# KINETIC THEORY

## Classical, Quantum, and Relativistic Descriptions

Richard L. Liboff

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# Preface

This text stems from a graduate course I introduced at Cornell a number of years ago. Since its inception in the mid-1960s, the field has expanded, predominantly in the directions of quantum and relativistic kinetic theory. Quantum kinetic theory is important to problems in the transport of particles, as well as radiation through material media, which finds application in solid-state and laser physics. Relativistic kinetic theory has grown important in certain plasma problems related to controlled thermonuclear fusion. It is also relevant to many problems in astrophysics. Classical kinetic theory is the foundation of fluid dynamics and thus is important to aerospace, mechanical, and chemical engineering.

The text is comprised of six chapters. In Chapter 1, the transformation theory of classical mechanics is developed for the purpose of deriving Liouville's theorem and the Liouville equation. Four distinct interpretations of the solution to this equation are presented. The fourth interpretation addresses Gibbs's notion of a distribution function that is the connecting link between the Liouville equation and experimental observation. The notion of a Markov process is discussed, and the central-limit theorem is derived and applied to the random-walk problem.

In Chapter 2, the very significant BBKGY hierarchy is obtained from the Liouville equation, and the first two equations of this sequence are applied in the derivation of conservation of energy for a gas of interacting particles. In nondimensionalizing this sequence, parameters emerge that differentiate between weakly and strongly coupled fluids. Correlation functions are introduced through the Mayer expansions. Examining a weakly coupled fluid comprised of particles interacting under long-range interaction leads to the Vlasov equation and the closely allied concept of a self-consistent solution. Prigogine's diagrammatic technique and related operator formalism for examining the Liouville equation are described. The Bogoliubov *ansatz* concerning the equilibration of a gas, as well as the Klimontovich formulation of kinetic theory, is also included in this chapter.

The Boltzmann equation is derived in Chapter 3 and applied to the derivation of fluid dynamic equations and the H theorem. Poincaré's recurrence theorem is proved and is discussed relative to Boltzmann's H theorem. Transport coefficients are defined, and the Chapman–Enskog expansion is developed. Results of this technique of solution to the Boltzmann equation are compared with experimental data and are found to be in good agreement for various molecular samples. Grad's method of solution of the Boltzmann equation involving expansion in tensor hermite polynomials is described. The chapter continues with a derivation of the Druyvesteyn distribution relevant to a current-carrying plasma in a dc electric field. In the last section of the chapter, the topic of irreversibility is returned to. Ergodic and mixing flows are discussed. Action-angle variables are introduced, and the notions of classical degeneracy and resonant domains in phase space are described in relation to the chaotic behavior of classical systems. A statement of the closely allied KAM theorem is also given.

In the first half of Chapter 4, the Vlasov equation is applied to linear wave theory for a two-component plasma comprised of electrons and heavy ions. Landau damping and the Nyquist criterion for wave instabilities are described. The chapter continues with derivations of other important kinetic equations: Krook–Bhatnager–Gross (KBG), Fokker–Planck, Landau, and Balescu–Lenard equations. A table is included describing the interrelation of the classical kinetic equations discussed in the text. The chapter concludes with a description of the widely used Monte Carlo numerical analysis in kinetic theory.

Quantum kinetic theory is developed in Chapter 5. A brief review of basic principles leads to a description of the density matrix, the Pauli equation, and the closely allied Wigner distribution. Various equivalent forms of the Wigner–Moyal equation are derived. A quantum modified KBG equation is applied to photon transport and electron propagation in solids. Thomas–Fermi screening and the Mott transition are also discussed. The Uehling–Uhlenbeck quantum modified Boltzmann equation is developed and applied to a Fermi liquid. The chapter continues with an overview of classical and quantum hierarchies of equations connecting reduced distributions. A table of hierarchies is included where the reader is easily able to view distinctions between these sets of equations. The Kubo formula, described previously in Chapter 3, is revisited and applied to the derivation of a quantum expression for electrical conductivity. The chapter concludes with an introduction to Green's function analysis and related diagrammatic representations.

The last chapter of the text addresses relativistic kinetic theory. The discussion begins with elementary concepts, including a statement of Hamilton's equations in covariant form. Stemming from a covariant distribution function in four-space, together with Maxwell's equations in covariant form, a relativistic Vlasov equation is derived for a plasma in an electromagnetic field. An important component of this chapter is the derivation and compilation of a table of Lorentz invariants in kinetic theory. The chapter continues with a derivation of the relativistic Maxwellian and concludes with a brief description of relativity in non-Cartesian coordinates.

Each chapter is preceded by a brief introductory statement of the subject matter contained in the chapter. Problems appear at the end of each chapter, many of which include solutions. Some such problems address self-contained descriptions of closely allied topics. In such cases, these problems are listed in the chapter table of contents under the heading *Topical Problems*. In addition to references cited in the text, a comprehensive list of references is included in Appendix D. Assorted mathematical formulas are included in Appendixes A and B. Reference to equations is as follows: Consider, for example, reference to (3.4) in Chapter 5. In this event the first numeral in (3.4) indicates that the equation is in Section 5.3. If mentioned in, say, Chapter 6, this same equation is referred to as (5.3.4). Reference to equations in problems is written, for example, as (P3.7). This is equation 7 in problem 3 in a given chapter. Reference to equations in an appendix is written, for example, (B.3), which is equation 3 in Appendix B. A list of symbols precedes the appendixes.

Stemming from the observation that science and society are inextricably entwined, a time chart is included (Appendix E) listing early contributors to science and technology of the classical Greek and Roman eras. The reader will note that a central figure in this display is the Greek philosopher, Democritus, who, at about 400 BCE, was the first to propose an atomic theory of matter. Readers of my earlier work [*Introduction to the Theory of Kinetic Equations*, Wiley, New York (1969)] will recall that it, too, included a time chart describing contributions to dynamics from the fifteenth to the nineteenth centuries.

We trust that this text will afford students and working scientists a firm foundation from which they may continue to more advanced topics in the field.

Deep appreciation is expressed to Kenneth Tennity for his support and confidence in this work. I am indebted to Catherine Kuhl for her diligence and care taken in the typing of the manuscript for this book.

Many others have contributed to the development of this work. I remain indebted to these kind individuals and would like here to express my sincere gratitude for their encouragement, support, and constructive criticism: Sidney Leibovich, Terrence Fine, Christof Litwin, Kenneth Gardner, Neal Maresca, K. C. Liu, Danny Heffernan, Edwin Dorchek, Philip Bagwell, Ronald Klein, Steve Seidman, S. Ramakrishna, G. George, William Morrell, Wayne Scales, Daniel Koury, Erich Kunhardt, Marvin Silver, James Hartle, Abner Shimony, Philip Holmes, Lloyd Hillman, Arthur Ruoff, L. Pearce Williams, Lloyd Motz, John Guckenheimer, Isaac Rabinowitz, and Ilya Prigogine.

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*Peace,*
*R. L. L.*

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# 1

# The Liouville Equation