is the same as maximising $\Gamma \cos^2 \theta$, and likewise the same as maximising

$$s_2 = \frac{1}{2} < 3 \cos^2 \theta - 1 > ,$$

which is the second zonal spherical harmonic.

Now, quite generally, when we are dealing with orientational distribution functions which have an axis of symmetry, S_2 will either be maximised or minimised on that axis. When it is maximised, meaning that the directions are preferentially parallel to the symmetry axis, the average direction is meaningful, and coincides with the symmetry axis. When it is minimised, the concept "average direction" may have become meaningless, e.g. if the molecular axes concerned are preferentially transverse to the symmetry axis, and S_2 has equal maximal values in every equatorial direction.

ORDER PARAMETERS

S₂, evaluated on the symmetry axis, is often called "the order parameter". It is an error to suppose that it should have the same value if determined, say for the greatest principal molecular axis of electric polarizability or the least principal axis of diamagnetic susceptibility, except when molecular symmetry causes these molecular axes to coincide. I should suppose that for typical asymmetric mesomorphogenic molecules, 10% discrepancies should be unsurprising, and for some of the potentially observable molecular axes much