

J. Van Bladel

# Relativity and Engineering

# Springer Series in Electrophysics

## Volume 15

Edited by Leopold B. Felsen

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# Relativity and Engineering

With 203 Figures

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## Preface

The main feature of this book is the emphasis on "practice". This approach, unusual in the relativistic literature, may be clarified by quoting some problems discussed in the text:

- the analysis of rocket acceleration to relativistic velocities
- the influence of gravitational fields on the accuracy of time measurements
- the operation of optical rotation sensors
- the evaluation of the Doppler spectrum produced by the linear (or rotational) motion of an antenna or scatterer
- the use of the Cerenkov effect in the design of millimeter-wave power generators
- the influence of the motion of a plasma on the transmission of electromagnetic waves through this medium.

A correct solution of these (and analogous) problems requires the use of relativistic principles. This remark remains valid even at low velocities, since first-order terms in  $(v/c)$  often play a fundamental role in the equations.

The "applicational" approach used in the text should be acceptable to *space engineers, nuclear engineers, electrical engineers*, and more generally, *applied physicists*. Electrical engineers, in particular, are concerned with relativity by way of the electrodynamics of moving bodies. This discipline is of decisive importance for *power engineers*, who are confronted with problems such as

- the justification of a forcing function  $(-D\Phi/Dt)$  in the circuit equation of a moving loop
- a correct formulation of Maxwell's equations in rotating coordinate systems
- the resolution of "sliding contact" paradoxes
- a theoretically satisfying analysis of magnetic levitation systems.

The discussion of these—and similar—topics forms the "raison d'être" of the present book. The treatment has been made as complete as possible, in particular as far as the survey of the literature is concerned. Limitation of space, however, forced us to refer several interesting items to the problems, which therefore deserve the reader's full attention. The same concern prompted us to

- omit a certain number of proofs and derivations (e.g., of Einstein's equations in Chap.8)
- ignore topics such as relativistic quantum mechanics
- skim over such fascinating aspects as cosmology, the systematic use of space-time diagrams, and the historical development of relativity.

The text has been written at the first-year graduate student level, and assumes an intermediate mathematical background, as well as a reasonable foundation in electromagnetic theory. The notes have been used by the author in graduate courses taught both at the University of Ghent and, under the auspices of the Brittingham Foundation, in 1974 at the University of Wisconsin. In order to facilitate acceptance by engineers, the text uses the SI system of units, and avoids non-essential notations such as the summation rule, which might confuse some readers.

The author has greatly benefitted from the expert advice generously offered by his colleagues H. Arzeliès, J. Bosquet, A.T. de Hoop, T.J. Higgins, and T. Shiozawa. Many errors were detected and corrected by D. De Zutter and R. De Smedt, while Viveca Van Bladel valiantly endeavored to improve the style of some of the more "literary" passages. Finally, some fifty-five authors quoted in the text kindly corroborated the reports of their efforts. May we thank them all here for their help.

Ghent, July 1984

*J. Van Bladel*

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# 1. Kinematics in Inertial Axes

In the second half of the nineteenth century experiments involving light rays started casting doubts on the validity of the law of addition of velocities. The experiments indicated, for example, that light propagates with the *same* speed with respect to all inertial frames, irrespective of the relative motion of the latter. The need to reexamine the velocity law led the physicists of that era to look critically at the basic tenets of Newtonian physics and, in particular, at the notions of length and time. Einstein succeeded in developing a theoretical structure which could accommodate the experimental evidence. The kinematic aspects of his theory are the subject of the present chapter.

## 1.1 The "Aether" in the Nineteenth Century

In Newton's physics there exists an absolute space: i.e., a set of axes  $K_0$  with respect to which all "true" motions should be measured. In this space a particle left to itself moves in a straight line with constant velocity. Further, this uniform motion (i.e., the *law of inertia*) also holds in all other rigid systems  $K$  which move with uniform velocity with respect to  $K_0$  (the *systems of inertia*). Consider two systems of inertia  $K$  and  $K'$ , moving with relative velocity  $\mathbf{w}$  (Fig.1.1). The relationship between the coordinates in  $K$  and  $K'$  is given, in prerelativistic physics, by the Galilean transformation

$$\begin{aligned} \mathbf{r}' &= \mathbf{r} - \mathbf{w}t, \\ t' &= t. \end{aligned} \tag{1.1}$$

In this transformation time has the same value in all systems of inertia. It is seen, from (1.1), that  $d^2\mathbf{r}/dt^2$  is equal to  $d^2\mathbf{r}'/dt^2$ . Since the mass in Newtonian mechanics is an invariant, the equation of motion

$$\mathbf{f} = m \frac{d^2\mathbf{r}}{dt^2} \tag{1.2}$$

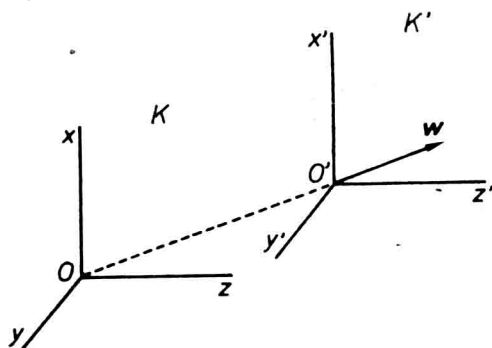


Fig.1.1. Two inertial frames in relative motion

goes over into

$$f' = f = m \frac{d^2 r'}{dt^2} . \quad (1.3)$$

A consequence of (1.1) is the well-known formula for the addition of velocities, viz.

$$v' = v - w . \quad (1.4)$$

Newton's concepts ran into difficulties in the late nineteenth century. It was commonly believed, at the time, that a medium (the *aether*) served as a substratum for the propagation of light, and that it penetrated into bodies like water in a sponge. Some physicists assumed that moving bodies dragged the aether locally (and partially) in their motion. Others believed that the aether was at absolute rest, and that the earth, for example, was swept by an aether "wind" in its motion through interstellar space. According to this view Galilean relativity, as represented by (1.1-3), applied only to classical mechanics, and electromagnetic phenomena had a *preferred* frame of reference  $K_0$ , in which the luminiferous aether was at rest. In consequence light should move with velocity  $c$  with respect to the aether, and its velocity at the surface of the earth should, according to (1.4), have a value different from  $c$ . An impressive series of experiments, some of which are discussed next, has shown the fallacy of this point of view.

## 1.2 Some Experimental Evidence

An experiment which cast particularly strong doubts on Newtonian physics was performed by Michelson and Morley in 1887. These physicists used the inter-

ferometer shown in Fig.1.2, in which  $M_1$  and  $M_2$  are mirrors, and  $L$  a half-silvered plate (a more detailed description of the equipment is found in [1.1]). Let  $v$  be the earth's translational velocity, oriented as shown. According to the aether model, rays 1 and 1' have velocities  $(c-v)$  and  $(c+v)$  with respect to the interferometer. The total "time of flight", from 0 to B and back to 0, is therefore

$$t_1 = \frac{OB}{c-v} + \frac{OB}{c+v} = 2 \frac{OB}{c} \frac{1}{1 - v^2/c^2} . \quad (1.5)$$

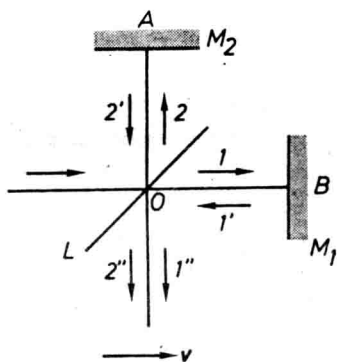


Fig.1.2. Sketch of the Michelson-Morley experiment

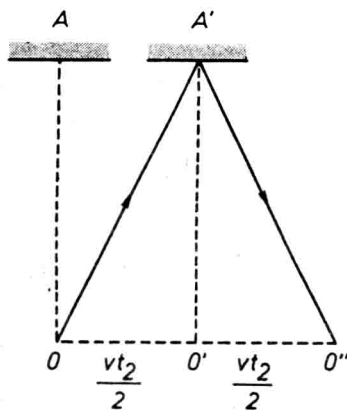


Fig.1.3. Relevant to the estimation of a travel time interval

To determine the time of flight of rays 2 and 2', we must take into account the perpendicular displacement of the interferometer during the time of flight. A look at Fig.1.3 shows that the travel time from 0 to A and back to 0 satisfies

$$t_2 = \frac{OA'}{c} + \frac{A'O''}{c} = \frac{2}{c} \sqrt{(OA')^2 + (vt_2/2)^2} . \quad (1.6)$$

The travel time  $t_2$  is therefore

$$t_2 = \frac{2}{c} \frac{OA}{\sqrt{1 - v^2/c^2}} . \quad (1.7)$$

As the translational velocity of the earth at the location of the experiment is of the order of  $300 \text{ ms}^{-1}$ , the ratio  $v/c$  is exceedingly small, and the difference of time intervals of flight can be written, to a good approximation, as



$$t_1 - t_2 = \frac{2}{c} (OB - OA) + \frac{v^2}{c^3} (2 OB - OA) \quad (1.8)$$

It follows that rays 1" and 2" are "out of step". Their phase difference can easily be displayed through observation of suitable fringes on a screen. Let now the interferometer be rotated through  $90^\circ$ . The roles of beams 1 and 2 are exchanged, and the fringe system should therefore move. Michelson did *not* observe any shift in the pattern. His observations have been checked most carefully, also in recent times [1.2], and his experiment repeated under a range of conditions and with ever increasing precision. The general conclusion holds that, to an accuracy of some  $30 \text{ m s}^{-1}$ , *the velocity of light is isotropic*. This is a most remarkable result if one remembers the countless confirmations of the law of addition of velocities in everyday's life.

Other experiments, such as Fizeau's (discussed in Sect.4.6), added to the puzzlement of the nineteenth-century physicist [1.3]. Further, numerous measurements showed that the Doppler effect depends only on the velocity of the source of light with respect to the *observer*, and not with respect to  $K_0$ . Finally, and this is particularly impressive, routine engineering practice shows that the electromagnetic induction associated with magnets and conductors in uniform motion depends on the *relative* velocity of these components, and not on their state of motion with respect to a hypothetical aether. All in all, the evidence at the time of Einstein's first memoir on relativity was clear: no experiment made at the earth's surface, using only terrestrial instruments, could reveal the *translational* motion of the earth with respect to the aether. A major creative effort was therefore needed to accommodate this "negative" evidence. Einstein's solution succeeded in doing so. Further, the validity of his approach has been supported by an impressive array of "positive" experiments.

### 1.3 Einstein's Relativity Postulates

At the turn of the century several schemes, based e.g. on the assumption of a Lorentz contraction, had been proposed to resolve the "paradoxes" mentioned above. These efforts, which form a fascinating chapter in the history of physics, are well-documented in the specialized literature, and are not described here [1.4]. Einstein's theory, described in a famous paper [1.5], does away with absolute motion and absolute space. Einstein postulates:

- 1) that the laws of electrodynamics and optics have the same form in all inertial frames;