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Propagation of Singularities for Fuchsian Operators

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TABLE OF CONTENTS

INTRODUCTION

The main purpose of this monograph is the study of Fuchsian systems of the form

$$(0.1) \qquad Pu = (t\partial_t I_N - A(t,x,D_t,D_x))u(t,x) = f(t,x) \ ,$$

where A is an $N \times N$ matrix of classical pseudodifferential operators (p d o) of order 0 defined on $\mathbb{R}^{n+1} = \mathbb{R}_t \times \mathbb{R}_x^n$. More precisely, we are interested in describing C^∞-singularities of the solutions of system (0.1) i.e. the set $WF(u) \setminus WF(Pu)$, where $WF(v)$ denotes the wave front set of the distribution v as defined in L. Hörmander [14] (for a vector-valued distribution $v = (v_1,\dots,v_N) \in \mathcal{D}'^N$ we put $WF(v) = \bigcup_{j=1}^{N} WF(v_j)$).

It is well known that the structure of the set $WF(u) \setminus WF(Pu)$ depends on the characteristics of the operator P, i.e.

$$(0.2) \qquad WF(u) \setminus WF(Pu) \subset \{(t,x,\tau,\xi) \in T^* \mathbb{R}^{n+1} \setminus 0 \mid t\tau = 0\} = \text{Char } P$$

Near a point $\rho_o = (t_o,x_o,\tau^o,\xi^o) \in \text{Char } P$ for which $t_o \neq 0$ or $t_o = 0$ and $\tau^o \neq 0$, $\xi^o \neq 0$, a complete description of $WF(u) \setminus WF(Pu)$ follows from the general results of J.J. Duistermaat - L. Hörmander [10]; in particular $WF(u) \setminus WF(Pu)$ is invariant under the action of the hamiltonian vector fields $H_\tau = \frac{\partial}{\partial t}$ and $H_t = -\frac{\partial}{\partial \tau}$ respectively.

Therefore we concentrate our analysis of $WF(u) \setminus WF(Pu)$ near the points of the two following disjoint subsets of Char P :

$$(0.3) \qquad \begin{aligned} \Sigma &= \{(t,x,\tau,\xi) \in T^* \mathbb{R}^{n+1} \setminus 0 \mid t = 0, \tau = 0\} \\ \dot{N}^* \mathbb{R}^n &= \{(t,x,\tau,\xi) \in T^* \mathbb{R}^{n+1} \mid t = 0, \xi = 0, \tau \neq 0\} \end{aligned}$$

In order to study singularities near a point $\rho_o \in \Sigma \cup \dot{N}^* \mathbb{R}^n$ a general policy consists in constructing a left microlocal parametrix for the system (0.1) i.e. an operator $E : \mathcal{D}'(\mathbb{R}^{n+1})^N \to \mathcal{D}'(\mathbb{R}^{n+1})^N$ such that $EP - I_N$ is a smoothing operator in a conical neighborhood of ρ_o. However it does not seem to be an easy task to construct parametrices directly for system (0.1). It turns out that it is more convenient to reduce (0.1) to a simpler equivalent canonical form and then construct a parametrix for the simplified system.

In carrying over this program an important role is played by the quantity:

$$(0.4) \qquad I^P_{\rho_o}(\zeta) = \det(\zeta I_N - A_o(\rho_o)), \ \zeta \in \mathbb{C} ,$$

where A_o denotes the principal symbol of the matrix A . The polynomial (0.4) shall be called the *indicial polynomial* of the system (0.1) at the point $\rho_o \in \Sigma \cup \dot{N}^* \mathbb{R}^n$. We point out that if (0.1) is an ordinary differential system (i.e. if A is a matrix of functions) then the polynomial $I^P_{\rho_o}(\zeta)$ coincides with the classical indicial polynomial encountered in the theory of ordinary differential systems with regular singularities (see e.g. B. Malgrange [21]). The precise canonical form of the system (0.1) in a conic neighborhood of ρ_o depends on whether the roots $\zeta_1(\rho_o), \dots, \zeta_N(\rho_o)$ of the *indicial equation* $I^P_{\rho_o}(\zeta) = 0$ differ by non zero integers. We shall say that the *Fuchs condition* $(F)_{\rho_o}$ is satisfied at ρ_o if

$$(F)_{\rho_o} : \zeta_i(\rho_o) - \zeta_j(\rho_o) \notin \mathbb{Z} \setminus \{0\} , \quad i,j = 1,\dots,N.$$

We now describe the results we obtain in the two different cases $\rho_o \in \Sigma$ and $\rho_o \in \dot{N}^* \mathbb{R}^n$.

a) <u>Case $\rho_o \in \Sigma$</u>

Let us first suppose that the Fuchs condition $(F)_{\rho_o}$ is satisfied by the system (0.1). It will be proved in Section 2 that there exists an intertwining matrix Q of classical p d o's of order 0, elliptic near ρ_o, such that in a conic neighborhood of ρ_o:

$$(0.5) \qquad Q^{-1} P Q \equiv \tilde{P} = t\partial_t I_N - \tilde{A}(x,D_x) ,$$

where $\tilde{A}$ is a $N \times N$ matrix of p d o's of order 0 which does not depend on t and D_t.

Note that $I^P_{\rho_o}(\zeta) = I^{\tilde{P}}_{\rho_o}(\zeta)$. We are thus reduced to analyze the singularities near ρ_o of the solutions to the system

$$(0.6) \qquad \tilde{P}u = (t\partial_t I_N - \tilde{A}(x,D_x))v(t,x) = g(t,x)$$

with $v = Q^{-1}u$, $g = Q^{-1}f$.

In the case $N = 1$ explicit microlocal parametrices for equation (0.6) have been constructed by N. Hanges [13]; these constructions may be readily extended to give parametrices for (0.6) in the general case $N \geq 1$. The results are reviewed in Section 1. As a consequence the following theorem can be proved.

THEOREM 1. Let $\rho_o = (0,x_o,0,\xi^o)$, $\xi^o \neq 0$, and define

$$(0.7) \quad \begin{cases} \gamma_1^{\pm} = \{(0,x_o,s,\xi^o) \in T^* \mathbb{R}^{n+1} \setminus 0 \quad | \pm s > 0\} \\ \gamma_2^{\pm} = \{(s,x_o,0,\xi^o) \in T^* \mathbb{R}^{n+1} \setminus 0 \quad | \pm s > 0\} \end{cases}$$

Let $Pu = f \in \mathcal{D}'(\mathbb{R}^{n+1})^N$ and suppose that $\rho_o \notin WF(f)$. Then

i) If for every $j \in \{1,2\}$ there is a choice of the sign + or - for which $\gamma_j^{\pm}$ does not intersect $WF(u)$ then $\rho_o \notin WF(u)$.

ii) If $I^P_{\rho_o}(\zeta) \neq 0$ for $\zeta = 0,1,2,\ldots$ and both γ_1^+ and γ_1^- do not intersect $WF(u)$ then $\rho_o \notin WF(u)$.

iii) If $I^P_{\rho_o}(\zeta) \neq 0$ for $\zeta = -1,-2,-3,\ldots$ and both γ_1^+ and γ_1^- do not intersect $WF(u)$ then $\rho_o \notin WF(u)$.

For $N=1$ this theorem was proved by N. Hanges [13]; for a less constructive proof see V.Ya. Ivrii [16], R. Melrose [22], and for a slightly different case see S. Alinhac [3].

We remark that in Section 1 we give some results on the structure of the solution u when the indicial polynomial has integer roots (for this case see also [16]). If the Fuchs condition $(F)_{\rho_o}$ is not satisfied the situation is more involved since we find a non-trivial obstruction to the reduction (0.5). In the absence of the Fuchs condition me make a tricky reduction inspired by M. Kashiwara - T. Oshima [19] and T. Ôaku [26] and an enlargement of the original system (0.1) to show that, if $Pu = f$, then there are vectors U,F with $WF(u) = WF(U)$, $WF(F) = WF(f)$ near ρ_o which satisfy near ρ_o a suitable Fuchsian system:

(0.8) $$P'U = (t\partial_t I_M - A'(x,D_x))U = F$$

where M is larger than N, A' is a $M \times M$ matrix of p d o's of order 0 which do not depend on t and D_t and $I^{P'}_{\rho_o}(\zeta)$ has the same roots as $I^P_{\rho_o}(\zeta)$. The precise construction of the system P' is quite technical and forms the core of Section 2 to which we refer the reader for further details.

As a consequence Theorem 1 holds for any system of the form (0.1).

For results of the same kind in the hyperfunction or analytic setting see M. Kashiwara, T, Kawai, T. Oshima [18], H. Tahara [28] and the recent work of T. Ôaku [26].

b) Case $\rho_o \in \dot{N}^* \mathbb{R}^n$

The last part of Section 2 is dedicated to finding a canonical form for the system (0.1) near points $\rho_\pm = (0, x_o, \tau = \pm 1, 0)$ of the conormal bundle of $\mathbb{R}^n_x$. If the Fuchs condition $(F)_{\rho_\pm}$ is satisfied at $\rho_\pm$, we show, as in N. Hanges [12], that the system P is microlocally equivalent near $\rho_\pm$ to the multiplication operator tI_N. Otherwise, using an idea of M. Kashiwara - T. Oshima [19], we show that (0.1) may be reduced via an elliptic intertwining operator to the canonical form

$$(0.9) \qquad P_\pm = t\partial_t I_N - B_\pm(x, D_t, D_x) .$$

Furthermore, $B_\pm$ is block upper triangular and $b_{ii,\pm}(x, D_t, D_x) = b_{ii,\pm}(x)$. As a consequence we obtain the following result

THEOREM 2. Consider the operator P defined in (0.1) as a mapping on $\mathcal{M}^N_{\rho_\pm}$, where $\mathcal{M}_{\rho_\pm}$ is the stalk over $\rho_\pm$ of the sheaf of microdistributions in $\mathbb{R}^{n+1}$. Then:

i) P is surjective, i.e. given $f \in \mathcal{D}'(\mathbb{R}^{n+1})^N$ there is a $u_\pm \in \mathcal{D}'(\mathbb{R}^{n+1})^N$ such that $\rho_\pm \notin WF(Pu_\pm - f)$

ii) Ker P is isomorphic to N copies $\mathcal{D}'_{x_o}(\mathbb{R}^n)$, the space of germs at x_o of distributions in the x variable.

When the Fuchs condition is satisfied and the roots of the indicial equation $I^P_{\rho_\pm}(\zeta) = 0$ are simple, more precise information on the structure of $\ker P \subset \mathcal{M}^N_{\rho_\pm}$ can be obtained (see the remark following Theorem 2.3 of Section 2). For closely related results in the hyperfunction setting see M. Kashiwara - T. Oshima [19] and H. Tahara [28].

In Section 3 we apply the previously developed theory of Fuchsian systems to the study of a scalar Fuchsian operator of order m and weight $m-k$, $1 \leq k \leq m$:

(0.11) $$P(t,x,D_t,D_x) = t^k P_m + t^{k-1} P_{m-1} + \dots + P_{m-k},$$

where $P_{m-j}(t,x,D_t,D_x)$ is a classical (pseudo) differential operator of order $m-j$ on $\mathbb{R}^{n+1}$, $j = 0,\dots,k$.

Operators of the form (0.11) are a particular case of those considered by M. Baouendi-C. Goulaouic [6]. The literature on Fuchsian differential operators is extensive. Besides the works cited previously and in the text, we mention some recent work on Fuchsian operators. Parabolic Fuchsian operators have been studied by C. Baiocchi-M. Baouendi [5] and elliptic Fuchsian operators are studied by P. Bolley - J. Camus [7]. Second order elliptic operators of Fuchs type are extensively treated in F. Treves [30]. For results on existence when $P_m(t,x,D_t,D_x)$ is strictly hyperbolic with respect to t see S. Alinhac [2] and N. Hanges [12]. J. F. Nourrigat [25] studies boundary value problems for strictly hyperbolic Fuchsian operators. G. Roberts [27] proves a Calderon type uniqueness theorem for operators of the form (0.11).

Here we assume that the hyperplane $t = 0$ is not characteristic for P_m and that the principal symbol p_m of P_m admits the factorization:

(0.12) $$p_m(t,x,\tau,\xi) = q(t,x,\tau,\xi)^r\, e(t,x,\tau,\xi) ,$$

where e is an elliptic factor, $q(t,x,\tau,\xi) = \tau - \lambda(t,x,\xi)$ with λ a smooth real function positively homogeneous of degree 1 in ξ and r is a positive integer. If $r \geqq 2$ we assume that the operator P satisfies a Levi condition with respect to the factor q (see J. Chazarain [8]).

Firstly we are interested in studying the singularities of a solution u of the equation $Pu = f$ near points $\rho_o \in T^* \mathbb{R}^{n+1} \setminus 0$ of the form $\rho_o = (0,x_o,\tau = \lambda(0,x_o,\xi^o), \xi^o)$, $\xi^o \neq 0$.

Note that the particular form of (0.11) implies that P satisfies a Levi condition with respect to the factor t if $k \geqq 2$. Since the Poisson bracket $\{t,q\} = -1$, points ρ_o belong to the non-involutory manifold

(0.13) $$\hat{\Sigma} = \{(t,x,\tau,\xi) \in T^* \mathbb{R}^{n+1} \setminus 0 \mid t = 0,\ \tau = \lambda(0,x,\xi), \xi \neq 0\}$$

which is characteristic for P.

In Section 3 we show that the Fuchs equation $Pu = f$ can be microlocally transformed into a Fuchsian system of the form (0.1) (with $N = \max(k,r)$) whose indicial polynomial at ρ_o can be explicitly computed in terms of the principal symbols $p_{m-j}(\rho_o)$ of the operators P_{m-j}, $j = 0,\dots,k$.

An application of Theorem 1 gives the following

THEOREM 3. Let P be as in (0.11) and satisfy the above hypotheses. Let $\rho_o \in \hat{\Sigma} \setminus WF(Pu)$, $u \in \mathcal{D}'(\mathbb{R}^{n+1})$. Denote by $\gamma_1(s)$ (resp. $\gamma_2(s)$) the integral curve of the hamiltonian field H_t (resp. H_q) with $\gamma_1(0) = \rho_o$ (resp. $\gamma_2(0) = \rho_o$) and let $\gamma_j^{\pm}$, $j = 1,2$ be the four open half-bicharacteristic curves with $\pm s > 0$. Then:

i) If for each $j \in \{1,2\}$ there is a choice of the sign + or - for which $\gamma_j^{\pm} \cap WF(u) = \emptyset$ then $\rho_o \notin WF(u)$.

ii) Suppose that $r \leqq k$ and the roots of the indicial polynomial of the associated system do not lie in $\{0,1,2,\dots\}$; if γ_1^+ and γ_1^- do not intersect $WF(u)$, then $\rho_o \notin WF(u)$.

iii) Suppose that $r \geqq k$ and that the roots of the indicial polynomial of the associated system do not lie in $\{-1,-2,-3,\dots\}$; if γ_2^+ and γ_2^- do not intersect $WF(u)$, then $\rho_o \notin WF(u)$.

The conclusions of part ii) and iii) of Theorem 3 are sharp, for $k \neq r$, in a sense clarified by the remarks following the proof of Theorems 3.1 and 3.2 of Section 3.

We conclude Section 3 by studying a Fuchsian differential operator of the form (0.11) near points $\rho_\pm = (0,x_o,\tau=\pm 1,\xi=0)$. Again we reduce P to an equivalent Fuchsian system of dimension k. The indicial polynomial of the so obtained system coincides with the classical indicial polynomial as defined in M. Baouendi - C. Goulaouic [6]. We then apply the results of Section 2 for Fuchsian systems to describe microlocally the kernel and the cokernel of P.

In Section 4 we come to the original motivation for this work: the study of propagation of singularities for (pseudo) differential operators with multiple non-involutive characteristics. Let

$$P = P_m + P_{m-1} + \dots \tag{0.14}$$

and for $j=1,2$, we let $q_j(t,x,\tau,\xi) = \tau - \lambda_j(t,x,\xi)$, where $\lambda_j(t,x,\xi)$ is a smooth real function positively homogeneous of degree 1 in ξ. Denote by $\Sigma_j = \{(t,x,\tau,\xi) \mid q_j(t,x,\tau,\xi) = 0\}$ and let $\Sigma = \Sigma_1 \cap \Sigma_2$. We suppose that near Σ

$$p_m(t,x,\tau,\xi) = (q_1(t,x,\tau,\xi))^k \, (q_2(t,x,\tau,\xi))^r \, e(t,x,\tau,\xi) \,, \tag{0.15}$$

where $e(t,x,\tau,\xi)$ is an elliptic symbol homogeneous of degree $m-(k+r)$ in (τ,ξ). Our second assumption is

$$\begin{cases} \text{if } k \geqq 2, P \text{ satisfies the Levi condition with respect to } q_1. \\ \text{if } r \geqq 2, P \text{ satisfies the Levi condition with respect to } q_2. \end{cases} \tag{0.16}$$

The third assumption is that Σ is symplectic, i.e.:

$$\{q_1,q_2\} \neq 0 \quad \text{on } \Sigma \,. \tag{0.17}$$

Under the above assumptions we show that if, e.g., $\{q_1,q_2\}(\rho_o) < 0$, then P is microlocally equivalent near Σ to a Fuchsian hyperbolic operator of order m and weight $m-k$. We can now state the main propagation result. Let $\rho_o \in \Sigma$ and for $j=1,2$, let $\gamma_j(s)$ be the integral curve of the hamiltonian field H_{q_j} such that $\gamma_j(0) = \rho_o$. Denote by $\gamma_j^{\pm} = \{\gamma_j(s) \mid \pm s > 0\}$ the four open half-bicharacteristics through ρ_o.

THEOREM 4. Let P be as in (0.14) and satisfy (0.15), (0.16), (0.17). Let $\rho_o \in \Sigma$ and $\rho_o \notin WF(Pu)$. If for every $j \in \{1,2\}$ there is a choice of the sign + or - such that $\gamma_j^{\pm} \cap WF(u) = \emptyset$, then $\rho_o \notin WF(u)$.

To obtain finer results on propagation of singularities for (0.14) corresponding to parts ii) and iii) of Theorem 3, one needs to know the coefficients of the indicial polynomial. Following the coefficients of the indicial polynomial does not seem to be an easy task due to the complexity of the Levi conditions; we give a complete detailed description in the case $1 \leqq k, r \leqq 2$, which includes the previous results of N. Hanges [13].

1. PRELIMINARIES AND REVIEW OF RESULTS OF N. HANGES

We state here some extensions to the vector-valued situation of results of N. Hanges [13].

To fix our notation let X be an open subset of $\mathbb{R}^n$ and let $\tilde{X} = (-T,T) \times X$ be a neighborhood of $\{0\} \times X$ in $\mathbb{R}^{n+1}$. The points of $T^*\tilde{X}$ will be denoted by (t,x,τ,ξ), $(t,x) \in \tilde{X}$, $(\tau,\xi) \in \mathbb{R}^{n+1}$, and we put $\dot{T}^*\tilde{X} = T^*\tilde{X} \setminus \tilde{X}$. If $i : X \longrightarrow \tilde{X}$ is the canonical immersion and $i^* : T^*_X\tilde{X} \longrightarrow T^*X$ is the associated map, we put $N^*X = (i^*)^{-1}(0)$, the conormal bundle of X in $\tilde{X}$.

If V is an open subset of $\tilde{X}$ (or X), by $L^m(V;p \times q)$, $m \in \mathbb{R}$, $p,q \in \{1,2,\dots\}$, we denote the space of $p \times q$ matrices $A = (A_{ij})$ of classical properly supported pseudo differential operators (p d o's) of order m defined on V (we omit $p \times q$ when $p = q = 1$). If E is a vector space, $E^p, p \in \{1,2,\dots\}$, denotes the product of p copies of E.

We consider now the system

$$(1.1) \qquad Pu = t\,\partial_t I_N\, u - B(x,D_x)u = f$$

where $B \in L^o(X;N \times N)$ and $u,f \in \mathcal{D}'(\tilde{X})^N$ (I_N being the identity matrix of $\mathbb{C}^N$).

To give a meaning to (1.1) we suppose that the vector $u = (u_1,\dots,u_N)$ satisfies the assumption $WF(u) = \bigcup_{j=1}^{N} WF(u_j) \subset \dot{T}^*\tilde{X} \setminus N^*X$.

Let Σ_1 (resp. Σ_2) be the hypersurface of $\dot{T}^*\tilde{X}$ defined by $t = 0$ (resp. $\tau = 0$) and put $\Sigma_o = \Sigma_1 \cap \Sigma_2$, $\Sigma = \Sigma_1 \cup \Sigma_2$,

$$\Sigma_1^\pm = \{(0,x,\tau,\xi) \in \Sigma_1 \mid \pm\tau > 0\}, \ \Sigma_2^\pm = \{(t,x,0,\xi) \in \Sigma_2 \mid \pm t > 0\}.$$

It is well known that $WF(u) \setminus WF(Pu) \subset \Sigma$ and that the structure of $(WF(u) \setminus WF(Pu)) \cap \cap (\Sigma_j \setminus \Sigma_o)$, $j = 1,2$, if $WF(u) \cap N^*X = \emptyset$, is taken care of by the results of Duistermaat - Hörmander [10].

The situation is more complicated near Σ_o , where we expect that a phenomenon of branching of the singularities may appear.

To give a precise description we introduce some relations on $\Sigma \times \Sigma$. If $\rho_o =$ $= (t_o, x_o, \tau^{(o)}, \xi^{(o)}) \in \Sigma_1$ (resp. Σ_2) define:

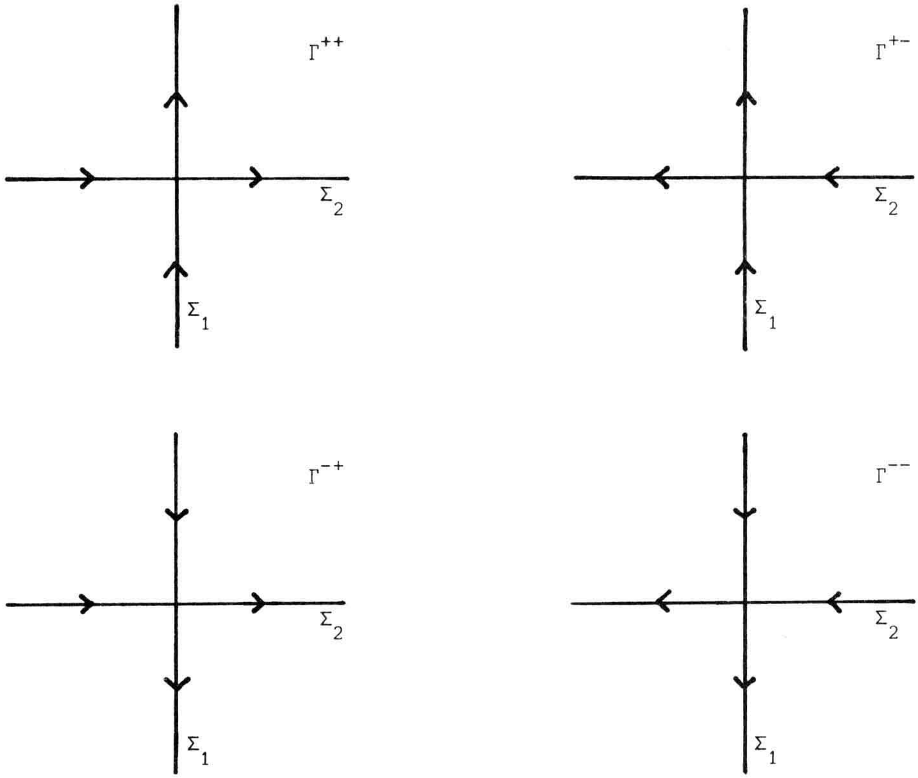
$$
(1.2)\qquad \begin{cases} \gamma_1^{\pm}(\rho_o) = \{(0, x_o, \tau, \xi^{(o)}) \in \Sigma_1 \mid \pm(\tau - \tau^{(o)}) \geqq 0\} \\ \gamma_2^{\pm}(\rho_o) = \{(t, x_o, 0, \xi^{(o)}) \in \Sigma_2 \mid \pm(t - t_o) \geqq 0\} \end{cases}
$$

We now define four relations Γ^{++} , Γ^{+-} , Γ^{-+} , Γ^{--} on $\Sigma \times \Sigma$ depending on the possible orientations of the bicharacteristics. Precisely, denoting by $\pi : \Sigma_j \longrightarrow \Sigma_o$, $j = 1,2$, the canonical projection, we put :

$$
(1.3)\qquad \Gamma^{++} = \begin{cases} (\rho = (t,x,\tau,\xi), \rho' = (s,y,\sigma,\eta)) \in \Sigma \times \Sigma \mid \pi(\rho) = \pi(\rho') = \hat{\rho}, \\ \rho \in \gamma_1^{+}(\rho') \text{ if } \rho' \in \Sigma_1^{+} , \ \rho \in \gamma_1^{+}(\rho') \cup \gamma_2^{+}(\hat{\rho}) \text{ if } \rho' \in \Sigma_1^{-} \cup \Sigma_o ; \\ \rho \in \gamma_2^{+}(\rho') \text{ if } \rho' \in \Sigma_2^{+} , \ \rho \in \gamma_2^{+}(\rho') \cup \gamma_1^{+}(\hat{\rho}) \text{ if } \rho' \in \Sigma_2^{-} \cup \Sigma_o \end{cases}
$$

$$
\Gamma^{+-} = \begin{cases} (\rho = (t,x,\tau,\xi), \rho' = (s,y,\sigma,\eta)) \in \Sigma \times \Sigma \mid \pi(\rho) = \pi(\rho') = \hat{\rho}, \\ \rho \in \gamma_1^{+}(\rho') \text{ if } \rho' \in \Sigma_1^{+} , \ \rho \in \gamma_1^{+}(\rho') \cup \gamma_2^{-}(\hat{\rho}) \text{ if } \rho' \in \Sigma_1^{-} \cup \Sigma_o ; \\ \rho \in \gamma_2^{-}(\rho') \text{ if } \rho' \in \Sigma_2^{-} , \ \rho \in \gamma_2^{-}(\rho') \cup \gamma_1^{+}(\hat{\rho}) \text{ if } \rho' \in \Sigma_2^{+} \cup \Sigma_o \end{cases}
$$

Γ^{-+} and Γ^{--} are defined as Γ^{+-} , Γ^{++} interchanging the roles of Σ_1 and Σ_2.



We have the following crucial result.

PROPOSITION 1.1. Let $\mathscr{R}$ be any one of the four relations $\Gamma^{\pm\pm}$ and let $\rho_o \in \Sigma_o$.

There exist a conic neighborhood V_{ρ_o} of ρ_o with $V_{\rho_o} \cap N^*X = \emptyset$ and a properly supported operator

$$E_{\mathscr{R}} : C_o^\infty(\tilde{X})^N \longrightarrow \mathscr{E}'(\tilde{X})^N$$

such that :