Perspectives in Control Theory

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Preface

The volume contains papers based on lectures delivered during the school "Perspectives in Control Theory" held in Sielpia, Poland on September 19–24, 1988.

The aim of the school was to give the state-of-the-art presentation of recent achievements as well as perspectives in such fields of control theory as optimal control and optimization, linear systems, and nonlinear systems. Accordingly, the volume includes survey papers together with presentations of some recent results. The special emphasis is put on:

— nonlinear systems (algebraic and geometric methods),

— optimal control and optimization (general problems, distributed parameter systems),

— linear systems (linear-quadratic problem, robust stabilization).

An important feature of the school (and consequently of the volume) was its really "international" character since it brought together leading control theoriests from West and East. All together the school was attended by 108 participants from 18 countries. During the school 21 one-hour invited lectures were delivered. Moreover, five half-an-hour talks were given and 30 contributions were presented in frames of poster sessions.

The school was organized and supported by:

— Institute of Mathematics of the Polish Academy of Sciences,

— Committee of Automatic Control and Robotics of the Polish Academy of Sciences,

— Institute of Automatic Control, Warsaw University of Technology (as Coordinator of the Basic Research Program R.P.I.02 "Theory of Control of Continuous Dynamic Systems and Discrete Processes").

The organizing committe consisted of: B. Frelek, B. Jakubczyk, T. Kaczorek, M. Kocięcki, K. Malanowski (vice-chairman), M. Niezgódka, A. Olbrot, C. Olech (chairman), W. Respondek (secretary), A. Sosnowski, A. Wierzbicki.

We would like to thank Ms. M. Wolińska for her excelent typing of some of the manuscripts.

B. JAKUBCZYK
K. MALANOWSKI
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Warsaw, August 1989.

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Jozsef Bokor, Hungarian Academy of Sciences

Hector O. Fattorini, University of California

Michel Fliess, Ecole Superieure d'Electricité

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Contents

REMARKS ON THE STABILIZABILITY OF NONLINEAR SYSTEMS BY SMOOTH FEEDBACK

Dirk Aeyels

Abstract

In this paper we discuss the known result that if a system is smoothly stabilizable then adding an integrator does not change this property. For this extended system stabilizing feedbacks depending on data explicitly available from the original system are proposed.

1. Introduction

Consider the following system

$$\dot{x} = f(x,u)$$

with $f: R^n x R \dashrightarrow R^n$ smooth, $f(0,0) = 0$.

Suppose that this system is smoothly stabilizable, i.e. there exists a smooth function $k: R^n \dashrightarrow R$ such that $k(0)=0$ and the origin is asymptotically stable for

$$\dot{x} = f(x,k(x)) \quad (1)$$

It is well known that the "extended" system

$$\dot{x} = f(x,y)$$
$$\dot{y} = u \quad (2)$$

is also smoothly stabilizable.

The proof of this result is given in [4], where additional references also containing the proof are given. It will be repeated in the next section.It will be seen that the feedback which stabilizes the extended system depends on a Lyapunov fuction for the stabilized system (1). This Lyapunov function exists by the inverse Lyapunov theorem [2], but an explicit expression is in general not available for implementation.

In this paper we discuss the stabilization of (2) by means of a procedure independent of the Lyapunov function corresponding to (1). The proposed feedback --although ensuring local stability--does not guarantee global stability. This will be shown by means of a counterexample.

However for some classes of nonlinear systems an indication will be given showing that the proposed feedback is globally stabilizing. The results concerning the global stabilization issue are incomplete. They will be the subject of a forthcoming paper.

2. Stabilization of the extended system

In this section we recall the proof (taken from [1]) that if the original system (1) is smoothly stabilizable, then the extended system is also smoothly stabilizable .

Assume that $\dot{x} = f(x,u)$ is smoothly stabilizable. Let

$$f_0(x) := f(x,k(x))$$

be the closed-loop system.
By the inverse Lyapunov theorem [2], there exists a positive definite function V such that $L_{f_0} V(x) < 0$ for all $x \neq 0$

Since f and k are smooth, there exists a smooth function g defined on R^{n+1} such that for all x,z

$$f(x,k(x)+z) = f_0(x)+zg(x,z)$$

Introduce the positive definite function on R^{n+1}:

$$W(x,y) := V(x)+ \tfrac{1}{2}(y-k(x))^2$$

Take the feedback for (2)

$$u(x,y) = -y+k(x)+ k(x).f(x,y)- V(x).g(x,y-k(x))$$

Take the derivative of W along trajectories of (2) with the feedback just defined

$$\begin{aligned}\text{Then } \dot{W} &= \nabla V(x).f(x,y)+(y-k(x))(\dot{y}-\nabla k(x).\dot{x}) \\ &= \nabla V(x).f(x,y)+(y-k(x)(-y+k(x)-\nabla V(x).g(x,y-k(x)) \\ &= \nabla V(x).f(x,y)-(y-k(x))^2-(y-k(x)).\nabla V(x)g(x,y-k(x))\end{aligned}$$

Since $f(x,y) = f(x,k(x))+(y-k(x))g(x,y-k(x))$

$$\dot{W} = L_{f_0} V(x)-(y-k(x))^2 < 0$$

for all nonzero (x,y). This assures stability.

It is remarked that the proposed feedback assures local stability of the extended system if the original system is locally stabilizable. It also assures global stability of the extended system if the original closed loop system is globally stable.

Notice that the expression for the feedback contains a term $\nabla V(x).g(x,y-k(x))$ which depends on a Lyapunov function V for the asymptotically stable system $\dot{x} = f(x,k(x))$. This renders the feedback u(x,y) hard to implement .

Therefore we consider (3) with the ∇V term left out ,i.e.

$$u(x,y) = -y+k(x)+\nabla k(x).f(x,y)$$

and investigate its stabilizing potential for the extended system.
In fact, under some extra conditions on f(x,u) (e.g. f(x,u) contains no linear terms in u) it follows rather immediately by means of the center manifold approach [1] that the new feedback stabilizes the extended system. The case f(x,u) containing no linear terms at all was communicated to me by Sontag & Sussmann.

We will show in what follows that -- without extra assumption on f -- the feedback locally stabilizes the extended system if the original closed loop system is asymptotically stable.

The proof which will be given in section 3 is an application of the center manifold approach and is rather straightforward.

3. Local stabilizability

Consider again the system

$$\dot{x} = f(x,u)$$

with $f: R^n xR \to R^n$.

Let u=k(x) be a smooth stabilizing feedback, i.e.

$$\dot{x} = f(x,k(x))$$

is asymptotically stable.
Consider the extended system

$$\dot{x} = f(x,y)$$
$$\dot{y} = u \qquad (2)$$

We will show by means of the center manifold approach [1] that (2) is locally stabilized by

$$u = -y+k(x)+\nabla k(x).f(x,y)$$

i.e.the system

$$\dot{x} = f(x,y)$$
$$\dot{y} = -y+k(x)+\nabla k(x).f(x,y) \qquad (3)$$

is asymptotically stable.
We perform a coordinate change:

x unchanged
$z = y-k(x)$

then (3) becomes

$$\dot{x} = f(x,k(x)+z)$$
$$\dot{z} = -z \qquad (4)$$

We want to show that (4) is asymptotically stable in $(x,z)=(0,0)$ knowing that

$$\dot{x} = f(x,k(x))$$

is asymptotically stable.
Rewrite (4) as

$$\dot{x} = Ax+bz+h(x,z)$$
$$\dot{z} = -z \qquad (5)$$

with A and b the appropriate Jacobians and h the higher order terms.

The system

$$\dot{x} = Ax+h(x,0) \tag{6}$$

is stable by assumption.

First, we blockdiagonalize (5) by means of a linear transformation

$$x_1 = Tx+r.z$$
$$z = z$$

In these coordinates, the system is

$$\dot{x}_1 = TAT^{-1}x_1+Th(T^{-1}x_1-T^{-1}rz,z)$$
$$\dot{z} = -z$$

It is remarked that

$$\dot{x}_1 = TAT^{-1}x_1+Th(T^{-1}x_1-T^{-1}rz,z)$$

with $z = 0$ is asymptotically stable in $x_1 = 0$ since (6) is asymptotically stable.

The matrix T can be taken such that TAT^{-1} consists of two diagonal blocks A_1 (with eigenvalues on the imaginary axis) and A_2 (with eigenvalues in the left half plane).
The system (4) is then represented by

$$\dot{x}_1 = A_1x_1+h_1(x_1,x_2,z)$$
$$\dot{x}_2 = A_2x_2+h_2(x_1,x_2,z)$$
$$\dot{z} = -z \tag{7}$$

with

$$\dot{x}_1 = A_1x_1+h_1(x_1,x_2,0)$$
$$\dot{x}_2 = A_2x_2+h_2(x_1,x_2,0) \tag{8}$$

asymptotically stable.