PROTTER / MORREY

COLLEGE CALCULUS WITH ANALYTIC GEOMETRY

College Calculus with Analytic Geometry

by

MURRAY H. PROTTER

University of California

Berkeley, California

and

CHARLES B. MORREY, JR.

University of California

Berkeley, California

ADDISON-WESLEY PUBLISHING COMPANY, INC.

Reading, Massachusetts · Palo Alto · London

Library of Congress Catalog Card No. 64-14734

College Calculus with Analytic Geometry

This book is in the

ADDISON-WESLEY SERIES IN MATHEMATICS

LYNN H. LOOMIS
Consulting Editor

Preface

At many colleges and universities it is customary to have two calculus courses. One course, planned for the majority of the students, treats topics in calculus and analytic geometry in the usual way, while the other, designed for honor students, is devoted to a rigorous and sophisticated development of the subject. While it is true that at the end of the third or fourth semester the honor students have covered all the material in the regular course (and indeed much more), it is usually the case that by the end of the first of these semesters the two courses have covered quite different selections of topics in calculus and analytic geometry. By the end of the first semester it invariably turns out that some students who are in the honor group more properly belong in the regular course; moreover, the best of the students who have been taking the standard material not only have the ability to master the more penetrating aspects of calculus but are often anxious to do so.

The task of shifting students from one course to the other after only one semester or less becomes rather difficult, and frequently the student who makes the transfer is penalized. Often even the talented beginner is reluctant to embark on the honors program: if he does not do well he discovers not only that his grade has suffered but also that his background is not adequate for him to transfer to the regular course. A shift in the opposite direction is a problem also. The student who has done well enough in the regular course to transfer to the honor section hesitates to do so because he has not studied the same material as those who have already been doing honors work for a full semester.

This text, together with *University Calculus*, by Charles B. Morrey, Jr.,* is designed to solve the problem described above. The topics taken up in this volume meet the needs of the majority of students taking the customary course (twelve semester hours) of calculus and analytic geometry at a college or university. This book leans heavily on the intuitive approach, gives many illustrative examples, emphasizes physical applications wherever suitable, and has a large selection of graded exercises. Definitions and theorems are stated with care and proofs of simple theorems are given in full. It is a companion to Morrey's *University Calculus*, which presents the same material (except for the last chapter) in the same order, but at a level suitable for the honor student. Morrey's text goes into the theory in much more detail, has complete proofs of many of the more difficult theorems, and in general gives a rigorous, soundly based treatment of calculus and analytic geometry. If both texts are used, one for the regular group and the other for the honors group, there should be little difficulty in shifting students back and forth depending on their ability to absorb rigorous mathematics.

---

* Reading, Mass.: Addison-Wesley Publishing Co., 1962.

At colleges where there are not enough students to form an honors group or where it is not the custom to do so, this text could be used, with Morrey's *University Calculus* recommended to those students who wish to do outside reading. Since both books use the same terminology and notation, the good student will be able to concentrate on mastering the theory. (When an unrelated book is used for outside reading the student may waste a great deal of time transposing the presentation into something he understands.)

Chapter 1 discusses inequalities, with emphasis on the solution of inequalities which contain the absolute-value symbol. Chapters 2 and 3 take up functions, functional notation, the elements of analytic geometry and, in particular, systems of linear inequalities in the plane.

Chapter 4, entitled "Preview of the Calculus," treats the fundamental notions of limit, differentiation, and integration in an informal way. This helps meet the need of many students of physics and engineering who are required to know as early as possible some of the elementary processes of calculus.

A rather thorough treatment of limits is given in Chapter 5. The definition of limit is illustrated geometrically, and the theorems on limits and continuity are stated and discussed.

Chapters 6 and 7 give a traditional development of the differentiation of algebraic functions and applications to problems of maxima and minima, related rates, and so forth.

The notion of area (Jordan content) is defined carefully in Chapter 8. This leads to the definition of integral and the Fundamental Theorem of Calculus. Applications are made to problems of liquid pressure, work, and so forth.

Chapters 9 and 10 resume the work on analytic geometry which was begun in Chapter 3. The methods of calculus are here used to great advantage. The amount of material on conics and related subjects is at least as great as that found in many texts devoted to analytic geometry alone.

The natural logarithm is defined by the integral and the exponential function is defined as its inverse. This approach is being used experimentally (and with apparent success) for students in eleventh-grade algebra and trigonometry classes under the SMSG program.

Vectors in the plane are the subject of Chapter 14 and vectors in space are treated in Chapter 18. Certain logical difficulties are avoided by defining a vector as an equivalence class of directed line segments. Furthermore, the discussion of equivalence classes puts this abstract concept in a natural setting. The material of these chapters is relatively independent of the rest of the book and could easily be omitted. On the other hand, for those who wish to introduce vectors early in calculus, much of Chapter 14 could be inserted after Chapter 6.

Chapter 17 is devoted to solid analytic geometry with coordinates used throughout. Once the student has mastered this material, the applications using vector terminology, which appear in Chapter 18, may be attacked with confidence.

The study of infinite series, taken up in Chapter 19, completes the customary elementary course in the calculus of functions of one variable. Chapters 20 and 21 are devoted to the initial topics in the calculus of functions of several variables.

Partial differentiation, line integrals and applications are taken up in Chapter 20. A definition of volume analogous to that given for area is discussed in Chapter 21. The elements of multiple integration with applications to area, volume, and mass are treated. In addition there are a number of physical applications to problems in center of gravity, moment of inertia, and so forth.

The last chapter is devoted to an elementary study of linear algebra. This chapter replaces the unit on differential equations in *University Calculus*. In recent years it has become evident that students in all the sciences have an urgent need for the elements of linear algebra. The presentation here is intended as a beginning study. Students who require or wish additional material would logically proceed to a course devoted entirely to linear algebra.

Although the subject of numerical analysis is not discussed, it is not entirely ignored. There are brief descriptions of the numerical implications of such topics as evaluation of integrals, computation of maxima and minima, solution of linear inequalities, etc. The purpose here is to make the student aware of the impact of digital computers on various branches of analysis. These remarks may be used as a springboard for a more detailed investigation of numerical analysis.

Berkeley, California
November 1963

M.H.P.
C.B.M., Jr.

Contents

CHAPTER 1

Inequalities

1. INEQUALITIES

In elementary algebra and geometry we study equalities almost exclusively. The solution of linear and quadratic algebraic equations, the congruence of geometric figures, and relationships among various trigonometric functions are topics concerned with equality. As we progress in the development of mathematical ideas — especially in that branch of mathematics of which calculus is a part — we shall see that the study of inequalities is both interesting and useful. An inequality is involved when we are more concerned with the approximate size of a quantity than we are with its true value. Since the proofs of some of the most important theorems in calculus depend on certain approximations, it is essential that we develop a facility for working with inequalities.

We shall be concerned with inequalities among real numbers, and we begin by recalling some familiar relationships. Given that a and b are any two real numbers, the symbol

$$a < b$$

means that *a is less than b*. We may also write the same inequality in the *opposite direction*,

$$b > a,$$

which is read *b is greater than a*.

The rules for handling inequalities come from our knowledge of arithmetic and are only slightly more complicated than the ones we learned in algebra for equalities. However, the differences are so important that we state them as four Rules of Inequalities, and they must be learned carefully.

1. If $a < b$ and $b < c$, then $a < c$. In words: *if a is less than b and b is less than c, then a is less than c.*
2. If c is any number and $a < b$, then it is also true that $a + c < b + c$ and $a - c < b - c$. In words: *if the same number is added to or subtracted from each side of an inequality, the result is an inequality in the same direction.*
3. If $a < b$ and $c < d$ then $a + c < b + d$. That is, *inequalities in the same direction may be added.* It is important to note that in general inequalities may *not* be subtracted. For example, $2 < 5$ and $1 < 7$. We can say, by addition, that $3 < 12$, but note that subtraction would state the absurdity that 1 is less than -2.

4. If $a < b$ and c is any positive number,

then $$ac < bc,$$

while if c is a negative number,

then $$ac > bc.$$

In words: *multiplication of both sides of an inequality by the same positive number preserves the direction, while multiplication by a negative number reverses the direction of the inequality.*

Since dividing an inequality by a number d is the same as multiplying it by $1/d$, we see that Rule 4 applies for division as well as for multiplication.

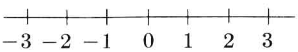

FIGURE 1–1

FIGURE 1–2

FIGURE 1–3

From the geometric point of view we associate a horizontal axis with the totality of real numbers. The origin may be selected at any convenient point, with positive numbers to the right and negative numbers to the left (Fig. 1–1). For every real number there will be a corresponding point on the line and, conversely, every point will represent a real number. Then the inequality $a < b$ could be read: *a is to the left of b.* This geometric way of looking at inequalities is frequently of help in solving problems. It is also helpful to introduce the notion of an *interval of numbers* or *points.* If a and b are numbers (as shown in Fig. 1–2), then the *open interval* from a to b is the collection of all numbers which are both larger than a and smaller than b. That is, an open interval consists of all numbers *between a and b.* A number x is between a and b if *both* inequalities $a < x$ and $x < b$ are true. A compact way of writing this is

$$a < x < b.$$

The *closed interval* from a to b consists of all the points between a and b, *including* a and b (Fig. 1–3). Suppose a number x is either equal to a or larger than a, but we don't know which. We write this conveniently as $x \geq a$, which is read: *x is greater than or equal to a.* Similarly, $x \leq b$ is read: *x is less than or equal to b,* and means that x may be either smaller than b or may be b itself. A compact way of designating a closed interval from a to b is to state that it consists of all points x such that

$$a \leq x \leq b.$$

An interval which contains the endpoint b but not a is said to be *half-open on the left.* That is, it consists of all points x such that

$$a < x \leq b.$$

Similarly, an interval containing a but not b is called *half-open on the right*, and we write

$$a \leq x < b.$$

Parentheses and brackets are used as symbols for intervals in the following way:

(a, b) for the open interval: $a < x < b$,
$[a, b]$ for the closed interval: $a \leq x \leq b$,
$(a, b]$ for the interval half-open on the left: $a < x \leq b$,
$[a, b)$ for the interval half-open on the right: $a \leq x < b$.

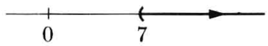

FIGURE 1–4

We can extend the idea of an interval of points to cover some unusual cases. Suppose we wish to consider *all* numbers larger than 7. This may be thought of as an interval extending to infinity to the right. (See Fig. 1–4.) Of course, infinity is not a number, but we use the symbol $(7, \infty)$ to represent all numbers larger than 7. We could also write: all numbers x such that

$$7 < x < \infty.$$

In a similar way, the symbol $(-\infty, 12)$ will stand for all numbers less than 12. The double inequality

$$-\infty < x < 12$$

is an equivalent way of representing all numbers x less than 12.

The first-degree equation $3x + 7 = 19$ has a unique solution, $x = 4$. The quadratic equation $x^2 - x - 2 = 0$ has two solutions, $x = -1$ and $x = 2$. The trigonometric equation $\sin x = \frac{1}{2}$ has an infinite number of solutions: $x = 30°, 150°, 390°, 510°, \ldots$. *The solution of an inequality* involving a single unknown, say x, *is the collection of all numbers which make the inequality a true statement*. Sometimes this is called the *solution set*. For example, the inequality

$$3x - 7 < 8$$

has as its solution *all* numbers less than 5. To demonstrate this we argue in the following way. If x is a number which satisfies the above inequality we can, by Rule 2, add 7 to both sides of the inequality and obtain a true statement. That is, we have

$$3x - 7 + 7 < 8 + 7, \qquad \text{or} \qquad 3x < 15.$$

Now, dividing both sides by 3 (Rule 4), we obtain

$$x < 5,$$

and we observe that *if* x is a solution, *then* it is less than 5. Strictly speaking, however, we have not *proved* that every number which is less than 5 is a solution.