

Understanding Quantum Physics

A User's Manual

Michael A. Morrison

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University of Oklahoma



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Preface

"I can't stand it, you know," he said,
"your paradoxes are too monstrous..."
"You are quite wrong," said Ambrose.
"I never make paradoxes; I wish I could."

—*The White People*
Arthur Machen

This is a book about physics, physics as natural philosophy. Its themes are embedded in the myriad beautiful connections among the physical concepts that govern the atomic and sub-atomic domain of matter. It is also a book about how these concepts manifest themselves in the mathematical machinery of quantum mechanics. In practice, these formulas are tools for solving problems involving microscopic systems; by applying them to a host of such problems you can begin to gain "quantum insight" into the nature of reality as understood in modern physics. Such understanding amounts to nothing less than a new way of thinking about the universe. My goal in this book is to provide an in-depth introduction to the concepts of quantum physics and their practical implementation.

Although definitely a physics text, this book contains several dollops of history, biography, and philosophy. I want you to appreciate that quantum physics is more than a collection of abstract laws and equations; it is a paradigm of reality at its most fundamental level, a paradigm with profound, provocative implications. And I want you to meet a few of the men and women who, early in the 20th century, created this towering intellectual achievement. So in these pages you'll have a chance to read some of their words.

I have tried to write a "user-friendly" textbook and an honest one. I have not dodged or hidden the difficulties of quantum mechanics or its applications. This is not *Quantum Physics Made Simple*—I don't believe quantum physics can be made *simple*. But I have tried to make it clear, sacrificing where necessary the elegance of brevity and rigor on the altar of clarity.

Nor is this book *Quantum Mechanics for Everybody*. You'll need a firm foundation in freshman and sophomore physics (mechanics, electricity and magnetism, and thermodynamics), a solid working knowledge of calculus and algebra, and at least a passing acquaintance with the physics of waves.

This book contains more material than one can cover in a single semester, so be selective in what you study. I and others who have used the book have found that by omitting some of the optional (starred) sections, undergraduates (at the junior level) can cover Volume I in about 15 weeks.

Since I began writing this book in 1981, many colleagues have asked, “Why write *another* quantum mechanics text?” A fair question. Neither the selection of topics nor their sequence in this book is particularly unconventional. But what I hope distinguishes it is its persistent emphasis on comprehension of the fundamentals of quantum physics and on how various aspects of this subject relate to one another, the sense in which quantum mechanics is a coherent intellectual structure. I care at least as much that you *understand* quantum mechanics as that you know how to *use* it to solve problems.

Consequently Parts I and II focus on explaining and motivating the postulates that underlie the machinery of quantum mechanics. You can cover this material rather quickly, but do take time to think about it. The postulates are the heart and soul of quantum physics. Our first applications, in Part III, are simple single-particle systems in one dimension. Such problems highlight the essentials of quantum physics but avoid the mathematical clutter and complexity of three-dimensional systems. These are not mere pedantic exercises; research physicists often use one-dimensional models to approximate real-world systems that are too complicated to solve or understand. So at the end of most chapters in Part III we'll take a look at such applications.

In Volume II we'll apply these fundamentals to the “real world” of three-dimensional, many-particle microscopic systems and will learn how to solve problems that are too difficult to admit exact solution. There we'll take up approximation methods such as perturbation theory and the variational method, three-dimensional properties such as orbital angular momentum, a new kind of angular momentum called spin, and many-particle systems, including atoms and molecules.

In both volumes I have provided considerable structure to help you organize your study. In addition to the customary chapters, sections, and subsections, they contain all sorts of asides, footnotes, boxed results, tables of formulae, etc. Think of this book as a layer cake with several levels of flavor. The most important material is in the top layer, the text. In the asides and footnotes you'll find all sorts of tasty morsels: supplementary information, extensions of material, suggested readings and references—things that would interrupt your train of thought were they in the text proper.

The best source of advice about how to study a book is not the author. Several students who used this manuscript in preliminary form, suggest the following: *Plan to go through each chapter several times. On your first reading, skip sections or subsections that seem difficult, planning to return to them later. The first time through, don't even read the asides and footnotes; come back to them also on subsequent readings. Above all, work the Questions, Exercises and Problems.*

The best advice I have ever read about how to study was written by the mathematician Paul R. Halmos in his “autobiography” *I Want to Be A Mathematician* (New York: Springer-Verlag, 1985), “[S]tudy actively. Don't just read [the text]; fight it! Ask your own questions, look for your own examples, discover your own proofs.” Halmos was writing about math, but his advice pertains as well to physics. If you follow it, you will come away from this book with a solid, working knowledge of quantum physics; if you ignore it, the prospects are grim. You can no more learn quantum mechanics without practice than you could learn how to ride a bike, play a piano, or swim without riding, playing, or swimming.

That's why you'll find in each chapter a host of Questions that will provoke you to think about the material and, at the end of each chapter, a wide selection of Exercises and Problems. Some of these anticipate material in subsequent chapters; others extend the content of the chapter you just read; still others provide opportunities to become familiar with new ideas through simple practice. I hope you will at least think through *all* the Questions and work *all* the Exercises. The Problems are a bit more challenging, but I've provided lots of help: most contain detailed directions, suggestions, helpful hints, etc. I cannot overemphasize the importance of your working as many of these as possible.

You'll also find throughout the book a host of very detailed examples. Rather than simply work these examples, I've tried to show you how to approach them, how to figure out how to work them. Please don't treat these as recipes in a cookbook. Rather, consider them as paradigms; each illustrates one of many possible ways to apply the principles you have studied. You will learn more and have more fun if instead of parroting what I do, you discover your own approaches to problem solving.

Of course, presenting examples in great detail takes a lot of space, and I've been able to include only a handful of the many useful illustrations of introductory quantum mechanics. So seek out others. Dig around in the many fine quantum books you'll find in your library (see the Bibliography at the end of this book). Better yet, make up your own examples. Again, Halmos's advice is invaluable: "A good stock of examples, as large as possible, is indispensable for a thorough understanding of any concept, and when I want to learn something new, I make it my first job to build one."

Each chapter also contains recommended readings, annotated lists of additional sources that will enhance your understanding and enjoyment of quantum physics. Please don't ignore these recommendations. Reading several explanations of important topics is good strategy for the study of any subject, but it's particularly important for quantum mechanics, which is so counter-intuitive that you need to see it from several vantage points.

This book grew out of my desire to convey a sense of the fascination and beauty of quantum physics. This won't be the only time you study this subject, nor should it be. Quantum mechanics is like a many-faceted jewel: every time you look at it you discover amazing new things. But I have the privilege of introducing the subject to you, so I particularly hope that this book communicates some of my enthusiasm for it. I believe that learning quantum physics can and should be a joyous activity. Although undeniably hard work, the study of quantum mechanics and its applications to atoms, molecules, solids, nuclei, and the whole rest of the microcosm can be a tremendously exciting intellectual adventure.

Finally, I want to know what you think. I hope that, if you feel so inclined, you'll write to me at the Department of Physics and Astronomy, University of Oklahoma, Norman, Oklahoma 73019 with your opinions of this book, your reaction to quantum physics, any errors you find in the text, or any suggestions you have for improvement should there be a second edition. But for now, settle down in a comfortable chair, get out some scratch paper, and dig in. You are about to be plunged into the Marvelous.

Michael A. Morrison
Norman, Oklahoma

Acknowledgments

One of the great pleasures of finishing a book is thanking those who helped in its creation. First and foremost I owe a great intellectual debt to the mentors who have helped me learn how to think about quantum physics: my professors at Rice University, G. King Walters, Neal F. Lane, and Thomas L. Estle, and my long-time friend and colleague Dr. Lee A. Collins. My other mentors I know only through their works. Like most later-generation textbooks, mine is in part a mosaic of variations on the best of what I have found in many sources, primarily in the vast library of quantum mechanics texts that preceded this one. I have acknowledged these sources collectively in the Bibliography at the end of this book and, where memory served, in footnotes to the text.

Closer to home, I am grateful to my fellow faculty members in the Department of Physics and Astronomy at the University of Oklahoma. During the many years since I began work on this book, they have consistently tolerated, encouraged, and supported me. In particular, Drs. Robert Petry, Thomas Miller, Deborah Watson, and Gregory Parker used various incarnations of this manuscript in classes and provided valuable feedback and brickbats. And Drs. Miller, Parker, Jack Cohn, Ryan Doezema, and Suzanne Willis read portions of the manuscript for technical accuracy.

Students played a crucial role in the latter stages of work on this book. I have tested all or part of the manuscript in five courses. Although space precludes my thanking all of the long-suffering students in those classes, I must single out a few—Thomas Haynes, Brian Argrow, José Rodriguez, Bok-hyun Yoon, and Bryan Biegel—for their extraordinary efforts on behalf of this project. Three graduate students, Wayne Trail, Bill Isaacs, and Brian Elza, carefully read the manuscript in proof and found errors I would not have believed were still in it. Finally, I'm indebted to three colleagues, Drs. Ron Olowin, Richard Henry, and Sheridan Simon, who provided solace and suggestions during our eleventh-hour search for a new title.

I and this book also benefited from the advice of a number of outside reviewers, including Drs. Robert Hallock, Sheridan Simon, John L. Stevens, Sanford Kern, H. A. Gersh, Stephan Estreicher, and Robert Silbey. Thanks folks: I didn't adopt all your suggestions, but I considered each one carefully.

Preparing this manuscript for production would have been impossible had it not been for superb technical help from two sources. Ms. Johnette Ellis provided assistance beyond the call of duty during the trying end-game months of the summer of 1987. And my wife Mary, who does not share my fondness for physics, nevertheless proofread this entire, vast manuscript.

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It is characteristic of all deep human problems
that they are not to be approached without
some humor and some bewilderment—
science is no exception.

—Freeman Dyson
Disturbing the Universe

In trying to achieve success
No envy racks our hearts,
For all we know and all we guess,
We mutually impart.

—Gilbert and Sullivan
Princess Ida

We shall go on seeking [truth] to the end,
so long as there are men on the earth.
We shall seek it in all manner of strange ways:
some of them wise,
and some of them utterly foolish.
But the search will never end.

—Arthur Machen
“With the Gods in Spring”

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CHAPTER 1

Introductory

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