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TECHNICAL MATHEMATICS WITH CALCULUS

Technical Mathematics

WITH CALCULUS

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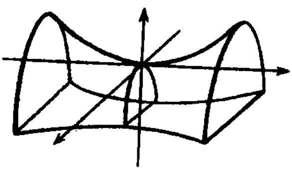

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TECHNICAL MATHEMATICS WITH CALCULUS

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Preface

Every textbook should have a definable purpose and a definable point of view. This particular text is intended to help students of engineering learn how to use the techniques of mathematics as a professional tool. The point of view adopted in this text, therefore, is that mathematics is an engineer's tool. The authors are quite well aware that there are other values of mathematics. They are aware that mathematics is a philosophy; it is a metaphysic; it is an art; and it is logic. However, these values of mathematics, important and worthwhile as they may be, are definitely subordinate to the central theme of this text, namely, that mathematics is an engineer's tool.

The authors are also perfectly well aware that there are many functions and many techniques which are not even hinted at in this volume. They have been omitted intentionally because experience has shown that these functions and techniques are not ordinarily encountered by the beginning engineer and engineering technician. The authors feel very strongly that the classroom time and textbook space can much more properly be devoted to the building up of skills in the use of more elementary techniques rather than to discussing the more theoretical aspects of mathematics, which will probably not be used until the student has developed considerable experience in his chosen field.

The text is the outgrowth of years of classwork with students at Wentworth Institute. These men are high school graduates who have had at least a year or two of algebra. While they expect to enter industry and work in intimate accord with artisans, they must also be at ease in the more precise atmosphere of engineering. Therefore, the authors have given a traditional and thorough treatment of basic algebra and trigonometry and have stressed the applications of these principles to a wide range of specific engineering situations. The calculus section is limited to those techniques of calculus with which the engineering student and the beginning young engineer are likely to be concerned. The more profound and subtle theory and mathematical thinking involved in the subject of limits, for example, have been intentionally left out.

The topics have been broken down into eight teaching units and three appendixes, as follows:

* Unit I	Slide Rule and Review of Arithmetic and Geometry
* Unit II	Basic Algebra
Unit III	Advanced Algebra and Logarithms
Unit IV	Introduction to Analytical Trigonometry
* Unit V	Numerical Trigonometry of the Right Triangle
Unit VI	Oblique Triangles and Applications of Numerical Trigonometry
Unit VII	Analytical Trigonometry
Unit VIII	Introduction to Calculus
* Appendix A	Computation Aids and Approximations
* Appendix B	Formulas of Geometry and Mensuration
* Appendix C	Solution of Higher-degree Equations

The starred units can be used in any order. Unit I is a mature review of arithmetic and geometry built around a study of the slide rule.

Unit II is a rather thorough review of basic algebra. As a review, it has been handled somewhat differently from a unit intended to introduce algebra.

Unit III continues where Unit II leaves off. This section is not a review and assumes that the student is studying these topics for the first time.

Unit IV introduces analytical trigonometry from the point of view of the electricity student and the more analytically inclined mechanics student.

Unit V is an elementary treatment of numerical trigonometry designed for the beginning student. This unit may be assigned on the opening day of school if desired.

Unit VI continues the study of numerical trigonometry and emphasizes the oblique triangle and trigonometric applications. The section of Chapter 14 on the use of logarithms in computation may be assigned in preparation for Unit VI without the earlier part of Unit V.

Unit VII is intended for electricity students, but Chapter 21 would apply as well for mechanics and building construction students.

Unit VIII is an introduction to the techniques of elementary calculus. The intention of the authors is here to introduce those techniques of calculus which will be useful to the engineering student in the pursuit of his major subjects. It is written for the student engineer and the student engineering technician. It definitely is not written for the physics major.

The Appendixes are used at appropriate times throughout the entire mathematics course.

The authors, in their courses, devote about a year and a half to the material in this book; but in courses that give a greater proportion of time to shopwork, the material may occupy a full two years.

The authors have been assisted in the preparation of the manuscript for this book by many persons.

C. W. Tudbury, now retired, was in charge of the mathematics department at Wentworth Institute when the manuscript was started. His successor, Clarence Paddock, maintained an equal interest until his retirement. Our present colleagues, J. A. Macdonald, A. M. Huyck, and R. C. Wheeler, have been particularly helpful. The following men have reviewed the entire manuscript except that portion on calculus and offered extremely valuable suggestions: Gordon Duvall of Ohio Mechanics Institute; D. H. Craighead of Ryerson Institute of Technology; Chauncey R. Kay of The Wyomissing Polytechnic Institute; LeRoy N. Young of Long Island Agricultural and Technical Institute; Professor T. J. Higgins of the University of Wisconsin.

To R. C. Pickett, president of Pickett and Eckel, Inc., the authors owe permission to photograph one of that company's slide rules. Lastly the authors wish to acknowledge the fine work of Janet Fiske, Mildred Meyer, George Morton, and James Nasson, who typed the manuscript.

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Contents

Preface v

UNIT ONE **SLIDE RULE AND REVIEW OF ARITHMETIC AND GEOMETRY**

Chapter 1 TREATMENT OF MEASURED DATA 1

Measured data · Rounding off numbers · Scientific notation

Chapter 2 THE SLIDE RULE 5

Scope of the slide rule · Description of the slide rule · Location of numbers on the scales; accuracy of the slide rule · Multiplication · Division · Location of decimal point in scientific notation · Combined multiplication and division · Proportion · Squares and square roots · Cubes and cube roots · Circumference and area of a circle · General suggestions for slide-rule operation

Chapter 3 TABLES AND INTERPOLATION 24

Extending the range of the tables · Interpolation between two points · Other helpful devices

Chapter 4 APPLICATIONS IN GEOMETRY 32

UNIT TWO **BASIC ALGEBRA**

Chapter 5 INTRODUCTION TO ALGEBRA 51

Explicit and literal numbers Absolute values · Addition · Subtraction · Multiplication · Division · Signs of operation · Algebraic expression · Some fundamental rules of exponents · Signs of grouping · Similar terms · Multiplying by a monomial · Multiplying one polynomial by another polynomial · Removing parentheses used to indicate multiplication · Dividing one monomial by another monomial · Division of a polynomial by a polynomial · Special products or expansions · Factoring · Properties of fractions · Least common multiple · Addition and subtraction of fractions · Multiplying two or more fractions · Dividing one fraction by another · Complex fractions

Chapter 6 LINEAR EQUATIONS IN ONE UNKNOWN 81

Equations · Operations on equations · Degree of an equation · To solve a linear equation · Formulas · Mathematical operations with dimensional units · Suggestions for the solution of applied problems involving linear equations

UNIT THREE **ANALYTIC GEOMETRY, MORE ADVANCED ALGEBRA, AND LOGARITHMS**

Chapter 7 FUNCTIONS AND GRAPHS 103

Constants and variables · Functions · Functional notation · Rectangular coordinates · Graph of a function · Functions not defined by equations

Chapter 8 SYSTEMS OF LINEAR EQUATIONS 127

Graphical solution of a system of two equations · Algebraic methods of solution · Elimination by addition or subtraction · Elimination by substitution · Solution by elimination by comparison · Literal linear equations · Equations linear in the reciprocals of the unknowns · Linear equations in more than two unknowns · Graphical solution of simultaneous linear equations

Chapter 9 EXPONENTS AND RADICALS 155

Laws of positive integral exponents · Roots and radicals · Principal roots · Rational and irrational numbers · Fractional exponents · Zero exponent · Negative exponents · Laws of radicals · Rationalizing the denominator in a radical · Reduction of the order of a radical · Addition and subtraction of radicals · Multiplication and division of radicals · Division by an irrational polynomial · Imaginary numbers · Operations with imaginary numbers · Complex numbers

Chapter 10 QUADRATIC EQUATIONS IN ONE UNKNOWN 180

Quadratic equations · Solution of incomplete quadratics · Solution of complete or affected quadratics · Solution by plotting · Solution by factoring · Solution by completing the square · Solution by the formula · Short graphical method · Literal quadratic equations ·

Precautions regarding roots · Vanishing roots · Extraneous roots · Checking · Summary of procedure for solving quadratic equations · Discriminant · Axis of symmetry—extreme value · Equations of the quadratic type · Applications leading to quadratic equations containing one unknown · Equations involving radicals · Applied problems in maxima and minima

Chapter 11 SIMULTANEOUS QUADRATIC EQUATIONS 210

Characteristic types of quadratic curves · Graphical solution · Algebraic solutions of quadratic systems · Solution of systems consisting of one linear and one quadratic equation · Equations of the form $ax^2 + by^2 = c$ · Elimination of constants · Reduction to simpler systems

Chapter 12 RATIO, PROPORTION, VARIATION 227

Quantities of the same kind · Proportions · Variation · Dimensionality of the proportionality constant · Applications

Chapter 13 THE BINOMIAL THEOREM—PROGRESSIONS 240

The binomial theorem · Expansion of a positive integral power of a binomial · The general term of the binomial expansion · Computation of powers and roots · Arithmetic progression · The nth term of an AP · The sum of the first n terms of an AP · Arithmetic means · Applications of arithmetic progressions · Geometric progression · The nth term of a GP · The sum of the first n terms of a GP · Geometric means · Infinite geometric progressions · Applications of geometric progressions

Chapter 14 LOGARITHMS 261

Definition of a logarithm · Graphical representation · Formation of a table of logarithms · Use of exponents in determining logarithms · Properties of logarithms · Multiplication · Division · Powers · Roots · Logarithm of unity · Reciprocal · Change of base · Systems of logarithms · Graphical derivation of table · Characteristics and mantissa · The antilogarithm of an expression · Role of logarithms in rough approximation · Using a table of logarithms · Interpolation · Computation using logarithms · Multiplication by the use of logarithms · Division by the use of logarithms · Combined multiplication and division · Cologarithms · Raising to a power by the use of logarithms · Extracting roots by the use of logarithms · Logarithmic computation of expressions involving addition and subtraction · Accuracy in logarithmic computations · Exponential equations · Aids to logarithmic computation · Relation of logarithms to the slide rule · Logarithmic computations on the slide rule

Chapter 15 MATHEMATICS OF INVESTMENT 303

Principal and interest · Simple interest · Compound interest · General formula · Present value · Nominal vs. effective rate · Use of binomial theorem · Annuities · Amount of annuity · Present value of an annuity · Time payments · Amortization and sinking funds · Perpetuities · Capitalized cost · Depreciation

Chapter 16 EXPONENTIAL FUNCTIONS—RATE OF GROWTH 326

Power function vs. exponential function · Compound-interest law · Significance of *e* · General formula · Natural logarithms · Use of the table · Relation between common and natural logarithms · Equivalent expressions

UNIT FOUR **INTRODUCTION TO ANALYTICAL TRIGONOMETRY**

Chapter 17 VECTORS AND TRIGONOMETRY 339

Vector and scalar quantities · Vector representation · Vectors and rectangular coordinates · Positive and negative angles · Units of angular measure · Angles of any magnitude · Addition and subtraction of angles · The quadrants · The coordinates of *P* in the four quadrants · Polar and rectangular coordinates · Components and projections · The trigonometric functions of first-quadrant angles · Functions of complementary angles in the first quadrant · The functions of 30, 45, and 60° · The zero divisor · Signed and unsigned numbers · The signs of the functions · The functions of zero degrees · The functions of 90° · The functions of 180° · The functions of 270 and 360° · Reducing the functions of any angle to functions of a first-quadrant angle · Variations in the functions · Inverse trigonometric functions · Some basic relations between the functions · Trigonometric identities

UNIT FIVE **NUMERICAL TRIGONOMETRY OF THE RIGHT TRIANGLE**

Chapter 18 THE RIGHT TRIANGLE 369

Angles · Arithmetical operations with degrees, minutes, and seconds · Unique determination of triangles · Solution of triangles · Trigonometric functions · Trigonometric tables · Use of table 18·1 · Trigonometric functions defined · The cofunctions · Reciprocal functions · Use of five-place trigonometric tables · Suggestions for solving right triangles · Interpolation in the five-place tables · Isosceles triangles · Regular polygons · Logarithmic solution of right triangles · Solution of oblique triangles without special formulas · Solution of an oblique triangle when three sides are given · Solution of an oblique triangle when two angles and a side are given · Solution of an oblique triangle when two sides and an angle are given · The slide-rule solution of right triangles · The slide-rule solution of oblique triangles · The solution of right triangles involving small angles

UNIT SIX **OBLIQUE TRIANGLES AND APPLICATIONS OF NUMERICAL TRIGONOMETRY**

Chapter 19 OBLIQUE TRIANGLES 415

The sine law · Derivation of the sine law · The cosine law · Derivation of the cosine law · The tangent law · Derivation of the tangent law · The half-angle laws · Derivation of the half-angle laws · Area of triangles · The ratio of sin ϕ to ϕ

Chapter 20 TRIGONOMETRIC APPLICATIONS AND REVIEW 444

Plane surveying · Measurement of angles · Mapping · Latitudes and departures · Azimuth angles · Interior angles · Calculation of areas · Measurement of distances in a vertical plane · Surveying around obstructions · Applications to machine- and tool-design problems · Solid figures

UNIT SEVEN **ANALYTICAL TRIGONOMETRY**

Chapter 21 VECTORS 471

Traverse courses as vectors · Vector addition · Polar and rectangular coordinates · Topographic and polar diagrams · Vector subtraction · The product of a vector and a scalar quantity · Velocity as a vector · Forces as vectors · Resolution of forces · Resolution of general vectors · The product of two vectors · Reciprocal relations

Chapter 22 TRIGONOMETRIC FORMULAS, IDENTITIES, AND EQUATIONS 501

Functions of the sum of two angles · Functions of the difference between two angles · Functions of twice an angle · Functions of half an angle · Trigonometric identities · Trigonometric equations

Chapter 23 GRAPHS OF THE TRIGONOMETRIC FUNCTIONS 514

Some periodic functions · Sine and cosine graphs · Average ordinates of the curves of sin ϕ and $\sin^2 \phi$ · Plotting the sine and cosine functions by geometric methods · Rotating vectors · Adding sine waves of the same frequency · Amplitude and phase relationships · The equivalence of graphical and algebraic methods · Adding harmonically related sine functions · Simple harmonic motion · Simple harmonic motion defined · Equations for coordinates of P · Phase · Parametric equations · Parametric equations for a sine curve · Lissajous figures

Chapter 24 COMPLEX NOTATION AND VECTOR ALGEBRA 542

The reference vectors · Addition of the reference vectors · Vector addition · Vector subtraction · Vector multiplication · Vector division · An alternative vector expression · The nth power of a vector · The nth roots of a vector · Demoivre's theorem · Euler's equation · The sine and cosine series · Rotating vectors · Some interesting side lights on Euler's equation · The hyperbolic sine and cosine

UNIT EIGHT AN INTRODUCTION TO CALCULUS

Chapter 25 GRAPHICAL METHODS 575

Functions · Dependence · Rates of change · Area under a straight-line graph · Calculation of work from a force-distance graph · Calculation of distance from a speed-time graph · Slope of nonlinear functions · Tangents defined · Average and instantaneous slopes and rates · The derivative · Approximate increments · Differential notation · Maxima and minima · Points of inflection · Derived curves · The area under a nonlinear curve · Summary

Chapter 26 DIFFERENTIATION 621

Average slopes and rates found by algebraic methods · Limits · General rule for differentiating a power function · The derivative of a sum or difference · The derivative of $y = u^n$ when u is a function of x · The derivative of the function of a function · Maxima and minima · The derivative of sin θ and cos ϕ · The derivative of sin u where u is a function of θ · The derivative of cos u where u is a function of θ · Differentiating the function $y = e^x$ · The derivative of e^u where u is a function of x · The derivative of ln x · The derivative of ln u where u is a function of x · The derivative of ka^u where a is any positive constant and u is a function of x · General proof for $dy/dx = knx^{n-1}$ · The derivative of a product and a quotient · The derivative of tan u · Flexion · Repeated differentiation · Derived curves

Chapter 27 INTEGRATION 674

Introduction to integration · The indefinite integral · The rule for $\int kx^n$ · The multiplied constant in integration · Evaluating the constant of integration · The importance of the differential du · Uniformly accelerated motion · Areas under graphs · The definite integral · The definite integral as the limit of a sum · Average ordinates · The integral of $ku^n\,du$ when $n = -1$ · The integral of sin $u\,du$ · The integral of $ke^u\,du$ · Volume by integration

Appendixes

A COMPUTATION AIDS AND APPROXIMATIONS 719

B FORMULAS OF GEOMETRY AND MENSURATION 739

C SOLUTION OF HIGHER-DEGREE EQUATIONS 758

Index 763

Answers 773

UNIT ONE

SLIDE RULE AND REVIEW OF ARITHMETIC AND GEOMETRY

Efficient computing methods applied to arithmetic, mensuration, and geometry—how to use the slide rule

Chapter 1 TREATMENT OF MEASURED DATA

In the first few chapters of this book we shall review some of the methods whereby much of the drudgery of computation may be eliminated and the effectiveness of the work increased. We shall discuss the efficient use of the slide rule, of tables, and (in the Appendix) of short cuts, approximations, formulas, and graph paper.

1·1 Measured Data. Many of the data with which the average technical man works are obtained experimentally. There is a definite limit to their reliability. The *reliability* of a number may be expressed in terms of either precision or accuracy. *Precision* is gauged by the position of the last reliable digit relative to the decimal point, whereas *accuracy* is measured by the number of significant figures. *Significant figures* are those known to be reliable and include any zeros not merely used to locate the decimal point.

For instance, if the diameters of several wires had been measured with a micrometer and found to be 0.118, 0.056, 0.008, and 0.207 in., one might say that these diameters had been measured to a precision of 0.001 in., and to an accuracy of three, two, one, and three figures, respectively.

Should the definition of significant figures seem somewhat arbitrary, let us consider the computation of the volume of a rectangular sheet of metal. Suppose that the measured length, width, and thickness are 165.2, 5.07, and 0.0021 in., respectively, and that these measurements are correct to the last digit given. Let us now compare the effect on the volume of changing the last digit of each measurement by one. It will be seen that such a change introduces respective errors of about one-sixteenth of 1 per cent, one-fifth of 1 per cent, and 5 per cent. The length, then, is the most accurate and the thickness the least accurate.

1·2 Rounding Off Numbers. Frequently a result will be rounded off because the last several digits either are in doubt or are not required in that particular computation. The operation of rounding off is governed by the following rule:

If the figures to be rejected represent less than half a unit in the last place to be retained, they are dropped. If they represent more than half a unit in the last place to be retained, the last retained digit is increased by one. If the rejected part represents just half a unit in the last place to be retained, the last retained significant digit is left even or raised to the nearest even number.

EXAMPLE 1

Number	Rounded off to Four figures	Three figures	Two figures
3.1416	3.142	3.14	3.1
14.815	14.82	14.8	15.
321.35	321.4	321	320
6,274.5	6,274	6,270	6,300

In addition and subtraction the precision of the answer corresponds to the least precise of the quantities involved. *Perform the addition or subtraction, and round off by eliminating any digits resulting from operations on broken columns on the right.*

EXAMPLE 2. Add:

```
175.6
  2.126
 13.04
  0.0028
190.7688 (Round off to 190.8.)
```

In multiplication and division the accuracy of the answer corresponds to the least accurate of the quantities involved. *Perform the multiplication or division and round off the answer to a number of significant figures equal to that in the least accurate quantity in the computation.*

EXAMPLE 3. Multiply:

$$3.14159 \times 47.82 = 150.2308338$$

Although the multiplicand has six significant figures, the multiplier has only four; therefore, we round off the product to four significant figures and get 150.2.

1·3 Scientific Notation. In scientific work very large or very small numbers are expressed as a number between 1 and 10 times an integral power of 10. Thus 2,580,000 would be written 2.58×10^6, and 0.0000258 would be written 2.58×10^{-5}. The magnitude of the number is revealed by a glance at the exponent (see Table 14·4, page 271).

Several other advantages in this notation will become apparent. Space is saved, a particularly important point in tabulating data. The labor of counting figures to the right or left of the decimal point —a labor attended by risk of error—is eliminated. The accuracy with which a quantity is known is indicated by the number of figures to the right of the decimal point. For example, when we consider the number 72,000, we cannot tell whether there are two, three, four, or five significant figures. No uncertainty exists when we write 7.2×10^4, 7.20×10^4, 7.200×10^4, or 7.2000×10^4.

The ease of dealing with large and small quantities in this manner is illustrated by the following problem.

Simplify the expression

$$\frac{400{,}000 \times 8{,}000{,}000 \times 0.0045}{60{,}000 \times 0.025 \times 100} = \frac{4 \times 10^5 \times 8 \times 10^6 \times 4.5 \times 10^{-3}}{6 \times 10^4 \times 2.5 \times 10^{-2} \times 10^2}$$

$$= \frac{4 \times 8 \times 4.5}{6 \times 2.5} \times 10^{(5+6-3)-(4-2+2)} = 9.6 \times 10^4 = 96{,}000$$

There are two instances in which we depart from the rule of expressing a quantity as a number between 1 and 10 times a suitable power of 10. If we were to extract the square root of 2.5×10^{-7}, we should write this as 25×10^{-8} in order to make the exponent of 10 divisible by the index of the root. The square root is readily seen to be 5×10^{-4}. Also, when quantities are to be added and subtracted, they must have the same exponents. Thus $4 \times 10^{-7} + 7 \times 10^{-5} = 4 \times 10^{-7} + 700 \times 10^{-7} = 704 \times 10^{-7} = 7.04 \times 10^{-5}$.

EXERCISE

1. Translate into ordinary notation: 5.18×10^6; 3.76×10^{-4}; 7.5×10^{-8}; 4.375×10^2.

2. Translate into scientific notation:

1 year = 31,500,000 sec (approx)
1 light-second = 186,000 miles
Wavelength of blue light = 0.000047 cm

3. $\dfrac{5 \times 10^7 \times 9 \times 10^3 \times 400}{4.8 \times 10^4} = ?$

4. $\dfrac{6 \times 10^2 \times 15 \times 10^4 \times 4 \times 10^{-1}}{8 \times 10^3 \times 25 \times 10^5} = ?$

5. $\dfrac{280{,}000 \times 16{,}000 \times 0.009}{2{,}100 \times 2{,}400{,}000 \times 0.04} = ?$

6. $\dfrac{0.12 \times 5{,}000 \times 33{,}000}{550 \times 18 \times 0.0002} = ?$

7. $\dfrac{1.2 \times 10^{5} \times 9 \times 10^{-2}}{16 \times 10^{3} \times 1.5 \times 10^{-4}} - 20{,}000 + \dfrac{7 \times 10^{6}}{2 \times 10^{2}} = ?$