

**HANDBOOK
OF
STATISTICAL TABLES**

HANDBOOK OF STATISTICAL TABLES

by

✓
D. B. OWEN

Sandia Corporation



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PREFACE

This book of tables is intended for three audiences: the student in statistics who needs some readily accessible tables to be used in conjunction with his courses in statistics; the practicing statistician, quality control man, or industrial engineer who wants a set of tables from which he can obtain answers with a minimum of interpolation and other calculation; and the research worker who will find in this collection many functions more extensively tabulated than ever before.

In writing the description of the tables, the temptation was strong to write too much. It was decided that the best procedure was to keep the examples to a minimum and, wherever possible, to try to pick unusual examples, unusual in the sense that the particular applications are not widely shown in textbooks of today. However, some examples were included merely to define certain parameters in terms of the quantities tabulated, and not to expound on the examples *per se*. Unless stated otherwise, when a sample is considered, it is to be assumed that the sample has been taken at random from the population, and that the observations are statistically independent of each other.

Concerning the selection of tables for this book, the choice has been dictated largely by two considerations: (a) amount of space taken by the table *vs.* its usefulness, and (b) a desire to make this compilation all-inclusive enough so that it can be used as a supplementary handbook for most courses in statistics, and so that at the same time it would contain as many as possible of the unusual tables of merit which are not given in other compilations now on the market. For example, short tables of the binomial and Poisson distributions have been included. The reader should not infer, however, that the Wilcoxon (Mann-Whitney) two-sample distribution (Sections 11.2 through 11.5), which covers many pages here, is more important than the Poisson distribution, which is represented by very few pages (Sections 9.3 and 9.4). The Poisson distribution has tables which are widely distributed, while the Wilcoxon (Mann-Whitney) distribution tables are not nearly as well distributed as they should be, considering their usefulness.

The references have been chosen either on the basis of special usefulness or, more often, on the basis that they appeared in the more recent literature, and referred back to the older literature. Hence, the research worker could, if he so desired, start with these current references and work back through the bibliographies to the older papers. In other words, a current paper was often chosen over an older paper, even though the older paper

PREFACE

may have been more pertinent to the example under discussion. The descriptions of the tables generally follow the tone and style set in the work from which the table was taken.

The tables have been reproduced directly from the output of digital computers wherever possible. This, of course, did not allow the flexibility in set-up of the tables that is possible with typescript, but it undoubtedly eliminated many transcription errors. The tables were checked against as many different sources as possible, and discrepancies were resolved wherever possible. In some cases, this was an agonizing process when tables which we thought were accurate were found to contain errors, and each error had to be resolved. Some tables were taken from journals and other sources. These are indicated by a number in square brackets following the table on the first page of each table. The number corresponds to an entry in the bibliography. We wish to express our appreciation to the authors, editors, and publishers who gave their permission to copy tables.

Many individuals helped with suggestions on this set of tables. If we tried to account for all of them, we would surely inadvertently leave someone out. Nevertheless, we will mention a few individuals whose suggestions have been most helpful: Dr. P. Olmstead, Professor Harold Ruben, Dr. L. H. Koopmans, Professor J. Rosenblatt, Professor W. Kruskal, Professor Z. W. Birnbaum, Dr. G. P. Steck, Dr. J. R. Blum, Dr. Marvin Zelen, and Dr. Churchill Eisenhart. Most of these people saw only a table of contents or a particular section of the book, and made their suggestions based on these. Any omissions or errors are clearly the author's responsibility. In addition, we wish to mention Mrs. Marjorie E. Endres who, over a two-year period, carried out many of the computations and checks, and supervised the work of a group of data reduction workers who prepared the tables. She has remained helpful and cheerful throughout the long periods of checking and rechecking that were applied to these tables.

The author, however, acknowledges the responsibility for the accuracy and extent of the tables. The author made the decisions, always, about what should be done and how it should be done.

Albuquerque, New Mexico
January, 1962

D. B. O.

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1. NORMAL DISTRIBUTION

1.1 The Normal Distribution and Related Functions

The various quantities given in this table are defined as follows:

$P(X)$ = the cumulative distribution function of a standardized normal random variable (a standardized normal random variable will be abbreviated as a random variable with an $N(0, 1)$ distribution)
 = $\Pr \{ \text{a normal random variable with mean zero and standard deviation one is less than or equal to } X \}$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^X \exp(-t^2/2) dt.$$

$Z(X)$ = the probability density of a standardized normal random variable

= the derivative of $P(X)$ with respect to X

$$= \frac{1}{\sqrt{2\pi}} \exp(-X^2/2).$$

$$Q(X) = 1 - P(X).$$

Q/Z = Mill's ratio.

$Z^{(1)}(X)$ = the first derivative of $Z(X)$ with respect to X

$$= \frac{-X}{\sqrt{2\pi}} \exp(-X^2/2).$$

$Z^{(2)}(X)$ = the second derivative of $Z(X)$ with respect to X

$$= \frac{(X^2 - 1)}{\sqrt{2\pi}} \exp(-X^2/2).$$

$Z^{(3)}(X)$ = the third derivative of $Z(X)$ with respect to X

$$= \frac{-X}{\sqrt{2\pi}} (X^2 - 3) \exp(-X^2/2).$$

Mill's ratio, Q/Z , and P/Z have applications in many places. Among them is the solution of the maximum likelihood equations for estimating

the biserial correlation coefficient [218].* Among the uses of $Z(X)$, $Z^{(1)}(X)$, $Z^{(2)}(X)$, and $Z^{(3)}(X)$ is the expansion of an arbitrary continuous probability density function $f(X)$ into a series of the form

$$f(X) = \sum_{j=0}^{\infty} c_j Z^{(j)}(X),$$

where

$$c_0 = 1, \quad c_1 = 0, \quad c_2 = \frac{1}{2}(\sigma^2 - 1), \quad c_3 = \frac{-1}{6} \mu_3, \quad \text{etc.},$$

where σ^2 is the variance of X , and μ_3 is the third moment about the mean of X . This is a Gram-Charlier series of Type A. See Reference [104], pp. 147-154, for some of the conditions under which an expansion of this type may be made. Haight [83] gives an index of probability distribution functions.

The value of $P(X)$ was computed on an IBM 704 using a Taylor's Series expansion about the nearest $X = 0(0.25)3.5$. The other functions are straightforward computations, given $P(X)$, and were done simultaneously with $P(X)$ on a digital computer. The results were checked against the tables given by Sheppard [172], pp. 104-110.

*Numbers in brackets refer to the Bibliography.

The Normal Distribution and Related Functions

$P(X) = \Pr\{N(0, 1)\text{r.v.} \leq X\}; Q = 1 - P; Z(X) = \text{density of}$

$N(0, 1)\text{r.v.}; Z^{(i)} = \text{ith derivative of } Z$

X	$P(X)$	$Z(X)$	Q/Z	P/Z	$Z^{(1)}(X)$	$Z^{(2)}(X)$	$Z^{(3)}(X)$
0.00	0.500000	0.398942	1.25331	1.2533	-0.00000	-0.39894	0.00000
0.01	0.503989	0.398922	1.24338	1.2634	-0.00399	-0.39888	0.01197
0.02	0.507978	0.398862	1.23356	1.2736	-0.00798	-0.39870	0.02393
0.03	0.511966	0.398763	1.22387	1.2839	-0.01196	-0.39840	0.03588
0.04	0.515953	0.398623	1.21430	1.2943	-0.01594	-0.39799	0.04781
0.05	0.519939	0.398444	1.20484	1.3049	-0.01992	-0.39745	0.05972
0.06	0.523922	0.398225	1.19550	1.3156	-0.02389	-0.39679	0.07159
0.07	0.527903	0.397966	1.18627	1.3265	-0.02786	-0.39602	0.08344
0.08	0.531881	0.397668	1.17716	1.3375	-0.03181	-0.39512	0.09524
0.09	0.535856	0.397330	1.16816	1.3486	-0.03576	-0.39411	0.10699
0.10	0.539828	0.396953	1.15926	1.3599	-0.03970	-0.39298	0.11869
0.11	0.543795	0.396536	1.15047	1.3714	-0.04362	-0.39174	0.13033
0.12	0.547758	0.396080	1.14179	1.3830	-0.04753	-0.39038	0.14190
0.13	0.551717	0.395585	1.13321	1.3947	-0.05143	-0.38890	0.15341
0.14	0.555670	0.395052	1.12474	1.4066	-0.05531	-0.38731	0.16484
0.15	0.559618	0.394479	1.11636	1.4186	-0.05917	-0.38560	0.17618
0.16	0.563559	0.393868	1.10809	1.4308	-0.06302	-0.38379	0.18744
0.17	0.567495	0.393219	1.09991	1.4432	-0.06685	-0.38185	0.19861
0.18	0.571424	0.392531	1.09183	1.4557	-0.07066	-0.37981	0.20968
0.19	0.575345	0.391806	1.08384	1.4684	-0.07444	-0.37766	0.22064
0.20	0.579260	0.391043	1.07594	1.4813	-0.07821	-0.37540	0.23150
0.21	0.583166	0.390242	1.06814	1.4944	-0.08195	-0.37303	0.24224
0.22	0.587064	0.389404	1.06043	1.5076	-0.08567	-0.37056	0.25286
0.23	0.590954	0.388529	1.05281	1.5210	-0.08936	-0.36798	0.26336
0.24	0.594835	0.387617	1.04527	1.5346	-0.09303	-0.36529	0.27373
0.25	0.598706	0.386668	1.03782	1.5484	-0.09667	-0.36250	0.28396
0.26	0.602568	0.385683	1.03046	1.5623	-0.10028	-0.35961	0.29405
0.27	0.606420	0.384663	1.02318	1.5765	-0.10386	-0.35662	0.30401
0.28	0.610261	0.383606	1.01599	1.5909	-0.10741	-0.35353	0.31381
0.29	0.614092	0.382515	1.00887	1.6054	-0.11093	-0.35035	0.32346
0.30	0.617911	0.381388	1.00184	1.6202	-0.11442	-0.34706	0.33295
0.31	0.621720	0.380226	0.99488	1.6351	-0.11787	-0.34369	0.34228
0.32	0.625516	0.379031	0.98801	1.6503	-0.12129	-0.34022	0.35145
0.33	0.629300	0.377801	0.98121	1.6657	-0.12467	-0.33666	0.36045
0.34	0.633072	0.376537	0.97448	1.6813	-0.12802	-0.33301	0.36927
0.35	0.636831	0.375240	0.96783	1.6971	-0.13133	-0.32927	0.37791
0.36	0.640576	0.373911	0.96126	1.7132	-0.13461	-0.32545	0.38638
0.37	0.644309	0.372548	0.95475	1.7295	-0.13784	-0.32155	0.39466
0.38	0.648027	0.371154	0.94832	1.7460	-0.14104	-0.31756	0.40275
0.39	0.651732	0.369728	0.94196	1.7627	-0.14419	-0.31349	0.41065
0.40	0.655422	0.368270	0.93567	1.7797	-0.14731	-0.30935	0.41835
0.41	0.659097	0.366782	0.92944	1.7970	-0.15038	-0.30513	0.42586
0.42	0.662757	0.365263	0.92329	1.8145	-0.15341	-0.30083	0.43317
0.43	0.666402	0.363714	0.91720	1.8322	-0.15640	-0.29646	0.44027
0.44	0.670031	0.362135	0.91118	1.8502	-0.15934	-0.29203	0.44717
0.45	0.673645	0.360527	0.90522	1.8685	-0.16224	-0.28752	0.45386
0.46	0.677242	0.358890	0.89932	1.8870	-0.16509	-0.28295	0.46034
0.47	0.680822	0.357225	0.89349	1.9059	-0.16790	-0.27831	0.46660
0.48	0.684386	0.355533	0.88772	1.9250	-0.17066	-0.27362	0.47265
0.49	0.687933	0.353812	0.88201	1.9443	-0.17337	-0.26886	0.47848

The Normal Distribution and Related Functions (cont.)

x	$P(x)$	$Z(x)$	Q/Z	P/Z	$z^{(1)}(x)$	$z^{(2)}(x)$	$z^{(3)}(x)$
0.50	0.691462	0.352065	0.87636	1.9640	-0.17603	-0.26405	0.48409
0.51	0.694974	0.350292	0.87078	1.9840	-0.17865	-0.25918	0.48948
0.52	0.698468	0.348493	0.86525	2.0043	-0.18122	-0.25426	0.49465
0.53	0.701944	0.346668	0.85977	2.0248	-0.18373	-0.24929	0.49959
0.54	0.705401	0.344818	0.85436	2.0457	-0.18620	-0.24427	0.50431
0.55	0.708840	0.342944	0.84900	2.0669	-0.18862	-0.23920	0.50880
0.56	0.712260	0.341046	0.84370	2.0885	-0.19099	-0.23409	0.51306
0.57	0.715661	0.339124	0.83845	2.1103	-0.19330	-0.22894	0.51710
0.58	0.719043	0.337180	0.83326	2.1325	-0.19556	-0.22375	0.52091
0.59	0.722405	0.335213	0.82812	2.1551	-0.19778	-0.21853	0.52448
0.60	0.725747	0.333225	0.82303	2.1780	-0.19993	-0.21326	0.52783
0.61	0.729069	0.331215	0.81799	2.2012	-0.20204	-0.20797	0.53094
0.62	0.732371	0.329184	0.81301	2.2248	-0.20409	-0.20265	0.53383
0.63	0.735653	0.327133	0.80807	2.2488	-0.20609	-0.19729	0.53648
0.64	0.738914	0.325062	0.80319	2.2731	-0.20804	-0.19192	0.53891
0.65	0.742154	0.322972	0.79835	2.2979	-0.20993	-0.18652	0.54110
0.66	0.745373	0.320864	0.79357	2.3230	-0.21177	-0.18110	0.54306
0.67	0.748571	0.318737	0.78883	2.3486	-0.21355	-0.17566	0.54480
0.68	0.751748	0.316593	0.78414	2.3745	-0.21528	-0.17020	0.54630
0.69	0.754903	0.314432	0.77949	2.4009	-0.21696	-0.16473	0.54758
0.70	0.758036	0.312254	0.77489	2.4276	-0.21858	-0.15925	0.54863
0.71	0.761148	0.310060	0.77034	2.4548	-0.22014	-0.15376	0.54945
0.72	0.764238	0.307851	0.76583	2.4825	-0.22165	-0.14826	0.55005
0.73	0.767305	0.305627	0.76137	2.5106	-0.22311	-0.14276	0.55043
0.74	0.770350	0.303389	0.75695	2.5392	-0.22451	-0.13725	0.55058
0.75	0.773373	0.301137	0.75257	2.5682	-0.22585	-0.13175	0.55052
0.76	0.776373	0.298872	0.74824	2.5977	-0.22714	-0.12624	0.55023
0.77	0.779350	0.296595	0.74394	2.6277	-0.22838	-0.12074	0.54973
0.78	0.782305	0.294305	0.73969	2.6581	-0.22956	-0.11525	0.54901
0.79	0.785236	0.292004	0.73548	2.6891	-0.23068	-0.10976	0.54808
0.80	0.788145	0.289692	0.73131	2.7206	-0.23175	-0.10429	0.54694
0.81	0.791030	0.287369	0.72718	2.7527	-0.23277	-0.09883	0.54559
0.82	0.793892	0.285036	0.72309	2.7852	-0.23373	-0.09338	0.54403
0.83	0.796731	0.282694	0.71904	2.8184	-0.23464	-0.08795	0.54227
0.84	0.799546	0.280344	0.71503	2.8520	-0.23549	-0.08253	0.54031
0.85	0.802337	0.277985	0.71106	2.8863	-0.23629	-0.07714	0.53814
0.86	0.805105	0.275618	0.70712	2.9211	-0.23703	-0.07177	0.53579
0.87	0.807850	0.273244	0.70322	2.9565	-0.23772	-0.06643	0.53324
0.88	0.810570	0.270864	0.69935	2.9925	-0.23836	-0.06111	0.53049
0.89	0.813267	0.268477	0.69553	3.0292	-0.23894	-0.05582	0.52757
0.90	0.815940	0.266085	0.69173	3.0665	-0.23948	-0.05056	0.52445
0.91	0.818589	0.263688	0.68798	3.1044	-0.23996	-0.04533	0.52116
0.92	0.821214	0.261286	0.68425	3.1430	-0.24038	-0.04013	0.51769
0.93	0.823814	0.258881	0.68057	3.1822	-0.24076	-0.03497	0.51404
0.94	0.826391	0.256471	0.67691	3.2222	-0.24108	-0.02985	0.51023
0.95	0.828944	0.254059	0.67329	3.2628	-0.24136	-0.02477	0.50624
0.96	0.831472	0.251644	0.66971	3.3042	-0.24158	-0.01973	0.50210
0.97	0.833977	0.249228	0.66615	3.3462	-0.24175	-0.01473	0.49779
0.98	0.836457	0.246809	0.66263	3.3891	-0.24187	-0.00977	0.49332
0.99	0.838913	0.244390	0.65914	3.4327	-0.24195	-0.00486	0.48871

The Normal Distribution and Related Functions

x	$P(x)$	$Z(x)$	Q/Z	P/Z	$z^{(1)}(x)$	$z^{(2)}(x)$	$z^{(3)}(x)$
1.00	0.841345	0.241971	0.65568	3.4771	-0.24197	0.00000	0.48394
1.01	0.843752	0.239551	0.65225	3.5222	-0.24195	0.00481	0.47903
1.02	0.846136	0.237132	0.64885	3.5682	-0.24187	0.00958	0.47398
1.03	0.848495	0.234714	0.64549	3.6150	-0.24176	0.01429	0.46879
1.04	0.850830	0.232297	0.64215	3.6627	-0.24159	0.01896	0.46346
1.05	0.853141	0.229882	0.63885	3.7112	-0.24138	0.02356	0.45801
1.06	0.855428	0.227470	0.63557	3.7606	-0.24112	0.02812	0.45243
1.07	0.857690	0.225060	0.63232	3.8109	-0.24081	0.03261	0.44673
1.08	0.859929	0.222653	0.62910	3.8622	-0.24047	0.03705	0.44092
1.09	0.862143	0.220251	0.62591	3.9144	-0.24007	0.04143	0.43499
1.10	0.864334	0.217852	0.62274	3.9675	-0.23964	0.04575	0.42895
1.11	0.866500	0.215458	0.61961	4.0217	-0.23916	0.05001	0.42281
1.12	0.868643	0.213069	0.61650	4.0768	-0.23864	0.05420	0.41657
1.13	0.870762	0.210686	0.61342	4.1330	-0.23807	0.05834	0.41023
1.14	0.872857	0.208308	0.61036	4.1902	-0.23747	0.06241	0.40380
1.15	0.874928	0.205936	0.60733	4.2485	-0.23683	0.06641	0.39728
1.16	0.876976	0.203571	0.60433	4.3080	-0.23614	0.07035	0.39067
1.17	0.879000	0.201214	0.60135	4.3685	-0.23542	0.07423	0.38399
1.18	0.881000	0.198863	0.59840	4.4302	-0.23466	0.07803	0.37724
1.19	0.882977	0.196520	0.59548	4.4931	-0.23386	0.08177	0.37041
1.20	0.884930	0.194186	0.59257	4.5571	-0.23302	0.08544	0.36352
1.21	0.886861	0.191860	0.58970	4.6224	-0.23215	0.08904	0.35656
1.22	0.888768	0.189543	0.58684	4.6890	-0.23124	0.09257	0.34955
1.23	0.890651	0.187235	0.58402	4.7569	-0.23030	0.09603	0.34248
1.24	0.892512	0.184937	0.58121	4.8260	-0.22932	0.09942	0.33536
1.25	0.894350	0.182649	0.57843	4.8966	-0.22831	0.10274	0.32820
1.26	0.896165	0.180371	0.57567	4.9685	-0.22727	0.10599	0.32099
1.27	0.897958	0.178104	0.57294	5.0418	-0.22619	0.10916	0.31375
1.28	0.899727	0.175847	0.57022	5.1165	-0.22508	0.11226	0.30648
1.29	0.901475	0.173602	0.56754	5.1928	-0.22395	0.11529	0.29917
1.30	0.903200	0.171369	0.56487	5.2705	-0.22278	0.11824	0.29184
1.31	0.904902	0.169147	0.56222	5.3498	-0.22158	0.12113	0.28449
1.32	0.906582	0.166937	0.55960	5.4307	-0.22036	0.12393	0.27712
1.33	0.908241	0.164740	0.55699	5.5132	-0.21910	0.12667	0.26974
1.34	0.909877	0.162555	0.55441	5.5973	-0.21782	0.12933	0.26235
1.35	0.911492	0.160383	0.55185	5.6832	-0.21652	0.13192	0.25495
1.36	0.913085	0.158225	0.54931	5.7708	-0.21519	0.13443	0.24755
1.37	0.914657	0.156080	0.54679	5.8602	-0.21383	0.13687	0.24015
1.38	0.916207	0.153948	0.54430	5.9514	-0.21245	0.13923	0.23276
1.39	0.917736	0.151831	0.54182	6.0445	-0.21104	0.14152	0.22537
1.40	0.919243	0.149727	0.53936	6.1394	-0.20962	0.14374	0.21800
1.41	0.920730	0.147639	0.53692	6.2364	-0.20817	0.14588	0.21065
1.42	0.922196	0.145564	0.53450	6.3353	-0.20670	0.14795	0.20331
1.43	0.923641	0.143505	0.53210	6.4363	-0.20521	0.14995	0.19600
1.44	0.925066	0.141460	0.52972	6.5394	-0.20370	0.15187	0.18871
1.45	0.926471	0.139431	0.52735	6.6447	-0.20217	0.15372	0.18145
1.46	0.927855	0.137417	0.52501	6.7521	-0.20063	0.15550	0.17423
1.47	0.929219	0.135418	0.52268	6.8619	-0.19906	0.15721	0.16704
1.48	0.930563	0.133435	0.52038	6.9739	-0.19748	0.15884	0.15988
1.49	0.931888	0.131468	0.51809	7.0883	-0.19589	0.16040	0.15277

The Normal Distribution and Related Functions (cont.)

X	P(X)	Z(X)	Q/Z	P/Z	Z ⁽¹⁾ (X)	Z ⁽²⁾ (X)	Z ⁽³⁾ (X)
1.50	0.933193	0.129518	0.51582	7.2051	-0.19428	0.16190	0.14571
1.51	0.934478	0.127583	0.51356	7.3245	-0.19265	0.16332	0.13869
1.52	0.935745	0.125665	0.51133	7.4464	-0.19101	0.16467	0.13172
1.53	0.936992	0.123763	0.50911	7.5709	-0.18936	0.16595	0.12481
1.54	0.938220	0.121878	0.50690	7.6981	-0.18769	0.16717	0.11795
1.55	0.939429	0.120009	0.50472	7.8280	-0.18601	0.16831	0.11114
1.56	0.940620	0.118157	0.50255	7.9607	-0.18433	0.16939	0.10440
1.57	0.941792	0.116323	0.50040	8.0964	-0.18263	0.17040	0.09772
1.58	0.942947	0.114505	0.49826	8.2350	-0.18092	0.17134	0.09111
1.59	0.944083	0.112704	0.49614	8.3766	-0.17920	0.17222	0.08456
1.60	0.945201	0.110921	0.49404	8.5214	-0.17747	0.17304	0.07809
1.61	0.946301	0.109155	0.49195	8.6694	-0.17574	0.17379	0.07168
1.62	0.947384	0.107406	0.48988	8.8206	-0.17400	0.17447	0.06535
1.63	0.948449	0.105675	0.48782	8.9752	-0.17225	0.17509	0.05910
1.64	0.949497	0.103961	0.48578	9.1332	-0.17050	0.17565	0.05292
1.65	0.950529	0.102265	0.48376	9.2948	-0.16874	0.17615	0.04682
1.66	0.951543	0.100586	0.48175	9.4600	-0.16697	0.17659	0.04081
1.67	0.952540	0.098925	0.47975	9.6289	-0.16521	0.17697	0.03487
1.68	0.953521	0.097282	0.47777	9.8016	-0.16343	0.17729	0.02903
1.69	0.954486	0.095657	0.47580	9.9782	-0.16166	0.17755	0.02326
1.70	0.955435	0.094049	0.47385	10.159	-0.15988	0.17775	0.01759
1.71	0.956367	0.092459	0.47192	10.344	-0.15811	0.17790	0.01200
1.72	0.957284	0.090887	0.46999	10.533	-0.15633	0.17799	0.00650
1.73	0.958185	0.089333	0.46808	10.726	-0.15455	0.17803	0.00110
1.74	0.959070	0.087796	0.46619	10.924	-0.15277	0.17802	-0.00422
1.75	0.959941	0.086277	0.46431	11.126	-0.15099	0.17795	-0.00944
1.76	0.960796	0.084776	0.46244	11.333	-0.14921	0.17783	-0.01456
1.77	0.961636	0.083293	0.46058	11.545	-0.14743	0.17766	-0.01959
1.78	0.962462	0.081828	0.45874	11.762	-0.14565	0.17744	-0.02453
1.79	0.963273	0.080380	0.45692	11.984	-0.14388	0.17717	-0.02937
1.80	0.964070	0.078950	0.45510	12.211	-0.14211	0.17685	-0.03411
1.81	0.964852	0.077538	0.45330	12.444	-0.14034	0.17648	-0.03875
1.82	0.965620	0.076143	0.45151	12.682	-0.13858	0.17607	-0.04329
1.83	0.966375	0.074766	0.44973	12.925	-0.13682	0.17562	-0.04774
1.84	0.967116	0.073407	0.44797	13.175	-0.13507	0.17512	-0.05208
1.85	0.967843	0.072065	0.44622	13.430	-0.13332	0.17458	-0.05633
1.86	0.968557	0.070740	0.44448	13.692	-0.13158	0.17399	-0.06047
1.87	0.969258	0.069433	0.44275	13.960	-0.12984	0.17337	-0.06452
1.88	0.969946	0.068144	0.44104	14.234	-0.12811	0.17270	-0.06846
1.89	0.970621	0.066871	0.43934	14.515	-0.12639	0.17200	-0.07231
1.90	0.971283	0.065616	0.43765	14.803	-0.12467	0.17126	-0.07605
1.91	0.971933	0.064378	0.43597	15.097	-0.12296	0.17048	-0.07969
1.92	0.972571	0.063157	0.43430	15.399	-0.12126	0.16966	-0.08323
1.93	0.973197	0.061952	0.43265	15.709	-0.11957	0.16881	-0.08667
1.94	0.973810	0.060765	0.43100	16.026	-0.11788	0.16793	-0.09002
1.95	0.974412	0.059595	0.42937	16.351	-0.11621	0.16701	-0.09326
1.96	0.975002	0.058441	0.42775	16.684	-0.11454	0.16607	-0.09640
1.97	0.975581	0.057304	0.42614	17.025	-0.11289	0.16509	-0.09944
1.98	0.976148	0.056183	0.42454	17.374	-0.11124	0.16408	-0.10239
1.99	0.976705	0.055079	0.42295	17.733	-0.10961	0.16304	-0.10523