Applications & Science of Neural Networks, Fuzzy Systems, & Evolutionary Computation III

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Applications and Science of Neural Networks, Fuzzy Systems, and Evolutionary Computation III

Bruno Bosacchi
David B. Fogel
James C. Bezdek
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Contents

Invited Paper

THE SOFTEST COMPUTER

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ABSTRACT

Neural networks, fuzzy systems, and evolutionary computation – the technologies featured in this conference – are often referred to collectively as "soft computing." Like "fuzzy logic," this has an unfortunate overtone, but we are stuck with the term. I argue that the intuition that governed this grouping of technologies is that they are all human mimetic. Then I offer an overview of how to construct an artifact that, like you, uses those approaches to exercise judgment and innovate out of unforeseen problems.

Keywords: soft computing, perception, artificial perception, consciousness, artificial consciousness

1. BASIC REQUIREMENTS FOR A THEORY OF MIND

There is no governing body to set these requirements, but I believe most of you will find these a reasonable place to start.

1. Mind must be simple. Einstein's advice applies. Everything must be made as simple as possible, but no simpler. I take this to exclude quantum mechanics, magic, supernatural intervention, *homunculus*, etc. It is not that those things are not possible. Rather, it is that they are to be regarded as last resorts to be abandoned anytime a simple explanation comes along. They are thus unstable.
2. Mind must be evolvable. As a product of evolution to which animals devote much energy and space, mind must serve a purpose. It must arise in a series of small steps each one of which conferred its own survival value to the animal inheriting it.
3. Mind must be buildable. If I cannot tell you how to build it, what right have I to assert I understand it?
4.

Those requirements exclude most prior approaches to mind, but not all. This work benefits from work by scientists like Walter Freeman, and Timo Jarvilehto, and even a few philosophers such as Daniel Dennett and John Searle. None of them would accept responsibility for this, but all of them would find much with which to agree.

2. PERCEPTION: A WARM-UP PROBLEM

For those who have heard or read some of my prior works in this area, I offer a different approach here. I will refer to the work of two geniuses now at work on mending minds and bodies- Paul Bach-y-Rita and V. S. Ramachandran. So far as I can tell, this is the first place where their works on perception have bee put into a single context – although they know and respect each other and each other's work. Both are experimental pioneers of human perception. Bach-y-Rita has concentrated on our perception of the outer world. Ramachandran has concentrated on our perception of our bodies.

To those who have not thought analytically about their perception of the outside world, the motto "Seeing is believing" makes sense. If I see it, it is real. This has been characatured as the doctrine of "Immaculate Perception." And it is not only wrong, but also exactly the opposite of the truth. A more accurate motto would be "Believing is seeing." We see what we believe is there in view of our expectations, concerns, memories, understanding, and sense data. We see a model of the world formed in our brains and designed not for accuracy but for utility. There is no way that you can actually perceive something out there. But your brain can solve a kind of inverse problem to model what is out there and you perceive that model. Space does not permit a full defense of that assertion here, but most readers will have figured that out for themselves long ago.

So where does vision occur? Not in the eye, as was once thought, but in the mind. So vision should be possible without eyes! In fact, Bach-y-Rita has given vision to people with no eyes! All they need is to get the relevant information into their brains, and almost any means will do. Using a small he4ad-mounted camera to gather data, he has inserted it through the skin via the abdomen, the back, and (most recently) the tongue (1). It really does not seem to matter. With practice, the mind automatically correlates this new information with the information it already knows how to obtain by touch and begins to see things "out there" in the world without the benefit of eyes. The patient constructs and perceives a mental model of the world.

But you also perceive your own body. Your brain has two body maps – one for sensing and one for acting. Like everything else in your brain, those models are interacting – what Edelman likes to call reentrant communication. Just as you can have visual illusions of the outside world, you can mismodel your body. Ramachandran was drawn to this field by the plight of amputees who, more often than not, experience the missing limb long after it is removed. His book on this is good reading (2). In it he goes far beyond phantom limbs. In fact he shows how to induce people into experiencing foreign objects as parts of themselves. How do you do that? The same way Bach-y-Rita's patients do. They cause a correlation between a familiar stimulus and the one they want the patient to perceive. The mind takes the new information into its model automatically when there is such a good correlation.

I wish I had the space to deal with this fully here, as it is a fascinating topic. But, alas, it is only a warm-up for the next topic, so I will quit here. Perhaps some of you will ask questions of me or of the real expert – you.

3. AN OUTLINE OF ARTIFICIAL ATTENTION

I hypothesize that the brain is continually updating both models – body and world. There is lots going on in both realms. Consciousness involves massive computation (the nature of which I have not yet described) on a very slow chemical-mechanical computer employing neurons, glial cells, and flowing chemicals. Yet it must be done quickly or you will not evade the predator or find a good mate. One of the many "tricks" nature employs is to filter the potential contents of consciousness. This is called attention. In performing a task, you generally attend to the information you need to perform it, but not always. Here are some circumstances that warrant notice:

1. You may be given more information than you can consciously attend. System designers make that mistake, because they assume a perfect human operator. When confronted with too much information, the attentional system simply ignores some of it. You do not perceive everything presented. An infamous case of such a situation caused an American sailor to shoot down an unarmed civilian Iranian airline. He had enough information displayed, but he attended only some of it. He never saw the information that would have prevented that tragedy.
2. You may consciously choose to attend to something. You can attend to your model of the thumb on your left hand very easily. In fact, you just did.
3. You usually attend to the unexpected. This brings us back to the correlation between sensory inputs noted earlier. If they do not correlate with our expectations, we attend them. But we "zero out" predicted events, e.g. the loud beating of the clock in your bedroom. You do not notice it, because you expect it. Once again, there is much too much to write in too little space. If you doubt that you attend to unpredicted events differently from predicted ones, try tickling yourself. You cannot do it, because your movements are predictable. But I could tickle you, because you are not predicting my movements in detail. Another out-of-norm experience is pain. You would notice your thumb if I struck it with a hammer.

Now consider the situation in which the system designer wants a human operator involved, but is sensible enough to want to avoid overloading his perceptual bandwidth. What can he do? He can implement what I call "Artificial Perception." That is an artificial filtering system between the operating system and the human that controls what is displayed to the human user. Under normal circumstances, it will display the normal data the operator needs automatically. But, as in human attention, the operator can choose to attend to anything and out-of-norm things will be brought to the operator's attention. Of course, it is not quite that simple. Sometimes there are conflicting goals, so choices must be made for the operator. Human attention does the same thing. A wonderful illustration is the attention of a soldier wounded in battle. Many other things seem even more important to his attentional system than the wound. So soldiers often report that they felt no pain from quite hideous wounds as they were in active advance or retreat. When the battle subsides, the soldier sudd3enly feels the pain.

I argue that Artificial Perception (using what I have said and many things I have no space to tell) is the ideal interface between a human and a complex system.

4. CONSCIOUSNESS

I only have space for a cursory survey of consciousness and I will do so in terms of the criteria suggested above. Let me go right to the purpose. Consciousness is the only known solution to a survival-critical problem: How do I build a system that has a good chance of dealing effectively not only with situations its ancestors have seen (through inherited behavior traits, instincts, etc.) and things the system has encountered in its "lifetime" (through learning) but also situations totally new to the system/organism and its ancestors? I include a wiring diagram for rudimentary consciousness at the end of this paper. Nature evolved consciousness bottom-up in this diagram. The last items are the most critical. *The role of consciousness is to take care of the model body in its model world as judged by its limbic system.* I will not go into details of the evolution or into such things as human consciousness (enriched tremendously by language and culture) or self consciousness. The kind of consciousness outline is likely present in salamanders. It is primitive, but real.

5. AN OUTLINE OF ARTIFICIAL CONSCIOUSNESS

To make a complex system conscious is not very difficult. We must provide it with a system (body) model and the sensory information to keep that model updated. Likewise for its situation or world model. Then we must provide it with means to evaluate its current and likely future states – a limbic system. We then give it a chooser module (a self), that uses the models and stored information to react in such a way as to optimize the likely good of the system. For those situations in which no stored responses are likely to be successful, we must provide it with ways to evolve new responses with better likely outcomes. Humans interact with conscious artifacts by changing their limbic systems. *When they want what you want, they become your conscious extensions.*

6. BACK TO SOFT COMPUTING

The Artificial Perception and Artificial Consciousness systems are, by extension of the definition given earlier soft computing systems. They mimic human computational processes. But instead of being useful tools (as are fuzzy, neural, and evolutionary methods), these are systems. In a sense, they are higher-order soft computing methods. Within them there will likely be conventional soft computing modules.

1. Fuzzy and neural methods [illegible] for the modeling. It is not important that the models use the same underlying methods as the realities modeled. It is more important that they be approximately correct most of the time and run very fast. To evolve new behaviors with any kind of reasonable speed, we need to exercise the models very rapidly.
2. Evolutionary methods (again accelerated for "real time" response) will be necessary for evolving new behaviors when it is determined that stored behaviors are likely to be inadequate.

So we have soft components within soft systems leading to new kinds of controls for complex systems. These controls combine the best capabilities of conventional controls with conscious systems and human intervention through the system's limbic system on the basis of the information the human operator draws from his Artificial Perception interface.

7. REFERENCES

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7. ACKNOWLEDGMENTS

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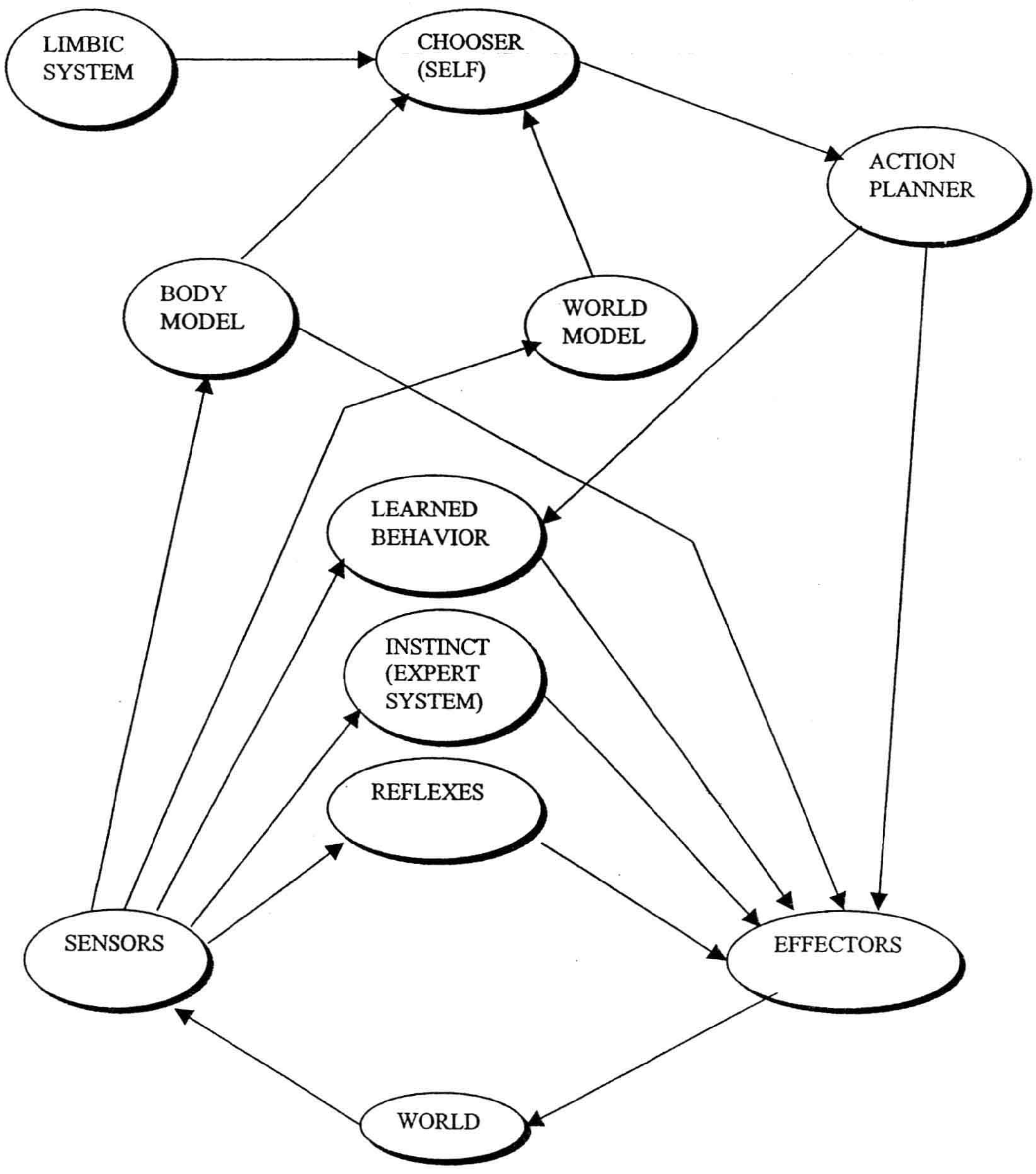
8. CONSCIOUS SYSTEM DRAWING



A Combination of an Autoassocative Morphological Memory and the Kernel Method

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ABSTRACT

We recently introduced a class of highly nonlinear associative memories called morphological associative memories (MAMs). Notable features of autoassociative morphological memories (AMMs) include optimal absolute storage capacity and one-step convergence. The fixed points can be characterized exactly in terms of the original patterns. Unfortunately, AMM fixed points include a large number of spurious memories.

In this paper, we use a combination of a basic AMM model and the kernel method in order to eliminate most of the spurious memories while leaving other AMM properties intact. Furthermore, our new AMM model is more tolerant to noise than a basic AMM model and less dependent on kernel selection than the original kernel method.

Keywords: Morphological associative memories, autoassociative morphological memories, fixed points, spurious memories, kernel method

1. INTRODUCTION

Morphological neural networks have been devised as a type of neural networks which is based on the mathematical theory of minimax algebra.[1,9] Thus, they are drastically different from "traditional" neural network models which perform linear operations followed by applications of nonlinear activation functions. Morphological neural networks were initially applied to a number of special problems.[2,13,14] A fairly comprehensive and rigorous basis for computing with morphological neural networks appeared in a paper by Ritter and Sussner.[9]

One of the first goals achieved in the development of morphological neural networks was the establishment of a morphological associative memory network (MAM). The basic recording strategies for MAMs are similar to correlation recording.[4] We have shown that MAMs exhibit many desirable characteristics.[10] We proved for example that the autoassociative morphological memory (AMM) does not impose any restrictions on the number of pattern associations to be encoded in the memory. Thus 2^n patterns can be stored in the binary case, where n is the length of memory. In comparison, McEliece et al. showed that the absolute (asymptotic) capacity of the Hopfield associative memory is $\frac{n}{2\log n}$ if with high probability the unique fundamental memory is to be recovered, except for a vanishingly small fraction of fundamental memories.[6] Evidently, the information storage capacity (number of bits which can be stored and recalled associatively) of the AMM also exceeds the respective number for certain linear matrix associative memories which was calculated by Palm.[8] Furthermore, the AMM converges in one step, i.e. every output pattern obtained after one application of the AMM is a fixed point or stable state of the memory.

In a recent paper, we succeeded in characterizing the fixed points of binary AMMs as well as the corresponding basins of attraction.[11] Our latest results on the fixed points of AMMs confirm our claim that the min-product version of the AMM is robust under erosive changes of the key patterns whereas the max-product version is robust under dilative changes of the key patterns. The kernel method, which exhibits a relatively small number of fixed points,

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has been used to deal with arbitrarily corrupted input patterns.[10] The kernel method's major drawback is the fact, that it is heavily dependent on the choice of kernel vectors.

In this paper, we present a new MAM model for autoassociation which consists of a combination of a basic AMM and the kernel method. Our new AMM model exhibits better error correction capabilities than the basic model and less dependence on kernel selection than the kernel method.

2. Computational Basis for Morphological Neural Networks

Most of the artificial neural networks which are currently in use specify functions composed of linear operations and nonlinear activation functions such as thresholding and sigmoidal functions. The underlying algebraic system used in these models is the *ring* $(\mathbb{R}, +, \cdot)$.

Morphological neural network computations are based on a certain algebraic lattice structure and consist of adding neural values and their synaptic strengths followed by forming the maximum ($\vee$) or, dually, the minimum ($\wedge$) of the results. For the purposes of this paper, it suffices to consider operations in the substructure $(\mathbb{R}, \vee, \wedge, +)$.

Several popular models of associative memories allow for a formulation in terms of matrices.[4,3,7,5] The model of associative memories described in this paper employs products of matrices which are defined in the algebraic structure of minimax algebra. Two types of matrix products exist in minimax algebra. For an $m \times p$ matrix A and a $p \times n$ matrix B with entries from $\mathbb{R}$, the matrix $C = A \boxed{\vee} B$, also called the *max product* of A and B, and the matrix $D = A \boxed{\wedge} B$, also called the *min product* of A and B, are defined by

$$c_{ij} = \bigvee_{k=1}^{p} (a_{ik} + b_{kj}), \quad d_{ij} = \bigwedge_{k=1}^{p} (a_{ik} + b_{kj}). \tag{1}$$

The algebraic structure $(\mathbb{R}, \vee, \wedge, +)$ provides for an elegant duality between matrix operations. For any real-valued $m \times n$ matrix A, we define the *conjugate matrix* A^* of A as the $n \times m$ matrix $A^* = -A'$, where A' is the transpose of A. It follows that

$$(A \wedge B)^* = A^* \vee B^* \text{ and } (A \boxed{\wedge} B)^* = B^* \boxed{\vee} A^* \tag{2}$$

The maximum and the minimum of two matrices occurring in Equation 2 are performed elementwise. Equation 2 implies that every statement about morphological neural networks induces a dual statement which simply arises by replacing each $\wedge$ symbol with a $\vee$ symbol and vice versa, and by reversing each inequality. As an example, the reader may find two true statements of minimax algebra in Equation 3 and the corresponding dual statements in Equation 4 below.

$$(A \boxed{\vee} B) \vee (A \boxed{\vee} C) = A \boxed{\vee} (B \vee C), \; A \boxed{\vee} (B \wedge C) \leq (A \boxed{\vee} B) \wedge (A \boxed{\vee} C) \quad \forall A \in \mathbb{R}^{m \times n}, \forall B, C \in \mathbb{R}^{n \times k}. \tag{3}$$

$$(A \boxed{\wedge} B) \wedge (A \boxed{\wedge} C) = A \boxed{\wedge} (B \wedge C), \; A \boxed{\wedge} (B \vee C) \geq (A \boxed{\wedge} B) \vee (A \boxed{\wedge} C) \quad \forall A \in \mathbb{R}^{m \times n}, \forall B, C \in \mathbb{R}^{n \times k}. \tag{4}$$

3. INTRODUCTION TO MORPHOLOGICAL ASSOCIATIVE MEMORIES

An associative memory is an input-output system that describes a relation $R \subseteq \mathbb{R}^m \times \mathbb{R}^n$. If $(\mathbf{x}, \mathbf{y}) \in R$, i.e. if the input $\mathbf{x}$ produces the output $\mathbf{y}$, then the associative memory is said to *store* or *record* the *memory association* $(\mathbf{x}, \mathbf{y})$. There are two basic approaches to record k vector pairs $(\mathbf{x}^1, \mathbf{y}^1), \ldots, (\mathbf{x}^k, \mathbf{y}^k)$, using a *morphological associative memory*. The first approach consists of constructing an $m \times n$ matrix W_{XY} as $W_{XY} = Y \boxed{\wedge} X^*$. The second, dual approach consists of constructing an $m \times n$ matrix M_{XY} of the form $M_{XY} = Y \boxed{\vee} X^*$. If the matrix W_{XY} receives a vector $\mathbf{x}$ as input, the product $W_{XY} \boxed{\vee} \mathbf{x}$ is formed. Dually, if the matrix M_{XY} receives a vector $\mathbf{x}$ as input, the product $M_{XY} \boxed{\wedge} \mathbf{x}$ is formed. As the reader may have already noticed, there are surprising similarities between the morphological associative memories described above and the *linear associative memory*.[4]

If $X = Y$ (i.e., $\mathbf{y}^\xi = \mathbf{x}^\xi$, for $\xi = 1, \ldots, k$), we obtain the *autoassociative morphological memories* (AMMs) W_{XX} and M_{XX}. In this special case, the recording scheme incorporated into W_{XX}, M_{XX} respectively, reminds of the recording scheme used for the synthesis of a Hopfield net.[3]

Example.

$$X = \begin{pmatrix} 1 & 3 \\ 0 & -1 \\ 4 & 1 \end{pmatrix}, \; W_{XX} = \begin{pmatrix} 0 & 1 & -3 \\ -4 & 0 & -4 \\ -2 & 2 & 0 \end{pmatrix}. \; W_{XX} \boxed{\vee} X = X = M_{XX} \boxed{\wedge} X \, . \tag{5}$$

Two fundamental theorems on MAMs have been established which are concerned with necessary and sufficient conditions for perfect recall of uncorrupted and corrupted patterns. The first theorem answers the existence question of perfect recall for sets of pattern pairs. The second theorem provides bounds for the amount of distortion of the exemplar patterns $\mathbf{x}^\xi$ for which perfect recall can be assured. Theorems 1 and 2 represent an important corollaries concerning autoassociative morphological memories which are far easier to state than the original theorems.

Theorem 1. The following statement holds for all $X \in \mathbb{R}^n$:

$$W_{XX} \boxed{\vee} X = X \text{ and } M_{XX} \boxed{\wedge} X = X \, . \tag{6}$$

In other words, the absolute storage capacity of AMMs is unlimited.

Definition. We say that distorted version $\tilde{\mathbf{x}}$ of the pattern $\mathbf{x}$ has undergone an ***erosive change*** or that $\tilde{\mathbf{x}}$ is an ***eroded version*** of $\mathbf{x}$ whenever $\tilde{\mathbf{x}} \leq \mathbf{x}$ and a *dilative change* or that $\tilde{\mathbf{x}}$ is a ***dilated version*** of $\mathbf{x}$ whenever $\tilde{\mathbf{x}} \geq \mathbf{x}$.

Theorem 2. If $W_{XX} \boxed{\vee} \tilde{\mathbf{x}}^\gamma = \mathbf{x}^\gamma$ then $\tilde{\mathbf{x}}^\gamma$ must be an eroded version of $\mathbf{x}^\gamma$. Similarly, if $M_{XX} \boxed{\wedge} \tilde{\mathbf{x}}^\gamma = \mathbf{x}^\gamma$ then $\tilde{\mathbf{x}}^\gamma$ must be a dilated version of $\mathbf{x}^\gamma$.

Suppose that $\mathbf{x}^1, \ldots, \mathbf{x}^k$ are "well-structured" patterns with k relatively small compared to n. We recently made the observation that W_{XX} is very robust in recalling patterns that are distorted by erosive noise and that M_{XX} is very robust in recalling patterns that are distorted by dilative noise.[12]

Example. Let X be the matrix whose columns correspond to the letter images of Figure 1. Specifically, consider the ten images $\mathbf{p}^1, \ldots, \mathbf{p}^{10}$ shown in Figure 1 (numbered from left to right and from top to bottom). Using the standard row-scan method, each pattern image $\mathbf{p}^\xi$ can be converted into a pattern vector $\mathbf{x}^\xi = \left(x_1^\xi, \ldots, x_{324}^\xi\right)$ by defining

$$x^\xi_{18(i-1)+j} = \begin{cases} 1 & if \;\; \mathbf{p}^\xi(i,j) = 1 \quad (= black\ pixel) \\ 0 & if \;\; \mathbf{p}^\xi(i,j) = 0 \quad (= white\ pixel) \, . \end{cases}$$

We used the ten pattern vectors $(\mathbf{x}^1, \ldots, \mathbf{x}^{10})$ in constructing the morphological memories W_{XX} and M_{XX}. As expected each individual pattern vector $\mathbf{x}^\xi$ was perfectly recalled in a single application of either W_{XX} or M_{XX}. The same ten patterns served as bipolar inputs to a discrete Hopfield net. Due to the considerable amount of overlap between the patterns, the Hopfield net failed to recall any of the patterns.

Figure 1.
The ten patterns used in constructing W_{XX} and M_{XX}. The output of W_{XX} and the output of M_{XX} is identical to the input pattern.

We conducted the experiments described in Figure 2 and in Table 1. We created eroded versions $\tilde{\mathbf{x}}^\gamma$ of some $\mathbf{x}^\gamma$, computed $W_{XX} \boxed{\vee} \tilde{\mathbf{x}}^\gamma$, and compared the results with $\mathbf{x}^\gamma$.

Figure 2.
Examples of corrupted input patterns which were perfectly recalled as the letter image "X" by the morphological memory W_{XX} (X consists of the patterns shown in Figure 1).

letter	"A"	"a"	"B"	"b"	"C"	"c"	"X"	"x"	"E"	"e"
$p = 0.1$	100	80	100	99	100	89	100	90	100	96
$p = 0.2$	96	63	100	97	100	75	96	81	100	92
$p = 0.3$	92	42	100	94	100	64	87	69	99	88

Table 2:
Experimental results after random elimination of black pixels with probablity p. The table shows the number of images in a sample of 100 images which were perfectly recalled in an application of W_{XX}.

4. REVIEW OF THE KERNEL METHOD

The previous section indicates that morphological associative memories outperform "conventional" associative memories such as correlation recording and the Hopfield net in many aspects. In fact, the morphological associative memory model reveals almost all the desired general characteristics of an associative memory with one notable exception: the network's inability to deal with patterns which include erosive and dilative noise at the same time.

The memory W_{XX} is suitable for application to patterns containing erosive noise and the memory M_{XX} is suitable for application to patterns containing dilative noise. Therefore, an intuitive idea to process noisy versions $\tilde{\mathbf{x}}^\gamma$ of $\mathbf{x}^\gamma$ containing both erosive and dilative changes is to use a combination of W_{XX} and M_{XX}. Specifically, the output of $M_{XX} \boxed{\wedge}\, \tilde{\mathbf{x}}^\gamma$ is multiplied (using $\boxed{\vee}$) by W_{XX} or, dually, the output of $W_{XX} \boxed{\vee}\, \tilde{\mathbf{x}}^\gamma$ is multiplied (using $\boxed{\wedge}$) by M_{XX}. The reasons why this approach fails will become evident in view of the results on fixed points and their basins of attraction which we provide in the next section.

Inspite of these difficulties, a modified approach can be applied to create a morphological associative recall memory that is robust in the presence of random noise (i.e. both dilative and erosive noise), even in the general situation when $X \neq Y$ and X and Y are not boolean. A memory M is defined as a memory which associates each input pattern $\mathbf{x}^\xi$ with an intermediate pattern $\mathbf{z}^\xi$. Furthermore, a memory W is defined such that each pattern $\mathbf{z}^\xi$ is associated with the corresponding output pattern $\mathbf{y}^\xi$. In other words, we obtain the following equations:

$$W \boxed{\vee} (M \boxed{\wedge} \mathbf{x}^\xi) = W \boxed{\vee} \mathbf{z}^\xi = \mathbf{y}^\xi \tag{7}$$

Under certain conditions depending on Z, the matrix M_{ZZ} can serve as M and the matrix W_{ZY} can serve as W. In this case, the $n \times k$ matrix $Z = (\mathbf{z}^1, \mathbf{z}^2, \ldots, \mathbf{z}^k)$ is called a *kernel* for (X, Y). Furthermore, for properly chosen vectors $\mathbf{z}^\xi$, we also have $M_{ZZ} \boxed{\wedge}\, \tilde{\mathbf{x}}^\xi = \mathbf{z}^\xi$ for most corrupted versions $\tilde{\mathbf{x}}^\xi$ of $\mathbf{x}^\xi$.

For all practical purposes, the vectors $\mathbf{z}^\xi$ can be thought of as sparse representations of $\mathbf{x}^\xi$, i.e. $\mathbf{z}^\xi$ are binary vectors such that $\mathbf{z}^\xi \leq \mathbf{z}^\xi$ and such that $\sum_{i=1}^n z_i^\xi$ is small. Let us recall the formal definition of a kernel below.

Definition. An $n \times k$ matrix $Z = (\mathbf{z}^1, \mathbf{z}^2, \ldots, \mathbf{z}^k)$ is called a *kernel* for (X, Y) if and only if the following two conditions are satisfied:

1. $M_{ZZ} \boxed{\wedge} X = Z$.
2. $W_{ZY} \boxed{\vee} Z = Y$.

Given a particular problem of associating binary patterns $\mathbf{x}^\xi$ with binary pattern $\mathbf{y}^\xi$, the kernel with respect to (X, Y) can be easily chosen by means of Theorem 3 below. Given a collection of kernel vectors whose choice is based on Theorem 3. The subsequent theorem determines the amount of corruption of the input patterns which is admissible in order to maintain perfect recall. In other words, given a corrupted version $\tilde{\mathbf{x}}^\gamma$ of $\mathbf{x}^\gamma$, under what conditions is

$$W_{ZY} \boxvee (M_{ZZ} \boxwedge \tilde{\mathbf{x}}^\gamma) = \mathbf{y}^\gamma. \tag{8}$$

Theorem 3. Let X, Y, Z be binary patterns with $Z \leq X$. If

$$\mathbf{z}^\gamma \wedge \mathbf{z}^\xi = \mathbf{0} \text{ and } \mathbf{z}^\gamma \not\leq \mathbf{x}^\xi \;\; \forall \gamma, \xi, \gamma \neq \xi \tag{9}$$

then Z is a kernel for (X, Y).

Theorem 4. If Z is a kernel for (X, Y) satisfying the conditions 9 then we have perfect recall for a distorted version $\tilde{\mathbf{x}}^\gamma$ of $\mathbf{x}^\gamma$, in other words $W_{ZY} \boxvee (M_{ZZ} \boxwedge \tilde{\mathbf{x}}^\gamma) = \mathbf{y}^\gamma$, if

[illegible] (10)

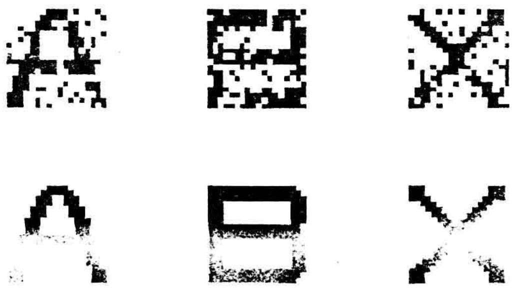

Figure 3.
An application of the kernel method. Each letter was corrupted by randomly reversing each bit with a probability of 0.15. The associative memory $\{input \to M_{ZZ} \to W_{ZY} \to output\}$ was trained using the ten exemplars shown in Figure 1. Presenting the memory with the corrupted patterns of the letters A, B, and X resulted in perfect recall (lower row).

5. FIXED POINTS O[illegible] AMM's A[illegible] THEIR [illegible]S OF ATTRACTION

One of the desired characteristics of an autoassociative memory is recall of the original patterns. In other words, every original pattern $\mathbf{x}^\xi$ should be a fixed point for every given set of patterns $\mathbf{x}^1, \ldots, \mathbf{x}^k$ with respect to a renewed application of the memory. Fixed points of AMMs are defined as follows.

Definition. A vector $\mathbf{x} \in \mathbb{R}^n$ is called a *fixed point* of W_{XX} if and only if $W_{XX} \boxvee \mathbf{x} = \mathbf{x}$. Similarly, $\mathbf{x}$ is a fixed point of M_{XX} if and only if $M_{XX} \boxwedge \mathbf{x} = \mathbf{x}$.

Ideally, an autoassociative memory only has the fundamental memories $\mathbf{x}^1, \ldots, \mathbf{x}^k$ and possibly a no-decision or ground state as fixed points. Let $S(W_{XX})$ denote the set of fixed points of W_{XX} and let $S(M_{XX})$ denote the set of fixed points of M_{XX}. By Theorem 1, both $S(W_{XX})$ and $S(M_{XX})$ include the original patterns. Experimentation reveals that apart from the fundamental memories various other fixed points exist.

The question arises as to what are the other elements of $S(W_{XX})$ and $S(M_{XX})$. The following theorem[10] provides some insight into this matter.

Theorem 5. For every $\mathbf{z} \in \mathbb{R}^n$, the pattern $W_{XX} \boxvee \mathbf{z}$ is a fixed point of W_{XX} and the pattern $M_{XX} \boxvee \mathbf{z}$ is a fixed point of M_{XX}

By Theorem 5, the set of all stable states can be found by probing W_{XX} and M_{XX} with every $\mathbf{z} \in \mathbb{R}^n$. Of course, this is a daunting task - even for the example above. Nevertheless, this method helped us to create the fixed points $\mathbf{y}^1$, $\mathbf{y}^2$, and $\mathbf{y}^3$ below.

Figure 4.
Images corresponding to fixed points of W_{XX} and M_{XX}. Note that the first image represents the maximum of "**X**" and "**a**".

Upon closer examination, the reader may detect the identities $\mathbf{y}^1 = \mathbf{x}^4 \vee \mathbf{x}^6$, $\mathbf{y}^2 = \mathbf{x}^1 \wedge \mathbf{x}^2$, and $\mathbf{y}^3 = (\mathbf{x}^3 \wedge \mathbf{x}^7) \vee \mathbf{x}^9$. Further experimentation leads to the following hypothesis. A pattern $\mathbf{y}$ is a fixed point of W_{XX} if and only if $\mathbf{y} = \mathbf{1}$ or if $\mathbf{y}$ can be written as an expression involving only $\mathbf{x}^1, \dots, \mathbf{x}^{10}$ and the symbols $\vee$, $\wedge$. Theorem 6 will confirm this hypothesis. Note that the condition $W_{XX} \in \{-1, 0\}^{n \times n}$ and $M_{XX} \in \{0, 1\}^{n \times n}$ is not stringent. The proof of sufficiency part of Theorem 6 is given in.[11] The proof of the other direction will appear in an upcoming article.

Definition. An expression involving only $\mathbf{x}^1, \dots, \mathbf{x}^k$ and the symbols $\vee$, $\wedge$ is called a *lattice polynomial* in $\mathbf{x}^1, \dots, \mathbf{x}^k$.

Theorem 6. Let $X = \{\mathbf{x}^1, \dots, \mathbf{x}^k\} \in \{0, 1\}^{n \times k}$. Suppose $W_{XX} \in \{-1, 0\}^{n \times n}$ and $M_{XX} \in \{0, 1\}^{n \times n}$. A pattern $\mathbf{y}$ is a fixed point of W_{XX} if and only if $\mathbf{y}$ is a lattice polynomial in $\mathbf{x}^1, \dots, \mathbf{x}^k$ or $\mathbf{y} = \mathbf{1}$. A pattern $\mathbf{z}$ is a fixed point of M_{XX} if and only if $\mathbf{z}$ is a lattice polynomial in $\mathbf{x}^1, \dots, \mathbf{x}^k$ or $\mathbf{z} = \mathbf{0}$.

The next theorem expresses the following fact: An application of W_{XX} to an arbitrary input pattern $\mathbf{x}$ produces the fixed point $\mathbf{y}$ which is closest to $\mathbf{x}$ in terms of a certain measure of distance. Let us first introduce some convenient notations.

For arbitrary $\mathbf{x} \in \mathbb{R}^n$, the symbol $\hat{\mathbf{x}}$ denotes the least upper bound ($l.u.b$) of $\mathbf{x}$ in $S(W_{XX}) \cup \{\mathbf{1}\}$, i.e. the smallest element $\mathbf{y}$ of $S(W_{XX}) \cup \{\mathbf{1}\}$ such that $\mathbf{y} \geq \mathbf{x}$. Similarly, $\check{\mathbf{x}}$ denotes the greatest lower bound ($g.l.b$) of $\mathbf{x}$ in $S(M_{XX}) \cup \{\mathbf{0}\}$, i.e. the largest element $\mathbf{z}$ of $S(M_{XX}) \cup \{\mathbf{0}\}$ such that $\mathbf{z} \leq \mathbf{x}$.

Theorem 7. The following equalities are satisfied for all $X \in \mathbb{R}^{n \times k}$ and $\mathbf{x} \in \mathbb{R}^n$.

$$W_{XX} \boxvee \mathbf{x} = \hat{\mathbf{x}} \ \text{ and } \ M_{XX} \boxwedge \mathbf{x} = \check{\mathbf{x}} \,. \tag{11}$$

In other words, $\hat{\mathbf{x}}$ acts as an attractor state for $\mathbf{x}$ when using W_{XX} and $\check{\mathbf{x}}$ acts as an attractor state for $\mathbf{x}$ when using M_{XX}.

Proof. The matrix W_{XX} has a zero diagonal by construction. Therefore, $W_{XX} \boxvee \mathbf{x} \geq \mathbf{x}$ for all $\mathbf{x}$. Since $\mathbf{x} \leq \hat{\mathbf{x}}$,

$$\mathbf{x} \leq W_{XX} \boxvee \mathbf{x} \leq W_{XX} \boxvee \hat{\mathbf{x}} = \hat{\mathbf{x}} \,. \tag{12}$$

The pattern $W_{XX} \boxvee \mathbf{x}$ is a fixed point of W_{XX} according to Theorem 5. Since $\hat{\mathbf{x}}$ is the smallest fixed point greater than or equal to $\mathbf{x}$, $W_{XX} \boxvee \mathbf{x}$ must be $\hat{\mathbf{x}}$.

6. ELIMINATION OF SPURIOUS MEMORIES

In the previous section, we established necessary and sufficient conditions for fixed points of W_{XX} and M_{XX}. Moreover, for every fixed point, we identified the corresponding basin of attraction. We characterized the set of fixed points of W_{XX}, M_{XX} respectively, as the set of lattice polynomials in $\mathbf{x}^1, \dots, \mathbf{x}^k$. Note that every lattice polinomial in $\mathbf{x}^1, \dots, \mathbf{x}^k$ can be written in conjuntive normal form, i.e. as an expression of the form