CSIR REPORT CENG 322

FORCED CONVECTIVE HEAT TRANSFER IN SINGLE PHASE FLOW OF A NEWTONIAN FLUID IN A CIRCULAR PIPE

An annotated summary of empirical correlations

D.G. ROGERS

CHEMICAL ENGINEERING RESEARCH GROUP — CSIR

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S Y N O P S I S

An extensive bibliography of empirical correlations for the Nusselt group for internal Newtonian pipe flow has been compiled to facilitate the design of heat transfer equipment. An index is provided for locating experimental heat transfer coefficients for particular fluids and flow conditions.

KEYWORDS: Heat transfer coefficient, turbulent, laminar, transition, review, internal flows, empirical correlations, fluid flow.

FILE: 660–43–2

WNNR VERSLAG CENG 322

HITTEOORDRAG BY GEDWONGE KONVEKSIE VAN 'N ENKELFASIGE NEWTONIESE VLOEIER IN 'N RONDE PYP

'n Opsomming van empiriese korrelasies met aantekeninge.

D G ROGERS

April 1980

SINOPSIS

Besonderhede van die empiriese korrelasies vir hitteoordrag by Newtoniese vloei in 'n pyp, wat in die vakliteratuur verskyn het, is versamel vir gebruik in die ontwerp van hitteruilers. 'n Indeks is voorsien om die eksperimentele hitteruilingskoëffisiënt vir bepaalde vloeistowwe en vloeitoestande op te spoor.

CONTENTS

1 INTRODUCTION

Order and Simplification
are the first steps towards
the mastery of a subject.

Thomas Mann

Reliable heat transfer coefficients for designing heat exchange equipment are often difficult to obtain and the design engineer may on occasion question the applicability of a chosen coefficient. This doubt may result in over conservative design methods being used and more expensive units than necessary being designed.

This bibliography was compiled as an aid to the designer to find an accurate heat transfer coefficient for the conditions for which he is designing, and to clarify the state of experimental data and correlations as a first step to improving the design process.

The heat transfer coefficient for the transfer of heat to or from a non-porous wall to a fluid is defined by the equation

$$q = h(\Theta_{wall} - \Theta_{\infty}) = -k_{fluid}\frac{\partial \Theta}{\partial y}\Bigg|_{y\,=\,wall}$$

The magnitude of the heat transfer coefficient, h, has been determined to depend substantially on the velocity field and it is thus convenient to subdivide heat transfer coefficients according to the flow pattern. The dependence of h on other factors such as the Prandtl number, Grashoff number and Peclet number does not enable a convenient subdivision to be made.

1.1 FLOW REGIMES

Internal forced flow is characterised by the three regions, laminar, turbulent and transitional flow, the latter occurring in the intermediate region between laminar and turbulent flow. The transitional flow may be further subdivided into laminar to turbulent transitional and turbulent to laminar transitional (also called reverse transition), depending on the history of the flow. The three fundamental regions are traditionally characterised by the Reynolds number as

Laminar $Re \leqslant 2\ 100$
Transitional $2\ 000 \leqslant Re \leqslant 10\ 000$
Turbulent $Re \geqslant 5\ 000$

These regions are loosely defined since the geometry and the heat transfer rate affect the transition process.

For internal pipe flow Metais and Eckert (1964) recognised the effect of free convection on the flow pattern and incorporated the GrPr product to further subdivide the flow into the regions

Forced convection turbulent
Free convection turbulent

Forced convection laminar
Free convection laminar

Mixed convection turbulent
Mixed convection laminar.

In this report only the three fundamental regions of laminar, turbulent and transitional flows and only experimental results and correlations are considered. For reviews on the theoretical models the texts of Shah and London (1978) for laminar flow models and Reynolds and Cebeci (1976) and Launder and Spalding (1972) for turbulent flow models are recommended. There is no specific text for transitional flows and this region is usually included in turbulent flow modelling.

1.2 LOCATING INFORMATION

Fluid and Equation indexes were prepared for locating original experimental data for particular fluids and flow conditions. For example, if a heat transfer coefficient is required for heating molasses in turbulent flow in a horizontal pipe, the Fluid index (Section 4.2) indicates that Friend and Metzner (1958) obtained experimental data; cross-referring to 1958 in the Equation index (Section 4.3) for the article by Friend and Metzner will give the data and an accurate heat transfer coefficient.

Alternatively the Equation index may be used for evaluating a given correlation for the Nusselt number (or other heat transfer group). For example, if the equation of Malina and Sparrow (1964) was in question, the entry in the Equation index will indicate how the equation fits given experimental data.

The Equation index also contains entries for which there is no cross reference in the Fluid index. These entries are largely analytical solutions, or correlations based on other researchers' data.

For literature other than English, the VDI—Wärmeatlas, Berechnungsblätter für den Wärmeübergang, is recommended as an interesting summary.

2 CHRONOLOGICAL SUMMARY OF EXPERIMENTAL HEAT TRANSFER COEFFICIENT CORRELATIONS

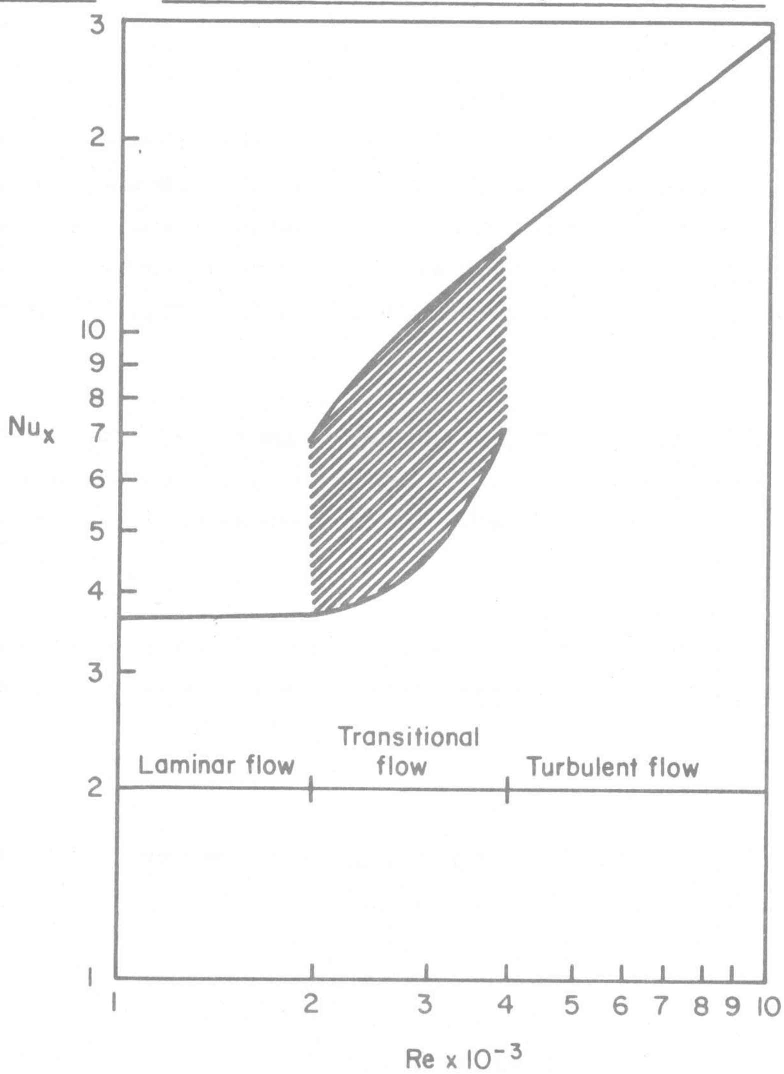
Osborne Reynolds (1874, 1884) was one of the first researchers to recognise and quantify the modes in which fluids flow in pipes. This he did as follows:

> *"... In the first place, it has shown that the property of viscosity or treacliness, possessed more or less by all fluids, is the general influence conclusive to steadiness, while on the other hand, space and velocity are the counter influence..."*

Reynolds therefore, divided fluid flow into two regions which have since been termed *laminar* and *turbulent* flow, which in isothermal conditions are distinguished by the Reynolds group, $\frac{\rho v d}{\mu}$.

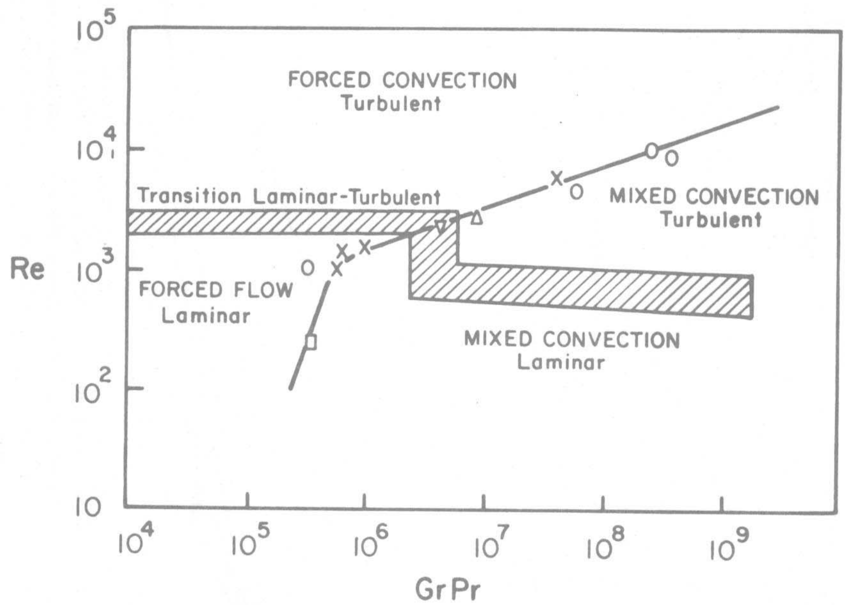
It was not until circa 1940 that an intermediate region of fluid flow which had a marked effect on the heat transfer, was discerned. This region was termed *transitional flow* and correlations of the form of Figure 1 were accepted; noteworthy is the lack of indication in the figure of how to select a Nusselt number in the transition region.

FIGURE 1 Nusselt numbers for transitional gas flow (Pr = 0,7)



Circa 1954 Eckert and Diaguila and later Metais and Eckert (1964) identified the effect of free convection on the extent and location of the three flow regions and presented results as in Figure 2.

FIGURE 2 Regimes of free, forced, and mixed convection for flow through horizontal tubes



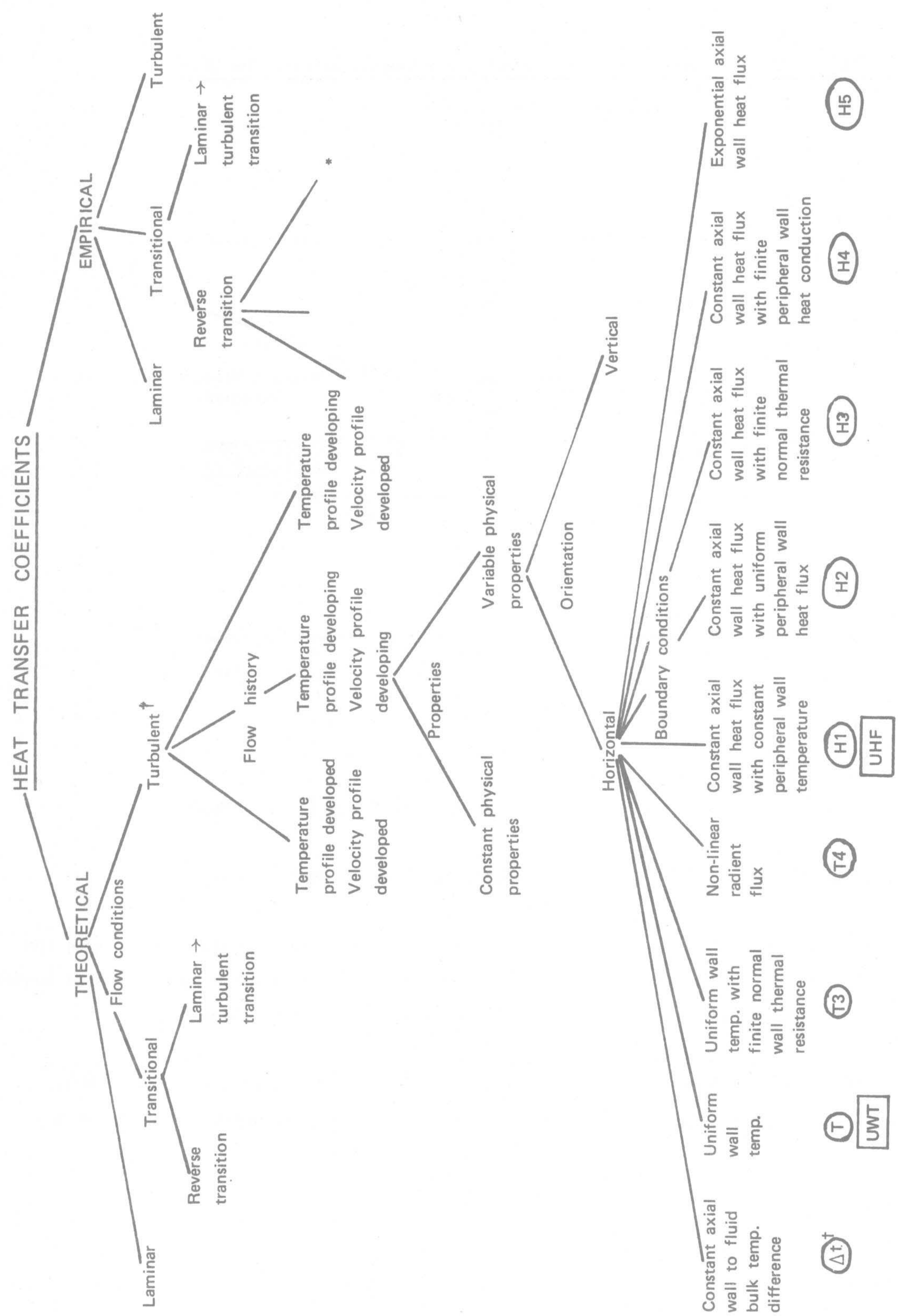
($10^{-2} < Pr\frac{d}{L} < 1$) Metais and Eckert (1964)

(The free convection limit was not established through lack of data).

However, correlations for determining the heat transfer in the transition region were still inadequate and circa 1970 Bankston further subdivided the region into the separate cases of *laminar to turbulent transition* and turbulent to laminar transition or *reverse transition.*

With these subdivisions of the flow conditions, Figure 3 has been constructed. It is envisaged that this Figure will be enlarged in the future as a more complete understanding is reached.

FIGURE 3 Schematic representation of flow division and boundary conditions for heat transfer in pipes



† Boundary conditions after Shah and London (1978)

‡ Asymptotes: Re → ∞, Pr → ∞; Re → ∞, Pr → 0 [Churchill (1977)] ; RePr → 0 [Lyon (1951)]

* All sublayers may be developed in the same manner as has been done for the turbulent, theoretical, heat transfer coefficients.

2.1 TURBULENT FLOW HEAT TRANSFER CORRELATIONS

This section summarises the most important points that may be extracted from the extended survey in section 3.

Since the available literature on heat transfer extends over a relatively long time span, it is often difficult to visualise the progression of the science. To facilitate the visualisation of the state of experimental turbulent flow heat transfer coefficient correlations Table 1 has been constructed. From this it is relatively easy to grasp the chain of thought through the time span.

In the other sections on laminar and transition flow heat transfer correlations, similar tables have been drawn up to facilitate visualisation.

TABLE 1 A chronological summary of turbulent flow heat transfer correlation methods

UWT = const. wall temp.
UHF = uniform heat flux

BOUNDARY CONDITIONS	DATE	
	1905 1909	Nusselt and Boussinesque from dimensional analysis suggest $Nu = \phi(Re)\,\psi(Pr)$
UWT Pr = 0,7	1917	Nusselt suggests $Nu = \phi(RePr)(^d/_L)^n$ for a developing velocity profile.
	1916 – 1919	Taylor proposes $\frac{1}{C_H} = \frac{2}{C_f}\left[1 + \frac{U_s}{U}(Pr - 1)\right]$ linking friction and heat transfer
	1922	McAdams and Frost take the viscosity at an average film temperature to align data.
$^L/_D > 35$	1924	McAdams and Frost suggest $Nu = \phi(Re)\,(1 + a\,^d/_L)^n$ eliminating the effect of infinite tube length on the Nu.
UWT Pr = 2,48 7,35	1924	Rice incorporates a temperature difference term, possibly to account for free convection.
	1929	Keevil and McAdams note the effect of the wall to bulk temperature difference to give different velocity profiles.

BOUNDARY CONDITIONS	DATE	
UWT $^{L}/_{D}$ 59 → 224	1931	Lawrence and Sherwood find that $^{d}/_{L}$ has only an effect for viscous liquids.
	1931	Drew, Hogen and McAdams suggest using the Gz number ($\equiv a\,RePr\,^{d}/_{L}$).
UWT	1933	Colburn suggests using film temperatures to align the data and suggests that for large Gr the group $(1 + a\,Gr^{1/3})$ should be included.
UHF $^{L}/_{D}$ = 48 Pr 2 → 5	1945	Bernardo and Eian note that the St correlated results better in the lower turbulent region.
	1950	Deissler concludes that the effect of fluid properties across the tube can be eliminated by evaluating the properties at a temperature close to the average of the wall and bulk temperatures.
	1951	Lyon notes that there is an expected minimum of the Nu as Pr → 0.
	1954	Deissler notes that increasing Pr eliminates the entrance effect and that the effect of variable viscosity can be eliminated by evaluating the viscosity at temperatures which are a function of the Pr.
UWT $^{L}/_{D}$ = 5 Pr = 0,7	1954	Eckert and Diaguila note the effect of GrPr on determining the limits of the flow regions.
UHF	1955	Hartnett defines the thermal entrance length and notes that at high Re the Pr has little effect.
UWT Pr = 0,7	1961	Jackson, Spurlock and Purdy experimenting with developing velocity profiles note an effect of $^{L}/_{d}$ but not of free convection. In their experiments the boundary layer did not fill the tube.
	1963	Petukhov and Popov suggest the equation $$Nu = \frac{^{f}/_{8}\,RePr}{a + b\sqrt{^{f}/_{8}}\,(Pr^{2/3} - 1)}$$

BOUNDARY CONDITIONS	DATE	
UHF $^L/_D$ = 30 Pr = 7 → 8	1964	Allen and Eckert use the wall to fluid temperature difference to correlate the local Nu.
UWT $^L/_D$ = 31 Pr = 0,71 → 5,52	1965	Kolář uses a turbulent $Re_t = \dfrac{u_m d}{\nu\sqrt{\frac{f}{8}}}$ to correlate data.
UHF $^L/_D$ = 21 Pr = 0,7 → 14,3	1967	Gowen and Smith show that the universal temperature profile is dependent on Pr and Re.
UWT $^L/_D$ = 80 Pr = 2 → 8	1971	Herbert and Sterns for vertical tubes note that the GrPr has little effect at high Re but increases as Re decreases.
	1972	Gross and Thomas note that inclusion of the $\frac{dP}{dx}$ term improves a theoretical model.
	1974	Mori, Sakakibara and Tanimoto find that for Gz = 50 the ratio of the wall thermal conductivity to that of the fluid and the wall thickness may be significant.

2.1.1 Conclusions

For heat transfer to a fluid with constant properties, a fully developed velocity profile and without free convection effects, the Nusselt number correlation is of the form $Nu = \phi(Re).\ \psi(Pr)$.

For free convection effects the term $(1 + a\ Gr^{1/3})$ may be included as a multiplier. [The effect of using the group as a multiplier is discussed by Brown and Thomas (1965)].

For a developing velocity profile the term $(^L/_D)^n$ or inclusion of a friction term $\sqrt{^f/_8}$ may be included as a multiplier. $(^L/_D)^n$ may be criticised as predicting an infinite Nu for an infinite tube length and $[1 + (^L/_D)^n]$ is sometimes used to eliminate this incongruity. The alternative use of the Graetz number is not to include entrance effects but to define the ratio of the rate of heat transfer by convection to the rate of heat transfer by conduction.

Variable fluid properties are accounted for by the ratio Nu/Nu_o as explained in section 2.4.

The most often cited equation is that of Petukhov and Popov (1963), with the variable fluid property correction term of Hufschmidt, Burck and Riebold (1966) which is

$$Nu = \frac{{}^{f}/_{8}\ RePr}{1{,}07 + 12{,}7\sqrt{{}^{f}/_{8}}\,(Pr^{2/3} - 1)} \cdot \left[\frac{Pr}{Pr_w}\right]^{0{,}11}$$

Alternatively the equations recommended by Metais and Eckert (1964) may be used.

2.2 LAMINAR FLOW HEAT TRANSFER CORRELATIONS

As in section 2.1, Table 2 has been constructed to facilitate visualisation.

TABLE 2 A chronological summary of laminar flow heat transfer correlations

BOUNDARY CONDITIONS	DATE	
UWT UHF	1885 1910	Graetz and Nusselt formulate an analytical solution for fully developed velocity and temperature profiles.
UWT UHF	1928	Lévêque extends the result to developing velocity profile giving $Nu = a(RePr\ {}^{d}/_{L})^{1/3}$
	1930	Dittus and Boelter include the term ΔT_{LM} to include variable physical properties.
UWT $Pr = 2 \rightarrow 8$	1930	Colburn and Hougen find that $Nu = aPr^{1/3}\,Gr^{1/3}$ with properties based on a film temperature, independent of the Re.
UHF ${}^{L}/_{D} = 150$	1931	Kirkbride and McCabe propose the form $Nu = a + \frac{b}{RePr\ {}^{d}/_{L}} + \frac{c}{(RePr\ {}^{d}/_{L})^{n}}$
	1933	Colburn includes the term $1 + a\ Gr^{1/3}$ to account for free convection and bases properties on a film temperature.
${}^{L}/_{D} = 90$	1936	Sieder and Tate suggest the property correction $Nu = a(RePr\,{}^{d}/_{L})^{1/3} \left[\frac{\mu b}{\mu w}\right]^{0{,}14}$

BOUNDARY CONDITIONS	DATE	
UWT Pr = 49 $^L/_D$ = 20 → 602	1942	Martinelli et al criticise the use of $1 + aGr^{1/3}$ as a multiplier as this would indicate an increasing effect of free convection with increasing Re, contrary to their observed results for vertical tubes. (See Brown and Thomas, 1965).
UWT Pr = 60 – 1000 $^L/_D$ = 40 → 193	1943	Kern and Othmer suggest $\frac{1 + aGr^{1/3}}{\log Re}$
	1959	Stephan suggests $Nu = f_n\,(^L/_d Pe)$ for constant properties and $Nu = f_n(^L/_d Re, Pr)$ for variable properties.
UHF Pr = 0,7 → 8	1961	Ede suggests $Nu = a + bGr^{1/3}$
$^L/_D$ = 72	1962	Oliver criticises $Gr^{\,d}/L$ as producing a term in D^4 and suggests $Gr^{\,L}/d$.
$^L/_D$ = 36 → 72	1965	Brown and Thomas find that for horizontal tubes the free convection increases with increasing Re.
UHF $^L/_D$ = 127 Pr = 0,7	1966	Mori et al find that free convection effect starts at $ReRa = 10^3$ and that the critical Re depends on the intensity of the secondary flow.
UHF Pr = 0,7 $^L/_D$ = 80	1966	McComas and Eckert notice that the free convection effect increases as the ratio of Gr to Re increases.
	1967	Iqbal and Stachiewicz analytically find that the tube inclination has little effect on the Nu.
UHF Pr = 2 →8 $^L/_D$ → 700	1968	Shannon and Depew notice that for $\frac{(GrPr)^{1/4}}{Nu_o} < 2$ the natural convection is negligible.
$^L/_D$ = 28	1971	Depew and August suggest using the group $Gz\ Gr^n\ Pr^m$

2.2.1 Conclusions

For a fully developed parabolic velocity profile, constant physical properties and no free convection effects, the Nusselt number expected from theory is for:

uniform wall temperature $Nu_\infty = 3{,}657$

and for uniform heat flux $Nu_\infty = \frac{48}{11} = 4{,}364$

For a developing velocity profile the term $(^L/D)^n$ or $[1 + (^L/D)^n]$ may be included as a multiplier. This latter form eliminates the problem of infinite tube lengths giving infinite heat transfer coefficients.

Free convection effects are not easily accounted for, however, the forms $1 + aGr^n$ and $\frac{1 + aGr^n}{\log Re}$ are accepted as reliable. In some instances the Grashoff group alone is included as a multiplier, and in the case of Ede (1961) the Nusselt group is correlated as a function of the Grashoff number only.

The extent or intensity of free convection may be expressed as a function of the ReRa product or as Ra^n/Nu_o.

Variable physical properties are included by using the $\frac{Nu}{Nu_o}$ form of correlation as given in section 2.4.

No single equation is recommended for all fluids and conditions, however, the most recent equation by Depew and August (1971)

$$Nu = 1{,}75[Gz + 0{,}12(GzGr^{1/3}\,Pr^{0{,}36})^{0{,}88}]^{1/3} \left[\frac{\mu b}{\mu w}\right]^{0{,}14}$$

may be reliable for most conditions.