To Eva

Choice, Welfare and Measurement

AMARTYA SEN

Contents

Preface

The papers included in this volume of essays deal with a number of related topics: choice, preference, rationality, welfare judgements, public decisions, social choice and social measurement. The papers are not arranged chronologically, but divided into five broad groups.

I have added a longish 'Introduction' to place the papers in their context. In fact, the 'Introduction' is not only concerned with the papers themselves but also with the related literature. The problems tackled in the different articles relate — in some cases closely — to each other. They also relate in various ways to the contributions of others. Also, many of the papers included here have provoked extensive discussions — in the form of extensions, applications and criticisms — and some of these discussions have clearly been much more valuable than the papers themselves. I have, therefore, gone well beyond the brief of a standard 'Introduction', and taken this opportunity of examining the underlying issues and the related developments.

One of the papers included here (Essay 7: 'Necessary and Sufficient Conditions for Rational Choice under Majority Decision') was written jointly with Prasanta Pattanaik, and I am grateful to him for permitting me to reproduce it here.

The idea of publishing a selection of essays came from René Olivieri of Basil Blackwell. His advice on what to select and how to arrange has been invaluable, and I am most grateful to him for his sagacious counsel.

A.K.S.

Introduction

1 Choice and Preference

1.1 Consistency and revealed preference

Preference may be seen as 'prior' to choice: we may try to choose what we prefer. This is indeed the natural sequence in reflective choice, seen from the first-person point of view. However, from the point of view of the outside observer the opposite sequence may be the natural one: we observe the person's choices and surmise his or her preferences from these choices. There are, of course, cases that run counter to each of these interpretations. There exist situations—or so we are told by people who keep watching themselves carefully—in which a person 'understands' what he or she 'really' prefers by observing his or her own choices (e.g., 'I didn't think I preferred sweet German wines until I noticed that I always choose them at wine parties!'). And, of course, we may learn about a person's preference by means *other than* observing his or her choices (e.g., through conversation), and on that basis we may advise or predict what the person should or would choose.

In all this both 'choice' and 'preference' are taken as 'primitive' concepts with meanings of their own. The correspondence of choice and preference is seen as an empirical matter, and this is indeed how the correspondence is viewed in the classic framework of demand theory.[1] It is, of course, possible to vary that empirical assumption without descending into incoherence. The picture is, however, quite different with the approach of 'revealed preference'.[2] Preference here is simply *defined* as the binary relation underlying consistent choice. In this case 'counter-preferential' choice is not empirically different, but simply impossible. *Non-preferential* choice is, of course, possible, since the choices may lack the consistency needed for identifying a binary relation of preference, but obviously it cannot be the case

1. See, for example, J. R. Hicks, *Value and Capital* (Oxford: Clarendon Press, 1939); H. O. A. Wold, *Demand Analysis* (New York: Wiley, 1963).

2. See P. A. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard University Press, 1947).

that such an identified preference relation exists *and* the choices are 'counter' to it.

The four papers in Part I explore and examine different aspects of the correspondence between choice and preference, and investigate some related issues, *e.g.*, linkages with individual welfare, characterization of normative analysis and conduct, and the behavioural foundations of economic theory. They address some of the problems that arise respectively in the definitional, empirical and normative linking of choice with preference.

The revealed preference approach—in its traditional formulation—has two technical limitations of some importance, and Essay 1 addresses itself to these. First, traditional revealed preference theory is almost exclusively concerned with 'transitive' preference relations. Thus the consistency of choice it demands is of an especially exacting variety which makes choices representable by a *transitive* binary relation. However, there are good grounds for expecting the introspective or observed preference relation to be not fully transitive, and indeed the case for admitting intransitivity of indifference has been forcefully argued in various contexts including demand theory.[3]

'Choice Functions and Revealed Preference' (Essay 1) discriminates between three cases, involving increasing regularity: (1) the choice function (telling us what is chosen from each subset) is representable by a binary preference relation (this is called 'normality', but 'binariness' would be a more direct description[4]), (2) *additionally* that the preference relation is quasi-transitive (transitive *strict* preference, not necessarily transitive indifference), and (3) *additionally* that the preference relation is fully transitive. The consistency requirements of each case are axiomatically identified.[5]

The second limitation concerns the restriction in traditional revealed preference theory that consistency of choice is demanded only over a class of convex polyhedra ('budget triangles' in the two-good case). While this is the form in which actual choices are faced by the consumer in competitive

3. See in particular W. E. Armstrong, 'The Determinateness of the Utility Function', *Economic Journal*, 49 (1939); D Scott and P Suppes, 'Foundational Aspects of Theories of Measurement', *Journal of Symbolic Logic*, 23 (1958), R. D Luce, 'Semiorders and a Theory of Utility Discrimination', *Econometrica*, 24 (1956); N Georgescu-Roegen, 'Threshold in Choice and the Theory of Demand', *Econometrica*, 26 (1958); T. Majumdar, *The Measurement of Utility* (London: Macmillan, 1962); P. C. Fishburn, 'Intransitive Indifference in Preference Theory: A Survey', *Operations Research*, 18 (1970); J. S. Chipman, L Hurwicz, M K. Richter and H. F Sonnenschein, *Preference, Utility and Demand* (New York, Harcourt, 1971).

4. This corresponds to the condition of 'rationalizability' investigated by M. K. Richter, 'Revealed Preference Theory', *Econometrica*, 34 (1966).

5. *Case* (1) does not require the preference relation to be transitive or quasi-transitive, but it does require it to be 'acyclic', i.e., free from strict preference cycles (e.g., x_1 preferred to x_2, , x_{r-1} preferred to x_r, and x_r preferred to x_1), which is a less exacting demand. Indeed, for a reflexive and complete binary relation R, the *necessary and sufficient* condition for it to generate a choice function, with a non-empty choice set for every finite, non-empty set, is the acyclicity of R (see Lemma 1*1 in my *Collective Choice and Social Welfare* (San Francisco: Holden-Day, 1970; reprinted, Amsterdam: North-Holland, 1980), p. 16)).

markets, the form can be quite different in non-competitive market situations,[6] and in choices other than that of pure consumption, e.g., of voters or of government bureaucracies.[7]

There is, in fact, a deeper methodological issue in the use of consistency axioms for choices over budget sets only. In so far as the consistency conditions represent *axioms of the system*, there is no reason why such consistency should not be demanded over all choices that could, in principle, arise—irrespective of whether such choices could be observed in market behaviour. However, in so far as the consistency conditions are taken as *hypotheses* to be tested, the issue of observation is important. But the possibility of actual testing of these consistency conditions of choice in markets is very limited.[8] Aside from other observational difficulties, there is a temporal problem. Over short periods people may seek variety (fish today and steak tomorrow is not inconsistent), but over longer periods tastes can easily change (apparent inconsistencies may then reflect instead a changing choice function). The popularity of an axiom such as the Weak Axiom of Revealed Preference is not really due to any decisive empirical support it has received—the tests have been very limited—but primarily due to its *intuitive* reasonableness as an axiom of choice behaviour. But as an *axiom* of choice behaviour—rather than as a hypothesis under testing—it is not at all clear why it should be assumed to apply only over choices that can, in fact, be observed (in this case, in individual behaviour in markets), rather than generally over all choices that can, in principle, arise.

If the consistency requirement is not exclusively confined to budget-set choices but applied to choices over all subsets,[9] the structure of revealed preference theory changes a great deal from its traditional format.[10] For

6 See T. Majumdar, 'Revealed Preference and the Demand Theorem in a Not-Necessarily Competitive Market', *Quarterly Journal of Economics*, 83 (1969)

7 For a far-reaching probe into the preferences revealed by governmental decisions, see K. Basu, *Revealed Preference of Governments* (Cambridge: Cambridge University Press, 1979).

8 See J. Kornai, *Anti-Equilibrium* (Amsterdam: North-Holland 1971); Essays 2 and 4 reprinted here; F. Hahn and M. Hollis (eds), *Philosophy and Economic Theory* (Oxford: Oxford University Press, 1979).

9. In fact, for the formal results presented in Essay 1, and many related ones, it is sufficient that the domain of the choice function includes all finite subsets (in most cases it is sufficient to include all pairs and triples), irrespective of whether other subsets are also included.

10. Non-budget-set choice functions were studied by Kenneth Arrow, 'Rational Choice Functions and Orderings', *Economica*, 26 (1959). Essay 1 extends Arrow's investigation and goes into motivational justification and also into factorization. See also H. S. Houthakker, 'On the Logic of Preference and Choice', in A. Tymieniecka (ed.), *Contributions to Logic and Methodology in Honor of J. M Bochenski* (Amsterdam: North-Holland, 1956); S. Afriat, 'Principles of Choice and Preference', Research Paper No. 160, Department of Economics, Purdue University, 1967; B. Hansson, 'Choice Structures and Preference Relations', *Synthese*, 18 (1968); P. K. Pattanaik, 'A Note on Democratic Decisions and the Existence of Choice Sets', *Review of Economic Studies*, 35 (1968), Essay 6 ('Quasi-transitivity, Rational Choice and Collective Decisions') reprinted in this volume; and my *Collective Choice and Social Welfare* (1970), chapter 1*.

example, the weak axiom of revealed preference can be shown to be *equivalent* to the strong axiom, and quite sufficient for transitivity of the generated preference relation, and in Essay 1 reproduced in this volume, a great many distinct conditions are shown to be exactly equivalent. The requirements of weaker regularity conditions of preference—in particular acyclicity and quasi-transitivity—are also similarly analysed.[11] The exercise is further pursued in Section 4 of Essay 8.

In the context of general choice functions, it is useful to factorize the requirements for the various regularity conditions of preference into constituent parts, and Essay 1 presents such factorizations.[12] One important distinction that emerges in Essay 1 is that between conditions—such as Property α—that insist on 'contraction consistency' (in the sense that they require that a chosen alternative must continue to be chosen as the 'menu' from which the choice is to be made is—in some particular way—contracted), and conditions—such as β and γ—that insist on 'expansion consistency' (in the sense that they require that a chosen alternative must continue to be chosen as the menu from which the choice is to be made is—in some particular way—expanded). In fact, in the context of Arrow-type impossibility theorems, this distinction proves to be crucial (as is shown in Essay 8, 'Social Choice Theory: A Re-examination', also reprinted here). Indeed, Arrow's impossibility result and related ones can be shown to be thoroughly dependent on contraction consistency and essentially independent of expansion consistency (see also Section 2.2 below).

Another distinction explored in Essay 1 that proves to be rather central to social-choice impossibility results is that between Samuelson's 'revealed preference' relation R (with $x\ R\ y$ if and only if x is chosen when y is available) and the pair-choice relation $\bar{R}$, sometimes called the 'base relation'[13] (with $x\ \bar{R}\ y$ if and only if x is chosen from the pair x, y). It emerges that the regularity properties of the base relation are immediately relevant to the impossibility results of the Arrow type, whereas those required of the revealed preference relation can in one sense be satisfied without violating Arrow's conditions and related ones (see Essay 8 reprinted here, and also Section 2.2 of this Introduction).

In these various ways Essay 1 is concerned with some of the most elementary

11. In recent years these and other regularity properties have come to be extensively investigated in the context of general choice functions by E. Bergstrom, J. H Blau, D. Blair and R. Pollak, G. Bordes, R. Deb, P. C. Fishburn, H. Herzberger, D. T. Jamison and L. J. Lau, S. Kanger, J. S. Kelly, D. Kelsey, A Mukherji, R. R. Parks, C. R. Plott, T. Schwartz, T. E. Smith, K. Suzumura, M Walker, R. Wilson, among others.

12. In recent years these and other types of factorization have been extensively investigated by D. Blair, G. Bordes, P. C. Fishburn, S. Fuchs-Seliger, H. Herzberger, J. S. Kelly, Y. Matsumoto, R. R. Parks, C. R. Plott, J. Richelson, T. Schwartz, M. Sertel and A. van der Bellen, M. Sjöberg, K. Suzumura, among others.

13. See H. G. Herzberger, 'Ordinal Preference and Rational Choice', *Econometrica*, 41 (1973).

tary issues of choice theory. However, it does not view preference as anything other than what transpires from choice, and it belongs to the tradition of seeing preference as tied completely to choice, dispensing with the need for an empirical or normative (and not just logical) investigation of the relation between choice and preference. The other three essays in Part I are concerned with different aspects of those substantive questions.

It is, however, worth mentioning that the technical results in Essay 1—and in contributions by others in a similar framework—are not all lost even when preference is given an existence of its own as an introspective concept. The results then need to be interpreted *either* (1) as dealing exclusively with the *binary relation of choice* which does not necessarily coincide with the introspective notion of 'preference', *or* (2) as dealing with the introspective notion of preference under empirical or normative assumptions guaranteeing 'preferential' choice. The former is, of course, quite the natural format for *institutional* social choice theory, and in that context no introspective social preference need be invoked. There is no obvious inadequacy there. The latter approach, on the other hand, fits in well with the traditional assumptions regarding personal behaviour, especially in economic theory. However, Essays 2, 3 and 4 question the sagacity of that common tradition, and in the next section the latter approach is discussed in the context of personal choice.

1.2 *Beyond consistency*

While consistency is taken in economic theory to be a necessary condition of rationality, it is usual to supplement that requirement by some substantive view as to what the individual would maximize. The regularity of consistent pursuit of self-interest is a frequently used assumption of rational behaviour. Though only a few authors have discussed this assumption explicitly (Edgeworth is one who did),[14] it is implicitly present in much of traditional economic theory. For example, in general equilibrium theory,[15] in establishing the correspondence of equilibria with Pareto optimality, 'preference' plays the dual role of determining individual decisions (it coincides with revealed preference in this role) and serving as the basis of Pareto optimality judgements (it reflects individual welfare in this role). Together this amounts to assuming that individual choices are guided exclusively by the requirements of maximizing the respective individual welfares.

That assumption may not be particularly unrealistic in some types of choices, but there is little evidence that all choices in *economic* matters fall in

14. F. Y. Edgeworth, *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (London, 1881).

15. See, for example, G. Debreu, *Theory of Value* (New York: Wiley, 1959); K J. Arrow and F. H. Hahn, *General Competitive Analysis* (San Francisco: Holden-Day, 1971; reprinted Amsterdam: North-Holland, 1980).

that category. Decisions regarding work ethics, job choice, where to live, whether to strike, etc., might well be partly influenced by values other than maximization of perceived individual welfare, Essays 2, 3 and 4 go into this question from various perspectives.

There is no point in repeating in this Introduction the arguments that are presented in the reprinted essays, but some general remarks might be useful.

First, it is important to distinguish between the assumption of individual-welfare maximization as a *rationality* condition and that as an *empirical* assumption as to how people actually do behave. Both the uses can be found in traditional economic theory. While both may be—and indeed are—questionable, they are not questionable on the same grounds. Essay 2 ('Behaviour and the Concept of Preference') is mainly concerned with the empirical question, whereas Essay 4 ('Rational Fools') goes into both the questions.[16]

Second, the interdependence between different people's welfare may make the pursuit of individual interests produce inferior results for all, *in terms of those very interests*. This problem, which is nowadays illustrated with the 'Prisoner's Dilemma' game, had been clearly perceived much earlier, and played an important part in—say—Hobbes's and Rousseau's treatment of the state.[17] Since the problem is quite central to many economic

16. On related issues see my *Collective Choice and Social Welfare* (1970), chapter 1; T Nagel, *The Possibility of Altruism* (Oxford: Clarendon Press, 1970), F. Hirsch, *Social Limits to Growth* (Cambridge, Mass.: Harvard University Press, 1976); H. Leibenstein, *Beyond Economic Man: A New Foundation for Microeconomics* (Cambridge, Mass.: Harvard University Press, 1976); T. Scitovsky, *The Joyless Economy* (Oxford: Oxford University Press, 1976); J. Elster, *Ulysses and the Sirens* (Cambridge: Cambridge University Press, 1979); A. O. Hirschman, *Shifting Involvements* (Princeton Princeton University Press, 1982); H. Margolis, *Selfishness, Altruism, and Rationality* (Cambridge. Cambridge University Press, 1982).

17. On this see W. J. Baumol, *Welfare Economics and the Theory of the State* (Cambridge, Mass.: Harvard University Press, 1955), and W. G. Runciman and A. K. Sen, 'Games, Justice and the General Will', *Mind*, 74 (1965). The 'prisoner's dilemma' was presented by R. D Luce and H. Raiffa, *Games and Decisions* (New York: Wiley, 1958) On related issues see A. K. Sen, 'On Optimizing the Rate of Saving', *Economic Journal*, 71 (1961); S.A. Marglin, 'The Social Rate of Discount and the Optimal Rate of Investment', *Quarterly Journal of Economics*, 77 (1963); M. Olson, *The Logic of Collective Action* (Cambridge, Mass. Harvard University Press, 1965); A. Rapoport and A. M. Chammah, *Prisoner's Dilemma. A Study in Conflict and Cooperation* (Ann Arbor: University of Michigan Press, 1965), A. K. Sen, 'Isolation, Assurance and the Social Rate of Discount', *Quarterly Journal of Economics*, 81 (1967); J. W. N. Watkins, 'Imperfect Rationality', in R. Borger and F. Cioffi (eds), *Explanation in the Behavioural Sciences* (Cambridge: Cambridge University Press, 1970); N Howard, *Paradoxes of Rationality* (Cambridge, Mass.: MIT Press, 1971); P. J. Hammond, 'Charity: Altruism or Cooperative Egoism?', in E. S. Phelps (ed.), *Altruism, Morality and Economic Theory* (New York: Russell Sage, 1975); K. Basu, 'Information and Strategy in Iterated Prisoner's Dilemma', *Theory and Decision*, 8 (1977); M. Black, 'The "Prisoner's Dilemma" and the Limits of Rationality', *International Studies in Philosophy*, 10 (1978); L. Levi, *The Enterprise of Knowledge* (Cambridge, Mass.: MIT Press, 1980); D. H. Regan, *Utilitarianism and Cooperation* (Oxford: Clarendon Press, ... 'Dilemma', *Proceedings of the British*

issues, it is important to consider the use of norms in economic behaviour (see Essays 2 and 3). Indeed, every member of the group might be better off with a norm involving systematic deviation from individual-welfare maximization, and Essay 3 ('Choice, Orderings and Morality') provides a format for analysing and thinking about such interdependence-oriented norms. The concept of meta-rankings (ranking of rankings) is introduced in this context.[18]

Third, while the Chinese attempt at doing away with incentives on an extremely broad front has been abandoned as a failure, the issues involved in that debate—discussed in Essay 3—remain important. Also, as argued in Essay 4, norm-based behaviour is both useful and—to a varying extent—actually used in many different spheres of economic activity—influencing the functioning of cooperatives, unions, business firms, and other organizations, both in socialist and capitalist economies.[19] Variability of such norms is, in fact, often invoked to explain international differences of work behaviour and productivity (e.g., in contrasting Japan and Britain), and while this is typically done in an *ad hoc* way, the underlying issue is a very general one, and requires more systematic treatment.

Fourth, the contrast between 'sympathy' and 'commitment' introduced in Essay 4 ('Rational Fools') is relevant in seeing how others figure in one's actions. Sympathy—including antipathy when it is negative—refers to one

18 For discussion, criticism, application and extension of the approach of meta-rankings, see K. Baier, 'Rationality and Morality', *Erkenntnis*, 11 (1977); A. K. Sen, 'Rationality and Morality A Reply', *Erkenntnis*, 11 (1977), A. K. Sen, 'Informational Analysis of Moral Principles', in R Harrison (ed.), *Rational Action* (Cambridge. Cambridge University Press, 1979), R E Goodin, 'Censored Utility Functions', Workshop on the General Will and Common Good, ECPR, Brussels, 1979 (Essex University), M. Hollis, 'Rational Man and Social Science', in R. Harrison (ed.), *Rational Action* (Cambridge. Cambridge University Press, 1979), M S McPherson, 'Mills Moral Theory and the Problem of Preference Change', *Ethics*, 92 (1982), and 'Want Formation, Morality and the Interpretative Dimension of Economic Inquiry', Research Paper RP–33, Williams College, 1979, G. A Gigliotti, 'Values, Tastes and Rights Respecting', Discussion Paper 80–20, and 'Higher Pleasures, Values and Tastes', Discussion Paper 80–21, Bureau of Economic Research, Rutgers University, 1980, T. Mayumdar, 'The Rationality of Changing Choice', *Analyse und Kritik*, 2 (1980), P K Pattanaik, 'A Note on the "Rationality of Becoming" and Revealed Preference', *Analyse und Kritik*, 2 (1980); G C Winston, 'Addiction and Backsliding. A Theory of Compulsive Consumption', *Journal of Economic Behaviour and Organization*, 1 (1980); M Hollis, 'Economic Man and the Original Sin', *Political Studies*, 29 (1981); L. Putterman, 'Incentives and the Kibbutz: Towards an Economics of Communal Work Motivation', Working Paper 81-24, Brown University, 1981; N. Bagent, 'Social Choice Correspondences', *Recherches Economiques de Louvain*, 46 (1980), and 'Rational Choice and the Taxation of Sin', *Journal of Public Economics*, 16 (1981); R. J. van der Veen, 'Meta-rankings and Collective Optimality', *Social Science Information*, 20 (1981); Hirschman, *Shifting Involvements* (1982), chapter 4; Margolis, *Selfishness, Altruism, and Rationality* (1982).

19 See, among others, A. K. Sen, 'Labour Allocation in a Cooperative Enterprise', *Review of Economic Studies*, 33 (1966), J. Vanek, *The General Theory of Labour-Managed Market Economies* (Ithaca, NY: Cornell University Press, 1970); B. Ward, 'Organization and Comparative Economics', in A Eckstein (ed.), *Comparison of Economic Systems* (Berkeley, Calif University of California Press, 1971), N E. Cameron, 'Incentives and Labour Supply in

person's welfare being affected by the position of others (e.g., feeling depressed at the sight of misery). Commitment, on the other hand, is concerned with breaking the tight link between individual welfare (with or without sympathy) and the choice of action (e.g., acting to help remove some misery even though one personally does not suffer from it). Sympathy alone does not require any departure from individual-welfare maximization: but commitment does involve rejection of that assumption.

Fifth, while 'commitment' may relate to the working of some universalized morality, it need not necessarily be so broad-based. Indeed, a sense of commitment to one's community, race, class, fellow-workers, fellow-oligopolists, etc., could be important in the choice of actions. Such relations already do, in fact, figure—typically in a rather *ad hoc* way—in various branches of economic theory.

Sixth, there are several related but different statements about a person's interests, actions, etc., that need to be distinguished, even though they are often identified in the literature:

(1) the person gets more satisfaction in state x than in state y (statement about satisfaction or pleasure);

(2) the person thinks that he or she is better off with x than with y (statement about introspective welfare);

(3) the person is better off with x than with y (statement about individual welfare which may or may not be introspective);

(4) the person prefers that x rather than y occurs (statement about the mental condition of preference, or desire, regarding states);

(5) the person would like to so choose that x rather than y occurs (statement about desired choice);

(6) the person believes that it would be right to so choose that x rather than y occurs (statement about normative judgement regarding choice);

(7) the person believes that it would be better if x were to occur rather than y (statement about normative judgement regarding states of affairs);

(8) the person so chooses that x rather than y occurs (statement about actual choice).

None of these statements logically entails any of the others, and it is a matter for substantive empirical or normative analysis to check how in particular cases any two of these statements link with each other.[20] The thoroughly methodical person who chooses with impeccable consistency but does not distinguish between different issues (such as those outlined above), has been characterized as the 'rational fool' (Essay 4). In one form or another, the rational fool is invoked a great deal in economic theory.

Seventh, one reason for the tendency in economics to concentrate only on the 'revealed preference' relation is a methodological suspicion regarding introspective concepts. Choice is seen as solid information, whereas introspection is not open to observation. This narrowly behaviourist view is critically scrutinized in Essay 2 ('Behaviour and the Concept of Preference'). Even as behaviourism this is peculiarly limited since *verbal* behaviour (or *writing* behaviour, including response to questionnaires) should not lie outside the scope of the behaviourist approach. Much of economic theory seems to be concerned with strong, silent men who never speak! One has to sneak in behind them to see what they are doing in the market, etc., and deduce from it what they prefer, what makes them better off, what they think is right, and so on. There is, of course, the problem of ascertaining the veracity of communication, e.g., in responses to questionnaires, but the difficulties of *strategic non-verbal* choice behaviour (departing from preference) are serious too.[21] It is argued in Essay 2 that in economic theory 'we have been prone, on the one hand, to overstate the difficulties of introspection and

Cooperative Enterprises', *Canadian Journal of Economics*, 6 (1973); A. K. Sen, *On Economic Inequality* (Oxford: Clarendon Press, 1973), chapter 4; J. E. Meade, 'Preference Orderings and Economic Policy', in A. Mitra (ed.), *Economic Theory and Planning: Essays in Honour of A. K. Dasgupta* (London: Oxford University Press, 1974); C. Riskin, 'Incentive Systems and Work Motivations: The Experience of China', *Working Papers for a New Society*, 1 (1974), E. S. Phelps (ed.), *Altruism, Morality and Economic Theory* (New York: Russel Sage, 1975); J. M. Montias, *The Structure of Economic Systems* (New Haven: Yale University Press, 1976); T. Wilson and A. S. Skinner (eds), *The Market and the State* (Oxford: Clarendon Press, 1976); M. D. Berman, 'Short-run Efficiency in the Labor-Managed Firm', *Journal of Comparative Economics*, 1 (1977); J. P. Bonin, 'Work Incentives and Uncertainty on a Collective Farm', *Journal of Comparative Economics*, 1 (1977); D. L. Chinn, 'Team Cohesion and Collective Labour Supply in Chinese Agriculture', *Journal of Comparative Economics*, 3 (1979); L. D. Israelson, 'Collectives, Communes, and Incentives', *Journal of Comparative Economics*, 4 (1980); L. Putterman, 'Voluntary Collectivization: A Model of Producers' Institutional Choice', *Journal of Comparative Economics*, 4 (1980); L. Putterman, 'On Optimality in Collective Institutional Choice', *Journal of Comparative Economics*, 5 (1981); T. Ishikawa, 'The Emulation Effect as a Determinant of Work Motivation', mimeographed, University of Tokyo, 1981; R.C.O. Matthews, 'Morality, Competition and Efficiency', *Manchester School*, 49 (1981). See also Michio Morishima's recent study, *Why has Japan 'Succeeded'? Western Technology and Japanese Ethos* (Cambridge: Cambridge University Press, 1982).

20. In addition to Essays 2 and 4, see also my 'Plural Utility', *Proceedings of the Aristotelian Society*, 81 (1980–81), and 'Rights and Agency', *Philosophy and Public Affairs*, 11 (1982).

21. The possibility (and under some assumptions, the ubiquity) of strategic distortions—originally discussed by Arrow, Dummet, Farquharson, Hurwicz, Samuelson and Vickrey—has recently been probed at great depth in the context of various types of choices, e.g., voting (the Gibbard–Satterthwaite theorem), resource allocation (Hurwicz's impossibility result), and there is an extensive literature in this area. I have discussed some of the more general issues in my 'Strategies and Revelation: Informational Constraints in Public Decisions', in J. J. Laffont (ed.), *Aggregation and Revelation of Preferences* (Amsterdam: North-Holland, 1979). Good technical accounts can be found in P. K. Pattanaik, *Strategy and Group Choice* (Amsterdam: North-Holland, 1978); H. Moulin, *The Strategy of Social Choice* (Amsterdam: North-Holland, forthcoming); B. Peleg, *Some Theoretic Analysis of Voting in Committees* (Cambridge: Cambridge University Press, forthcoming); J. J. Laffont and E. Maskin, 'The Theory of Incentives: An Overview', mimeographed, Université des Sciences Sociales de Toulouse, 1981.

communication, and on the other, to underestimate the problems of studying preferences revealed by observed behaviour'. Finally, even if it were the case that market choice provides the only solid basis of information, it would still be illegitimate to equate fundamentally different questions just on grounds that we have information on one but not on the others. Happily, silent choice is not the only source of information, and Essays 2 and 4 discuss the possibility of expanding the informational base for studying preference, welfare and norms.

2 Preference Aggregation

2.1 Restricted preferences

Modern welfare economics has been deeply influenced by Kenneth Arrow's 'general possibility theorem', showing the impossibility of aggregating individual preference orderings into a social order satisfying certain conditions of reasonableness.[22] The four essays in Part II deal with that and related aggregation problems. Arrow defines a social welfare function as a functional relation which specifies one social ordering R for any set (in fact, n-tuple) of individual preference orderings—one per person (with n people in the society): $R = f(R_1, \ldots, R_n)$. Arrow's four conditions demand, respectively that: (1) the domain of the function should include any conceivable n-tuple of individual preference orderings (unrestricted domain); (2) if everyone prefers any x to any y, then that x is socially preferred to that y (weak Pareto principle); (3) no individual is a dictator in the sense that whenever he prefers any x to any y, it must be the case that x is socially preferred to y (non-dictatorship); and (4) the social ranking of any pair (x, y) depends on individual rankings of that pair only (independence of irrelevant alternatives).[23] An intuitively explained proof of Arrow's theorem can be found in a later essay in this volume, viz., Essay 15 ('Personal Utilities and Public Judgements: Or What's Wrong with Welfare Economics?'), pp. 331–4, which discusses that proof in the context of pursuing a critique of the informational basis of traditional welfare economics—an issue that will be taken up later in this Introduction.

One possible solution, which Arrow himself has explored, is that of dropping the condition of unrestricted domain. The method of majority decision clearly satisfies the three conditions other than unrestricted domain,

22. K. J. Arrow, *Social Choice and Individual Values* (New York: Wiley, 1951; 2nd edition, 1963).

23. This is the second version of Arrow's 'impossibility theorem', presented in the second edition of his book. The independence condition is defined here in purely relational terms (like Arrow's other conditions), though Arrow himself used a choice-functional form, which happens to be exactly equivalent, in his framework, to the above relational statement.

but yields intransitive social preferences for some n-tuples of individual preference orderings. Duncan Black and Kenneth Arrow showed that if the individual preference combinations are 'single-peaked', then the majority relation must be transitive, if the number of individuals happens to be odd.[24] Single-peakedness is a domain restriction that permits only those individual preference combinations such that the alternatives can be arranged in a way that would make everyone's utility curve (preference intensity) have one peak only. Other sufficient conditions for transitive majority rule were identified by Inada, Vickrey and Ward.

Essay 5 offered a generalization of all these conditions in the form of 'value restricted' preferences. There is no particular point in discussing in this Introduction the exact content of value restriction, or how it relates respectively to the other conditions of which it is a generalization, since all this is discussed extensively in Essay 5.[25]

However, three limitations of the result in Essay 5 are worth noting. First, the result concerns full transitivity of strict preference whereas regularity properties such as quasi-transitivity or acyclicity are adequate for there being a majority *winner* in every finite subset (acyclicity is, in fact, exactly necessary and sufficient for that). Transitivity is not the only interesting issue, and indeed in the context of choice it is in an obvious sense a much less interesting issue than acyclicity of the majority preference relation. Second, the peculiar—almost eerie—requirement in the conditions proposed by Arrow, Inada, Vickrey and Ward that the number of individuals (more strictly the number of non-indifferent individuals) be odd is also present in the generalization offered in Essay 5, and this makes this whole route or solution rather *ad hoc*. Third, value restriction is only a sufficient condition, not also a necessary one. Other sufficiency conditions can be found.

24. R. D. Black, *The Theory of Committees and Elections* (Cambridge: Cambridge University Press, 1958); Arrow, *Social Choice and Individual Values* (1951), chapter VII.

25. There have been many further contributions on related lines. Aside from Essay 6 in this volume published in 1969, see K. Inada, 'On the Simple Majority Decision Rule', *Econometrica*, 37 (1969); K. Inada, 'Majority Rule and Rationality', *Journal of Economic Theory*, 2 (1970); P. K. Pattanaik, *Voting and Collective Choice* (Cambridge: Cambridge University Press, 1971); P. C. Fishburn, *The Theory of Social Choice* (Princeton: Princeton University Press, 1973); R. Saposnik, 'On Transitivity of the Social Preference Relation under simple Majority Rule', *Journal of Economic Theory*, 10 (1975); M. Salles, 'A General Possibility Theorem for Group Decision Rules with Pareto Transitivity', *Journal of Economic Theory*, 11 (1975); C. R. Plott, 'Axiomatic Social Choice Theory: An Overview and Interpretation', *American Journal of Political Science*, 20 (1976); J. S. Kelly, *Arrow Impossibility Theorems* (New York: Academic Press, 1978); J. M. Grandmont, 'Intermediate Preferences and Majority Rule', *Econometrica*, 46 (1978); W. Gaertner and A. Heinecke, 'On Two Sufficient Conditions for Transitivity of the Social Preference Relation', *Nationalökonomie*, 37 (1978); S. Slutsky, 'A Characterisation of Societies with Consistent Majority Decision', *Review of Economic Studies*, 44 (1977). There have been many other interesting contributions as well.

The first two problems are dealt with in Section 3 of Essay 6 ('Quasi-transitivity, Rational Choice and Collective Decisions') where it is shown that value restriction is sufficient for *acyclic* majority relations, i.e., for the existence of majority winners, *irrespective* of whether the number of individuals is odd or even.

The third limitation is removed in Essay 7 ('Necessary and Sufficient Conditions for Rational Choice under Majority Decision'), written jointly with Prasanta Pattanaik. For this class of restrictions, necessary and sufficient conditions for acyclicity of the majority relation are identified (this turns out to be satisfying either 'value restriction', or at least one of two other identified conditions, viz., 'extremal restriction' and 'limited agreement'). Necessary and sufficient conditions are found also for the special case of 'strict' (i.e., antisymmetric) individual preferences (value restriction is exactly the necessary and sufficient condition in this case). The necessary and sufficient condition for fully transitive majority preference is also identified (extremal restriction in this case).²⁶ Essay 7 also shows that the adequacy of value restriction and limited agreement extend well beyond the majority rule, and they apply to whole classes of rules with certain general characteristics.

It is clear that these results can be interpreted as being 'comforting' typically only in those choice situations in which the set of alternatives is rather limited, e.g., choosing between a few candidates in an election, or deciding in an assembly between some alternative proposals. In the economic problems of allocation and distribution involving a rich commodity space, there is little chance that the required conditions will be fulfilled. Indeed, it is easily checked that even for the elementary problem of the distribution of a given cake between three or more persons (with each preferring more cake for himself), the majority preference relation will be intransitive, and furthermore will violate the milder requirement of acyclicity. (It follows, of course, immediately that these far-from-pathological preferences will violate the identified restrictions since these restrictions are—by virtue of the *sufficiency* parts of the theorems—adequate for guaranteeing acyclicity of the

26. This exercise of finding necessary and sufficient domain restrictions for transitivity of social preference can be extended from the case of the majority relation to that of any rule satisfying Arrow's Pareto principle, non-dictatorship and independence. See E. Kalai and E. Muller, 'Characterization Functions and Nonmanipulable Voting Procedures', *Journal of Economic Theory*, 16 (1977); E. Kalai and Z. Ritz, 'Characterization of Private Alternative Domains Admitting Arrow Social Welfare Functions', *Journal of Economic Theory*, 22 (1980); E. Maskin, 'Social Welfare Functions on Restricted Domain', *Review of Economic Studies*, forthcoming; G. Chichilnisky and G. Heal, 'Necessary and Sufficient Conditions for a Resolution of the Social Choice Paradox', mimeographed, University of Essex, 1981.

majority relation.²⁷) As a social welfare function to be used in welfare economics, the method of majority decision offers very little. Indeed, the *necessity* parts of the theorems help to show the severity of the required restrictions and their typical unsatisfiability in welfare economic problems. While the domain conditions such as value restriction require only 'a comparatively limited measure of agreement' (as it is put in Essay 5), that agreement is much too ambitious for welfare economics, and offers scope for optimism only in some other contexts, e.g., in some political decision problems involving aggregation of individual judgements concerning the relative merits of a few alternative candidates (or proposals). Essay 7 serves to bring out the demanding preconditions of consistent majority decisions through identifying the *necessity* requirements.

The failure of majority rule to deal consistently with welfare economic problems is not really a cause for mourning. As argued in Essay 8 ('Social Choice Theory: A Re-examination') the method of majority decision is a most peculiar way of dealing with conflicts of interest. Even in ranking just one pair of alternative social states, in which context the problem of intransitivity or cyclicity does not arise, majority rule is a terribly gross method. To illustrate, in the cake division problem, with any given division of the cake, take away half the share of the worst-off person and divide the loot among the rest. We have just made a majority 'improvement'. If we are ambitious and want *more* social improvement, we repeat the exercise! The majority rule cannot really serve as the basis of welfare economic judgements dealing with interest conflicts, and this can be seen even without considering the question of consistency at all. (The basic issue here, which relates to the 'informational basis' of majority rule, and which applies to all Arrovian social welfare rules, is considered more generally in Section 3.1.)

The method of majority decision, then, just isn't a plausible social welfare function for welfare economics, and it wouldn't have been of much interest to welfare economics even if it were consistent. Types of aggregation problems are distinguished in Essay 8, and among the distinctions considered is the one between aggregating individual *interests* (as in, say, the cake-division problem) and aggregating individual *judgements*, e.g., regarding public welfare or institutional policy (as in, say, committee decisions or

27. G. H. Kramer shows explicitly—rather than by implication—that 'the Sen–Pattanaik conditions' will be violated over a wide class of welfare economic problems ('On a Class of Equilibrium Conditions for Majority Rule', *Econometrica*, 41 (1973)). There have been a great many contributions on related themes involving majority rule over commodity spaces representing allocation and distribution possibilities. Similar grounds for 'pessimism' hold also for the corresponding conditions for *any* Arrovian rule, e.g., the conditions identified by Maskin, Kalai and Muller, Kalai and Ritz, and Chichilnisky and Heal.

political elections). The gross method of aggregation used by majority rule—it is argued in Essay 8—is less plausible for the former than possibly for the latter. It is, therefore, of some interest that the preference restrictions shown to be necessary and sufficient for consistent majority decision in Essay 6 are also less plausible for the former than for the latter.

2.2 Social intransitivity and non-binariness

Essay 6 ('Quasi-transitivity, Rational Choice and Collective Decisions') explores a different way of avoiding Arrow's impossibility problem. Rather than restricting the domain of the social welfare function, it is possible to expand 'the range' of it, by dropping the insistence that social preference be fully transitive. Since full transitivity is unnecessary for preference-based choice, the weaker requirement of acyclicity being adequate for it (discussed already in Section 1.1, 'Consistency and revealed preference'), it is not entirely unreasonable in the context of social choice to demand that Arrow's requirement of transitivity of social preference be replaced by the milder condition of acyclicity.[28] This relaxation leads to a more general type of function, called a 'social decision function' (SDF), of which a social welfare function is a special case.[29] It is demonstrated in Essay 6 that the four conditions proposed by Arrow can all be satisfied for a social decision function, since all the Arrow conditions can be met even with *quasi-transitive* social preference, dropping only the insistence on transitivity of indifference.

This was seen in Essay 6 as a technical result of some interest—suggesting the need for further investigation—but very far from a 'resolution' of the Arrow problem. 'Two notes of caution' were emphasized 'lest we jubilate too much at the disappearance of Arrow's impossibility result for social decision functions.' The first of the two notes of caution pointed out that the

28. See A. K. Sen, 'Planner's Preferences: Optimality, Distribution and Social Welfare', presented at the International Economic Association Roundtable Conference at Biarritz in 1966, published in J. Margolis and H. Gutton (eds), *Public Economics* (London: Macmillan, 1969), and P. K. Pattanaik, 'A Note on Democratic Decisions and the Existence of Choice Sets', *Review of Economic Studies*, 35 (1968).

29. There is a slight ambiguity here. In fact, in Essay 6 a 'social decision function' was characterized as a rule 'the range of which is restricted to only those binary relations *R* each of which generates a choice function over the entire *X* [the set of social states]' (p. 125). It is only with the further assumption that *X* is finite that this becomes equivalent to the definition used here requiring the generated binary relation *R* to be reflexive, complete and acyclic. The difference is not, in fact, very important in the present context since the relevant arguments do not deal with infinite sets. Indeed, while the concept of a 'social decision function' was first introduced in Essay 6, the more common definition of it by now is the variant used here (see, for example, J. H. Blau and R. Deb, 'Social Decision Functions and the Veto', *Econometrica*, 45 (1977)).

four Arrow conditions were proposed by him as being 'necessary for a reasonable social choice mechanism; he did not claim this set to be *sufficient* for it' (Essay 6, p. 128). It was pointed out that the example in terms of which the theorem asserting the satisfiability of the Arrow conditions for a social decision function is proved was, in fact, 'unattractive to most of us', and other conditions must be imposed for a reasonable social choice procedure. The actual example involved everyone having a 'veto' in the sense that if anyone strictly preferred any x to any y that would make x to be socially at least as good as y. Allan Gibbard soon proved that for all social decision functions satisfying Arrow's four conditions and yielding quasi-transitive social preference at least one person must have a veto,[30] and thus the noted 'unattractiveness' is, in fact, inescapable. A non-veto condition—in the same spirit as Arrow's non-dictatorship requirement (but stronger)—would re-establish the impossibility.

That veto result re-occurs in subsequent contributions even without quasi-transitivity but with additional conditions (such as 'positive responsiveness') imposed on social decision functions.[31] There was indeed rather limited reason for jubilation at the technical disappearance of Arrow's impossibility. However, the precise role of transitivity and other regularity conditions of social preference in generating Arrow-type impossibility results got fairly thoroughly investigated as a consequence of this line of inquiry, and appropriate axiomatic derivations of various types of *special power* were developed in this context.[32]

The second 'note of caution' introduced in Essay 6 points to the possibility that 'we might wish to impose certain rationality conditions on the choice

30. 'Intransitive Social Indifference and the Arrow Dilemma', unpublished manuscript, 1969, reported in my *Collective Choice and Social Welfare* (1970), pp 49–50. In fact, Gibbard proved the existence of an 'oligarchy', with every member of the oligarchy having a veto and all members of the oligarchy together being decisive. The example used in the proof of the possibility of a social decision function satisfying all of Arrow's conditions (theorem V in Essay 7) corresponds to an 'oligarchy' of all individuals. Ashok Guha established the existence of a hierarchy of oligarchies, on the basis of slightly stronger assumptions; see A. Guha, 'Neutrality, Monotonicity and the Right of Veto', *Econometrica*, 40 (1972), and J. H. Blau, 'Neutrality, Monotonicity and the Right of Veto: A Comment', *Econometrica*, 44 (1976).

31. See A. Mas-Colell and H. Sonnenschein, 'General Possibility Theorem for Group Decisions', *Review of Economic Studies*, 39 (1972); D. J. Brown, 'Acyclic Aggregation over a Finite Set of Alternatives', *Cowles Foundation Discussion Paper No. 391*, 1975; B Hansson, 'The Existence of Group Preferences', *Public Choice*, 28 (1976); J. H. Blau and R. Deb, 'Social Decision Functions and the Veto', *Econometrica*, 45 (1977)

32. In addition to the oligarchy and veto theorems referred to above, important results have been established recently about partial vetoes and group vetoes on the basis of acyclity itself without the additional assumptions needed for the existence of an individual vetoer (see D. H. Blair and R. A. Pollak, 'Acyclic Collective Choice Rules', mimeographed, University of Pennsylvania, 1980, and D. Kelsey, 'Acyclic Social Choice', M Phil thesis, Oxford University, 1981).

functions generated by the SDF' (p. 129). Such 'rationality' conditions might make the underlying social preference transitive, in which case the Arrow result will, of course, re-emerge fully.[33] The insistence on 'rationality' (consistency) conditions becomes particularly significant when social choice takes the form of a choice function which need not necessarily be based on—or be representable by—a binary relation ('preference').

In Essay 8 a 'functional collective choice rule' is defined as a functional relation that determines a choice function $C(\cdot)$ for the society for any n-tuple of individual preference orderings: $C(\cdot) = f(R_1, \ldots, R_n)$. This is essentially equivalent to a 'social choice function' as defined by Peter Fishburn,[34] and differs from it only in presentation, in separating out the 'choice function'—in the usual sense—for the society $C(\cdot)$ as the 'value' of the functional collective choice rule. (In contrast, social welfare functions, social decision functions, etc., are examples of *relational collective choice rules*: $R = f(R_1, \ldots, R_n)$, with the 'value' of the function f being a binary preference relation R.) Consistency or 'rationality' conditions required of social choice can make $C(\cdot)$ 'binary'. Less demandingly, weaker consistency conditions (e.g., some conditions obtained in Essay 1 through factorization) can permit the use of some—but not all—properties associated with the binariness of choice, and some of these more limited properties can be, in many cases, adequate for re-establishing the impossibility results (see Essay 8).

Schwartz, Plott, Campbell and Bordes have demonstrated that it is possible to have a functional collective choice rule satisfying Arrow's conditions, appropriately redefined, and fulfilling some consistency properties as well.[35] There is an element of ambiguity in what is to count as appropriate redefinition. Take, for example, the weak Pareto principle, which in the case of Arrow's social welfare function (or a social decision function) is defined as requiring that if everyone prefers some x to some y, then x is *socially preferred* to y. If, however, the notion of social preference is dropped and replaced by

33. Arrow's dictatorship result can, in fact, be established even without full transitivity, using only 'semiorder' properties and weaker conditions; see D. H. Blair and R. A. Pollak, 'Collective Rationality and Dictatorship: The Scope of the Arrow Theorem', *Journal of Economic Theory*, 21 (1979); and J. H. Blau, 'Semiorders and Collective Choice', *Journal of Economic Theory*, 21 (1979).

34. P. C. Fishburn, *The Theory of Social Choice* (Princeton: Princeton University Press, 1973). A 'social choice function' specifies the chosen subset of any non-empty subset S of the set of alternative states, for any n-tuple of individual preference orderings: $C(S, (R_1, \ldots, R_n))$.

35. See T. Schwartz, 'On the Possibility of Rational Policy Evaluation', *Theory and Decision*, 1 (1970); C. R. Plott, 'Path Independence, Rationality and Social Choice', *Econometrica*, 4 (1973); D. E. Campbell, 'Democratic Preference Functions', *Journal of Economic Theory*, 12 (1976); G. Bordes, 'Consistency, Rationality and Collective Choice', *Review of Economic Studies*, 43 (1976). See also S. Bloomfield, 'An Axiomatic Formulation of Constitutional Games', Technical Report No. 71-18, Operations Research House, Stanford University, 1971; and R. Deb, 'On Schwartz's Rule', *Journal of Economic Theory*, 6 (1977).

that of a choice function for the society, this condition can be translated in several different ways. One simple translation—indeed the one that most authors have used—demands in this case (when everyone prefers x to y) that x alone must be chosen from the *pair* (x, y). This can be seen as a condition on the 'base relation' of social choice (see Section 1.1 above). Alternatively, the Pareto principle could have been applied to the 'revealed preference relation' of social choice, requiring that if everyone prefers x to y, then y must not be chosen from *any* set that contains x. We can refer to these two alternative interpretations as the 'pair interpretation' and the 'general interpretation' respectively of the weak Pareto principle in the context of social choice.

The truth of the claims made by Schwartz, Plott, Campbell and Bordes can be easily checked by taking an example of a suitable social choice procedure. Take the majority rule and convert any majority preference cycle into an indifference class in the context of choice from any set containing all the elements involved in the cycle. If, for example, x beats y, which beats z, which again beats x, then—in choosing from the triple (x, y, z) or from any set containing that triple—proceed to find the subset of best elements treating x, y and z as socially indifferent. Schwartz, Campbell and Bordes show that not only do rules of this type satisfy all of Arrow's conditions, as they interpret them, but also such rules have many other attractive properties, including full transitivity of the 'revealed preference relation' associated with the generated choice functions.

Despite its ingenuity and technical merits, this line of resolution has, I believe, an inescapable defect. It works only because the Arrow problem is reinterpreted in the choice context in such a way that some of Arrow's conditions—in particular the Pareto principle and non-dictatorship—are made to work exclusively on the *base relation* thanks to choosing the 'pair interpretation', while some of the other Arrow conditions—in particular regularity in the form of transitivity (or quasi-transitivity, acyclicity, etc., in extensions of the Arrow result)—are made to work exclusively on the *revealed preference relation*. If this 'schizophrenia' is removed, Arrow-type impossibilities would re-occur; see Essay 8.

In fact, there are three distinct ways in which the impossibility results can be revived:

(1) Impose all the requirements on the base relation, including demanding transitivity (or quasi-transitivity, acyclicity, etc.) of that relation.

(2) Impose all the requirements on the revealed preference relation, including choosing the 'general interpretation'—as opposed to the 'pair interpretation'—of the Pareto principle and non-dictatorship.

(3) Maintain the 'schizophrenia'—imposing some conditions on the base relation and some on the revealed preference relation, but directly impose consistency conditions on choices from different sets—in particular impose 'contraction consistency' (see Section 1.1) which will tie up the base relation to the revealed preference relation.

Essay 8 ('Social Choice Theory: A Re-examination') shows how robust the Arrow-type impossibility problems are in preference aggregation.[36] The scepticism expressed in Essay 6 ('Quasi-transitivity, Rational Choice and Collective Decisions'), while proposing this line of inquiry, has unfortunately proved to be quite justified.

3 Welfare Comparisons and Social Choice

3.1 Informational basis of preference aggregation

In aggregating individual preferences, Arrow's social welfare functions and the related rules (social decision functions, social choice functions, functional collective choice rules, etc.) have to make do with rather limited information, viz., interpersonally non-comparable, ordinal utilities (or orderings). This informational base is expanded in Essays 9 to 16, and that was also one of the main thrusts in my *Collective Choice and Social Welfare* (chapters 6–9). It is perhaps useful to go a little into the motivation behind these exercises.

The Arrow informational format does not permit the use of cardinal information regarding individual utilities. This, however, proves to be not really a serious limitation, since the Arrow impossibility result can be generalized to cover cases in which the individual preferences are expressed as cardinal utility functions (see Theorem 8*2 in *Collective Choice and Social Welfare*). However, the absence of interpersonal comparability of individual utilities is a binding limitation, and its removal does indeed permit many rules satisfying all of Arrow's conditions, redefined for such a broader framework (see chapters 7 and 9 in *Collective Choice and Social Welfare*). It can be argued that it is the imposed poverty of the utility information that dooms Arrow's aggregation exercise to failure.

But this remark is a bit misleading since the informational limitation of the Arrow format relates not merely to the poverty of the utility information but also to the eschewal of non-utility information. Most actual public judgements make extensive use of non-utility information, varying from relative incomes and ownerships to the description of who is doing what to whom. Taking note of non-utility information is not, of course, explicitly ruled out by social welfare functions, etc., despite treating individual

36. See also Sen, *Collective Choice and Social Welfare* (1970), pp. 81–2; J. A. Ferejohn and D. M. Grether, 'On a Class of Rational Social Decision Procedures', *Journal of Economic Theory*, **8** (1974); P. C. Fishburn, 'On Collective Rationality and a Generalized Impossibility Theorem', *Review of Economic Studies*, **41** (1974); D. Blair, G. Bordes, K. Suzumura and J. S. Kelly, 'Impossibility Theorems without Collective Rationality', *Journal of Economic Theory*, **13** (1976); J. A. Ferejohn and D. M. Grether, 'Weak Path Independence', *Journal of Economic Theory*, **14** (1977); Y. Matsumoto, 'Choice Functions: Preference, Consistency and Neutrality', D. Phil. thesis, Oxford University, 1982.

preference orderings or ordinal utilities as the basis of social judgement or choice. This is because the description of the social state incorporates non-utility information, and the social welfare functions, etc., need not be 'neutral' to the nature of the social state. However, such a neutrality is, in fact, precipitated by the combination of the conditions that Arrow—and following him others—have typically used, viz., the Pareto principle, independence, and unrestricted domain. While this consequence was implicitly recognized in the literature for a long time, since Arrow's own proof of his impossibility theorem proceeded by way of some neutrality,[37] the neutrality results were formally established by Guha, Blau, d'Aspremont and Gevers.[38] Essay 15 ('Personal Utilities and Public Judgements: Or What's Wrong with Welfare Economics?') examines the genesis and the exact role of this type of 'neutrality'.

A strong form of such neutrality, which has been called 'welfarism', has been present in the moral philosophical literature for centuries. Indeed, the utilitarian formula of judging the states of affairs by simply summing the individual utilities—used not only by Bentham but also by such economists as Edgeworth, Marshall, Pigou, Robertson and others—illustrates a straightforward case of welfarism. In a less restrictive form, welfarism is present in the general formula for social welfare that can be found in, say, Samuelson's *Foundation of Economic Analysis*, or Graaff's *Theoretical Welfare Economics*: $W = f(u)$, when u is the vector of individual utilities. This is often called—misleadingly I think—'individualism' (as if individuals must be seen as no more than *locations* of their pleasures or utilities!). Arrow's social welfare functions, constrained by his conditions, produce a slightly weaker form of welfarism—called 'strict ranking welfarism' (see Essay 15)—which requires exclusive reliance on utility information in the special case in which individual preferences happen to be strict, involving no indifference.

The consequences of these information restrictions are indeed severe for public judgements, and this general issue has been discussed in Essays 8, 11 and 15.[39] It may be useful to illustrate the nature of the limitation with an example. Consider again the cake division problem and take two different cases (Figure 1). In case A, person 1 is very rich while 2 and 3 are poor,

37. See Arrow, *Social Choice and Individual Values* (1963) pp. 98–100; and Sen, *Collective Choice and Social Welfare* (1970), Lemma 3*a. See also J. H. Blau, 'The Existence of Social Welfare Functions', *Econometrica*, **25** (1957).

38. Guha, 'Neutrality, Monotonicity and the Right of Veto' (1972); Blau, 'Neutrality, Monotonicity and the Right of Veto: A Comment', (1976); C. d'Aspremont and L. Gevers, 'Equity and Informational Basis of Collective Choice', *Review of Economic Studies*, **46** (1977). See also Essay 11.

39. See also my 'Informational Basis of Alternative Welfare Approaches: Aggregation and Income Distribution', *Journal of Public Economics*, **3** (1974), and 'Informational Analysis of Moral Principles', in R. Harrison (ed.), *Rational Action* (Cambridge: Cambridge University Press, 1979).
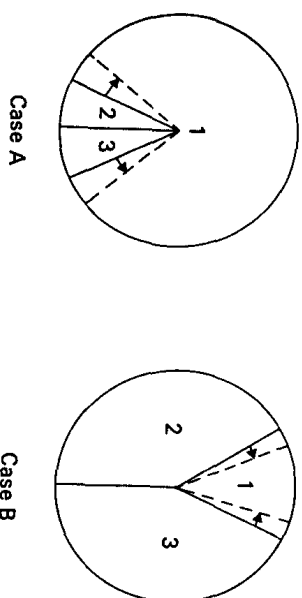
whereas in case B, person 1 is very poor while 2 and 3 are rich. In both cases we consider a redistribution—cutting out a bit from person 1's share of the cake and dividing that gain between 2 and 3. If each person prefers more cake to himself (i.e., if we make the standard assumption for the cake division exercise), then persons 2 and 3 prefer the change while person 1 disprefers it, in both cases A and B. Now the question: are the two cases of redistribution exactly similar? In the Arrow format they have to be. Suppose we want to say that the redistribution is more justified in case A than in case B, how would we distinguish the two cases? It is tempting to point out that person 1 is *worse off* than the others in case B but not so in case A. But if by 'worse off' we mean having lower utility, then that type of statement is ruled out by the absence of interpersonal comparisons. If, on the other hand, by worse off we mean having less cake and thus being poorer, that type of non-utility information cannot be taken into account given the feature of strict-ranking welfarism, insisting on exclusive reliance on utility information only. Indeed, in the Arrow format the two cases are *informationally identical*, and exactly the same judgement must be made about the change in both the cases, since the individual preference orderings are identical in the two cases.[40]

Arrow's remarkable achievement was to show—though he did not put it this way—that in such an informational format there are no consistent non-dictatorial rules. It does not belittle the outstanding importance of this elegant and far-reaching logical result—it has in fact been the prime mover of a whole discipline—to note that an informational format that cannot distinguish between cases A and B is quite unsuitable *anyway* for welfare economics. More information is needed to deal with interest conflicts. The individual preference orderings are identical in the two cases, is also impossible.

Case A   Case B

FIGURE 1

<hr/>

40. Majority rule would, of course, support the redistribution in both cases. That is not needed in the Arrow format generally. What is needed, however, is treating the two cases alike. Either the redistribution is better in both cases, or worse in both, or indifferent in both

This informational problem is not quite so compelling in the case of aggregation of conflicting *judgements* (e.g., in combining different people's judgements on whether it would be better for Britain to come out of the EEC, or on whether marijuana should be legalized), as opposed to aggregating conflicting *interests*. Essay 8 explores the distinctions between different aggregation types and shows the varying relevance of Arrow's—and related—results, depending on the precise nature of the aggregation exercise that is involved.

3.2 *Utility information: invariance, intersection and partial comparability*

Essays 9 to 12 deal primarily with enriching the utility information, while Essays 13 to 16 are mainly concerned with the use of non-utility information. Since the Arrovian social welfare functions and related structures do not have room for richer utility information, it is necessary to get a suitable format for putting more utility information into the process of social judgement or choice. The procedure used for this purpose in *Collective Choice and Social Welfare* is to define a 'social welfare functional' SWFL as a function of the *n*-tuple of individual utility functions:[41] $R = F(U_1, \ldots, U_n)$, and then to constrain it through identifying different *n*-tuples of utility functions that are 'informationally identical'. The methods are explained and explored in Essays 9 and 11.

Depending on the measurability assumption of individual utility (e.g., ordinality, cardinality, etc.), each person has a *family* of utility functions, e.g., in the case of ordinality each member of a family of utility functions of a person is a positive monotonic transformation of all other members (and the family includes *all* such transformations). If there is no interpersonal comparability, we can pick any *n*-tuple of individual utility functions—one from each person's family. However, interpersonal comparability can be seen as consisting in tying up different people's utility functions with each other, reducing this freedom of choice, e.g., we can't simply blow up the representation of my utility function a million-fold, keeping yours unchanged (overwhelming your utility gains and losses in the utilitarian calculus). So interpersonal comparability specifies some *subset*—called the 'comparability set'—of the set of *n*-tuples of utility functions permitted by the measurability assumption, and all *n*-tuples in a comparability set are informationally identical. Thus, depending on the exact measurability and comparability assumptions that are chosen, for any real utility situation a subset of all possible alternative *n*-tuples of utility functions (in the form of alternative *n*-tuples of

<hr/>

41. Note that this does not imply that each state must be judged by the utility *vector* associated with that state, i.e., welfarism is not implied. Welfarism can, of course, be produced by additional restrictions imposed on SWFLs, see Essay 11.

utility functions) are arrived at. A SWFL is thus supplemented by statements on informationally equivalent *n*-tuples of utility functions.

From here we can go in one of two directions. Either we can admit only those rules that yield the *same* social ordering (or choice) for every *n*-tuple in the informationally identical set. This all-or-nothing approach was explored in Chapter 8 of *Collective Choice and Social Welfare* and is explored at some length in Essay 11. Or, alternatively, we can admit any rule whatsoever, but accept only that *partial* ordering of social states which is common to (i.e., is the intersection of) all the social orderings generated by the rule, respectively for all the members of the comparability set. Essay 9 explores this more permissive 'intersection' approach. To illustrate the contrast, for non-interpersonally-comparable individual utilities, the utilitarian rule is simply inadmissible under the all-or-nothing approach, but will yield the Pareto partial ordering in the more permissive approach (since even with no interpersonal comparability a state that is higher in everyone's utility function must yield a higher utility-sum no matter which particular utility representations we choose).

The more permissive 'intersection' approach also allows the use of *partial* interpersonal comparability. This concept was introduced in Essay 9, and reflected the possibility that utility comparisons may be neither impossible, nor—on the other hand—terribly exact. To take up an example presented in Essay 9, we might not be able to put Emperor Nero's utility functions into a one-to-one correspondence with every other Roman's utility function, but we might nevertheless find it absurd to multiply Nero's utility function by a suitably large number—keeping the utility functions of the others unchanged—to produce the result that there was indeed a net gain in the utility-sum from the burning of Rome while Nero played the fiddle.

Partial comparability may either represent a limitation of information regarding the true situation, or be interpreted as reflecting an intrinsic element of vagueness in the very nature of utility comparisons.[42] Essay 11 analyses how partial comparability can be accommodated within a formal structure of utility aggregation, and Essay 12 makes a few further remarks on that problem.[43]

42. There is some similarity, in this respect, between the partiality of utility comparisons and Issac Levi's notion of 'indeterminate probabilities' (see I. Levi, 'On Indeterminate Probabilities', *Journal of Philosophy*, 71 (1974), and *The Enterprise of Knowledge* (Cambridge Mass: MIT Press, 1980))

43. See also my *Collective Choice and Social Welfare* (1970), chapter 7*; C. Blackorby, 'Degrees of Cardinality and Aggregate Partial Orderings', *Econometrica*, 43 (1975); B. J. Fine, 'A Note on "Interpersonal Aggregation and Partial Comparability"', *Econometrica*, 43 (1975), K. Basu, *Revealed Preference of Governments* (Cambridge: Cambridge University Press, 1979), Th. Bezembinder and P. van Acker, 'A Note on Sen's Partial Comparability Model', Department of Psychology, Katholieke Universiteit, Nijmegen, 1979.

If partial comparability is viewed not as an intrinsic quality of utility comparison, but as a reflection of limitation of information, e.g., available to a person making a political judgement, or to a planner taking policy decisions, then there is a clear similarity between that notion and Abba Lerner's concept of 'ignorance' on the part of the central planners as to who has which particular utility function out of a given set of (interpersonally comparable) utility functions.[44] Lerner had shown that with the assumption of 'equal ignorance', which implies that any person has the same probability of having any given utility function as any other person, the optimal distributional rule for a given total income is equal division, provided (i) the utilitarian formula of social welfare is accepted, and (ii) we maximize the expected social welfare.

Milton Friedman pointed out the limitation of the utilitarian formula in dealing with problems of income distribution *when people's 'capacities to enjoy' in fact differ*, since we may not be indifferent to *the distribution of* total personal utilities.[45] Essay 10 shows that Lerner's result does not require the utilitarian formula, and indeed any concave group welfare function—no matter how concerned with the distribution of total utilities—will yield Lerner's result. Thus, Friedman's criticism, while valid for Lerner's own proposition, does not affect the case for equal division of income no matter what importance we decide to attach to the question of 'distribution' of total personal utilities. The same consequence holds—in fact with even weaker assumptions—if the approach is focused not on maximizing the *expected value of social welfare*, but on maximizing the *minimal value of social welfare*.[46]

However, while the assumption of 'equal ignorance' imposes a pattern of symmetry on the information we have regarding different people's utility functions, there is no such assumption in the general format of partial comparability. The purpose in this case is to catch a common attitude towards interpersonal comparisons of utility, which involves neither Pigovian precision,[47] nor Robbinsian rejection.[48]

44. A. P. Lerner, *The Economics of Control* (London: Macmillan, 1944).

45. M. Friedman, 'Lerner on the Economics of Control', *Journal of Political Economy*, 55 (1947); reprinted in his *Essays in Positive Economics* (Chicago: Chicago University Press, 1966).

46. Gordon Tullock, in a characteristically spirited note ('On Mathematics as Decoration', *Papers in Economic Criticism*, May 1975), has attacked this last result as trivial. It might well be trivial (the proof is just a few lines anyway), but Tullock's diagnosis is based on a confusion between (1) maximizing the *minimal value of social welfare*, and (2) maximizing the *minimal income that any person receives* ('in the Sen article there is another mathematical proof which is that if you divide the given sum of money equally among people, the minimum amount that any one of those persons can receive is higher than if you divide the money unequally', Tullock, p. 23)! In my response to Professor Tullock, I had to confess my enhanced admiration for his ability to write so much and so well without—evidently—being able to read.

47. A. C. Pigou, *The Economics of Welfare* (London: Macmillan, 1920).

48. L. Robbins, 'Interpersonal Comparisons of Utility', *Economic Journal*, 48 (1938).

3.3 Axioms, implications and interpretations

In the format of social welfare functionals it is possible to study various common—and uncommon—social welfare rules (e.g., utilitarianism, Rawlsian maximin,[49] and various other formulas for judging social welfare). That was indeed one of the main purposes in getting away from the informational straitjacket of Arrow's social welfare functions (see Chapters 7 to 9 of my *Collective Choice and Social Welfare*).[50] There have been a number of outstanding contributions in this general area in recent years, providing—among other things—alternative axiomatizations of various social welfare rules.[51]

While cardinality of individual utility functions without interpersonal comparability has no effect on Arrow's impossibility result, interpersonal comparability of the 'ordinal' sort, even without cardinality, does indeed remove the impossibility. An example satisfying all the Arrow conditions in the framework of ordinal comparability is Rawls's rule of judging the welfare of the society by the welfare level of the worst-off individual. This can be made compatible with the stronger version of the Pareto principle by being defined in the lexicographic form—often called 'leximin'—so that if the worst-off persons in two states are equally badly off, then we compare the second worst-off persons, and so on (see chapter 9 of my *Collective Choice and Social Welfare*). While the Arrow conditions are all satisfied by the Rawlsian 'leximin', they do not uniquely define that rule. Peter Hammond, Steven Strasnick, Claude d'Aspremont, Robert Deschamps, Louis Gevers,

49. John Rawls, *A Theory of Justice* (Cambridge, Mass.: Harvard University Press, 1971). See also his 'Social Unity and Primary Goods', in A. Sen and B. Williams (eds), *Utilitarianism and Beyond* (Cambridge: Cambridge University Press, 1982).

50. See also my 'Informational Basis of Alternative Welfare Approaches: Aggregation and Income Distribution' (1974); 'Rawls versus Bentham: An Axiomatic Examination of the Pure Distribution Problem', *Theory and Decision*, 4 (1974); 'Welfare Inequalities and Rawlsian Axiomatics', *Theory and Decision*, 7 (1976); and Essay 11.

51. See P. J. Hammond, 'Equity, Arrow's Conditions and Rawls' Difference Principle', *Econometrica*, 44 (1976); S. Strasnick, 'Social Choice Theory and the Derivation of Rawls' Difference Principle', *Journal of Philosophy*, 73 (1976); d'Aspremont and Gevers, 'Equity and Informational Basis of Collective Choice' (1977); Deschamps and L. Gevers, 'Leximin and Utilitarian Rules: A Joint Characterisation', *Journal of Economic Theory*, 17 (1978); E. Maskin, 'A Theorem on Utilitarianism', *Review of Economic Studies*, 45 (1978); R. Deschamps and L. Gevers, 'Separability, Risk-bearing and Social Welfare Judgements', in J.-J. Laffont (ed.), *Aggregation and Revelation of Preferences* (Amsterdam: North-Holland, 1979); Gevers, 'On Interpersonal Comparability and Social Welfare Orderings'; E. Maskin, 'Decision-Making under Ignorance with Implications for Social Choice', *Theory and Decision*, 11 (1979); K. W. S. Roberts, 'Possibility Theorems with Interpersonally Comparable Welfare Levels', *Review of Economic Studies*, 47 (1980); K. W. S. Roberts, 'Interpersonal Comparability and Social Choice Theory', *Review of Economic Studies*, 47 (1980).

Eric Maskin and Kevin Roberts have produced various alternative ways of tightening the conditions to pinpoint the 'leximin' rule exactly.

Essay 11 provides, among other things, a rather different axiomatic derivation of 'leximin'. Leximin—like the maximin—has the *appearance of* an 'extremist' criterion in giving priority to the interest of the worst-off even if it goes against the interests of a very large number of others. Its greater plausibility in two-person conflicts has been asserted, and certainly it is not unreasonable to give priority to the interests of the worse-off among exactly two persons when all others are indifferent. Theorem 8 ('Rawls from Inch to Ell') in Essay 11 shows that for any social welfare functional with unrestricted domain and independence, leximin for two-person conflicts logically entails leximin *in general*—no matter how many people are involved in the interest conflict.[52]

While the axiomatic study of interpersonal comparisons and their uses helps to clarify the ways through which utility information can be utilized for social judgement or choice, there are important interpretational issues underlying all these exercises. What do interpersonal comparisons stand for? Are they just value judgements, as is often alleged? If so, how can we systematize these judgements for further use? Or are they factual matters? If so, how do we obtain these facts for our exercises? Essay 12 ('Interpersonal Comparisons of Welfare') investigates these foundational problems, and explores various alternative avenues. Moving far away from the once-fashionable view that interpersonal comparisons are 'meaningless', it is argued in Essay 12 that 'the central problem in the theory of interpersonal comparisons of welfare seems to be an embarrassment of riches—there are many reasonable ways of making such comparisons and they need not coincide'.

4 Non-utility Information

4.1 Pareto versus rights

Essay 13 ('The Impossibility of a Paretian Liberal') was instrumental in starting a line of inquiry that has led to a rather voluminous literature. The impossibility result indicates a conflict between individual liberty (in the form of a person being 'decisive' over certain personal matters) and the Pareto principle (asserting the priority of unanimous preference rankings). The formal result shows the inconsistency of three conditions, viz.,

52. Peter Hammond has provided an alternative proof of this theorem, and in the process proved a very important general property relating two-person equity norms to *n*-person equity norms, of which 'Rawls from Inch to Ell' can be seen as a special case ('Equity in Two Person Situations: Some Consequences', *Econometrica*, 47 (1979)).

unrestricted domain, the weak Pareto principle, and a condition of 'minimal liberty',[53] requiring that at least two persons have some 'personal sphere' each, such that if either of them prefers some x to some y in his or her own personal sphere, then x is socially preferred to y.[54] A necessary—though quite plausibly not sufficient—condition for a pair (x, y) to be in an individual i's personal sphere is that the states x and y differ from each other exclusively in a matter that is 'personal' to i, e.g., whether i reads or does not read a particular book.

Various proposals for avoiding the conflict have been suggested in the literature, and they have respectively involved alternative ways of weakening each of the three conditions: unrestricted domain,[55] minimal liberty,[56] and

53. In Essay 13 the condition was called 'minimal liberalism', though it was also stated: 'The term "liberalism" is elusive and is open to alternative interpretations. Some uses of the term may not embrace the condition defined here. What is relevant is that Condition L represents a value involving individual liberty that many people would subscribe to. Whether such people are best described as liberals is a question that is not crucial to the point of the paper' (p. 286). In Essay 14 the condition was renamed as 'minimal libertarianism'. Neither term is very satisfactory. The expression 'minimal liberty'—used here—has the advantage of focusing directly on 'a value involving individual liberty that many people would subscribe to', rather than on the advocacy of that value in liberalism or libertarianism.

54. For different types of examples of this conflict, see my Collective Choice and Social Welfare, chapter 6; Allan Gibbard, 'A Pareto-Consistent Libertarian Claim', Journal of Economic Theory, 7 (1974); Jonathan Barnes, 'Freedom, Rationality and Paradox', Canadian Journal of Philosophy, 10 (1980); J. Fountain, 'Bowley's Analysis of Bilateral Monopoly and Sen's Liberal Paradox in Collective Choice Theory: A Note', Quarterly Journal of Economics, 95 (1980); E. T. Green, 'Libertarian Aggregation of Preferences: What the "Coase Theorem" Might Have Said', Social Science Working Paper 315, California Institute of Technology, 1980.

55. See, among others, J. H. Blau, 'Liberal Values and Independence', Review of Economic Studies, 42 (1975); C. Seidl, 'On Liberal Values', Zeitschrift für Nationalökonomie, 35 (1975); F. Breyer, 'The Liberal Paradox, Decisiveness over Issues, and Domain Restrictions', Zeitschrift für Nationalökonomie, 37 (1977); F. Breyer and G. A. Gigliotti, 'Empathy and the Respect for the Rights of Others', Zeitschrift für Nationalökonomie, 40 (1980); D. Austen-Smith, 'Necessary and Sufficient Conditions for Libertarian Collective Choice Rules', mimeographed, University of York, 1981; E. Maskin, B. Nalebuff and A. Sen, unpublished notes on the impossibility of the Paretian liberal.

56. See, among others, K. Ng, 'The Possibility of a Paretian Liberal: Impossibility Theorems and Cardinal Utility', Journal of Political Economy, 79 (1971); A. Gibbard, 'A Pareto-Consistent Libertarian Claim', Journal of Economic Theory, 7 (1974); P. Bernholz, 'Is a Paretian Liberal Really Impossible?', Public Choice, 19 (1974); J. H. Blau, 'Liberal Values and Independence' (1975); D. E. Campbell, 'Democratic Preference Functions', Journal of Economic Theory, 12 (1976); J. S. Kelly, 'Rights Exercising and a Pareto-Consistent Libertarian Claim', Journal of Economic Theory, 13 (1976); J. S. Kelly, 'The Impossibility of a Paretian Libertarian Claim', Economica, 43 (1976); J. Aldrich, 'The Dilemma of a Paretian Liberal: Some Consequences of Sen's Theorem', Public Choice, 30 (1977); J. A. Ferejohn, 'The Distribution of Rights in Society', in H. W. Gottinger and W. Leinfelner (eds), Decision Theory and Social Ethics: Issues in Social Choice (Dordrecht: Reidel, 1978); E. Karni, 'Collective Rationality, Unanimity and Liberal Ethics', Review of Economic Studies, 45 (1978); D. C. Mueller, Public Choice (Cambridge: Cambridge University Press, 1979); R. Gardner, 'The Distribution of Rights in Society', and W. Leinfelner (eds), R. Gardner, 'The Strategic Inconsistency of the Paretian Liberal', Public Choice, 35 (1980); F. Breyer and R. Gardner, 'Liberal Paradox,

weak Pareto principle.[57] Essay 14 examines and evaluates some of the proposals which had by then (1976) been put forward, but there have been many others since then. There have also been many interesting extensions of the result to other areas, in particular to group rights (including federalism).[58]

One reason for the attention that Essay 13 has received is the interest that normative 'rights' have recently aroused. The paper departed from the traditional formulations of welfare economics in trying to make room for rights, and it identified a conflict even with the allegedly mildest of the welfare-economic principles, viz., the weak Pareto principle. The lessons drawn from the conflict have, however, varied a great deal. For example, in a powerfully argued contribution, Robert Nozick has claimed that the lesson to be drawn from this result is the impossibility of reflecting rights through a 'social ordering',[59] and he has outlined an alternative, deontological

Game Equilibrium, and Gibbard Optimum', Public Choice, 35 (1980); K. Suzumura, 'Liberal Paradox and the Voluntary Exchange of Rights Exercising', Journal of Economic Theory, 22 (1980); W. Gaertner and L. Krüger, 'Self-supporting Preferences and Individual Rights: the Possibility of Paretian Libertarianism', Economica, 47 (1981); L. Krüger and W. Gaertner, 'Alternative Libertarian Claims and Sen's Paradox', Economics Discussion Paper 81, University of Bielefeld, 1981; K. Suzumura, 'Equity, Efficiency and Rights in Social Choice', Discussion Paper 155, Kyoto Institute of Economic Research, 1981; J. L. Wriglesworth, 'Solution to the Gibbard and Sen Paradoxes Using Information Available from Interpersonal Comparisons', mimeographed, Lincoln College, Oxford, 1981.

57. See, among others, M. J. Farrell, 'Liberalism in the Theory of Social Choice', Review of Economic Studies, 43 (1976); K. Suzumura, 'On the Consistency of Libertarian Claims', Review of Economic Studies, 45 (1978); P. J. Hammond, 'Liberalism, Independent Rights and the Pareto Principle', forthcoming in the Proceedings of the 6th International Congress of Logic, Methodology and Philosophy of Science, 1979; D. Austen-Smith, 'Restricted Pareto and Rights', Journal of Economic Theory, forthcoming; Rawls, 'Social Unity and Primary Goods' (1982); P. Coughlin and A. Sen, unpublished notes on conditional Pareto principles.

58. See R. N. Batra and P. K. Pattanaik, 'On Some Suggestions for Having Non-binary Social Choice Functions', Theory and Decision, 3 (1972); D. N. Stevens and J. E. Foster, 'The Possibility of Democratic Pluralism', Economica, 45 (1978); J. L. Wriglesworth, 'The Possibility of Democratic Pluralism: A Comment', Economica, 49 (1982). For extensions in a different direction, see A. Weale, 'The Impossibility of Liberal Egalitarianism', Analysis, 40 (1980); and I. S. McLean, 'Liberty, Equality and the Pareto Principle: A Comment on Weale', Analysis, 40 (1980). Gibbard's 'A Pareto-Consistent Libertarian Claim' (1974), pursues problems of internal consistency of libertarianism.

59. R. Nozick, 'Distributive Justice', Philosophy and Public Affairs, 3 (1973), and Anarchy, State and Utopia (Oxford: Blackwell, 1974), pp. 164–6. On related matters, see C. K. Rowley and A. T. Peacock, Welfare Economics: A Liberal Restatement (London: Martin Robertson, 1975); James Buchanan, 'An Ambiguity in Sen's Alleged Proof of the Impossibility of the Paretian Liberal', mimeographed, Virginia Polytechnic, 1976; C. R. Perelli-Minetti, 'Nozick on Sen: A Misunderstanding', Theory and Decision, 8 (1977); B. Barry, 'Lady Chatterley's Lover and Doctor Fischer's Bomb Party: Liberalism, Pareto Optimality and the Problem of Objectionable Preferences', presented at the Ustaoset Conference on the Foundations of Social Choice Theory, 1981; P. Gärdenfors, 'Rights, Games and Social Choice', Noûs, (1981); R. Sugden, The Political Economy of Public Choice (Oxford: Martin Robertson, 1981); P. J. Hammond, 'Utilitarianism, Uncertainty and Information', in A. Sen and B. Williams (eds), Utilitarianism and Beyond (Cambridge: Cambridge University Press, 1982); B. Chapman, 'Individual Rights and Collective Rationality: Some Implications for Economic Analysis of Law', Hofstra Law Review, 10 (Winter 1982).