AN INTRODUCTION TO QUANTUM FIELD THEORY

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GEORGE STERMAN

Institute for Theoretical Physics,
State University of New York at Stony Brook

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1993

Printed in Great Britain at the University Press, Cambridge

A catalogue record of this book is available from the British Library

Library of Congress cataloguing in publication data

Sterman, George.
An introduction to quantum field theory / George Sterman.
p. cm.
Includes bibliographical references and index.
ISBN 0-521-32258-8. – ISBN 0-521-31132-2 (pbk)
1. Quantum field theory. I. Title.
QC174.45.S78 1993
530.1'43–dc20 92-29556 CIP

ISBN 0 521 32258 8 hardback
ISBN 0 521 31132 2 paperback

For Elise, Adrienne
and my parents

Preface

The search for the underlying structure of physical reality is as old as speculative thought. Our deepest experimental insights to date are expressed in the language of quantum field theory, in terms of particles that interact at points in space–time, subject to the constraints of special relativity. The theoretical developments that lead to this portrait are the subject of this book. Its aim is to provide a self-contained introduction to relativistic quantum field theory and its applications to high-energy scattering. Some of the methods described predate quantum theory, while others are quite recent. What makes them vital is not only their considerable success thus far, but also the very limitations of that success.

There is every reason to believe that quantum field theory is not a closed chapter. A great deal of freedom remains in the choice of particles and their interactions within the field-theoretic description of fundamental processes. The 'standard model', which describes elementary processes as they are known at this time, is a grab bag of matter and forces, in which breathtaking theoretical elegance coexists with seemingly senseless arbitrariness. Whatever the next step in our understanding of elementary processes, however, the elements of quantum field theory will remain relevant to their description.

Quantum field theory is a vast subject, and an introductory presentation necessarily involves choices of emphasis and of omission. My approach begins with the fundamental considerations of space–time and internal symmetry. These issues are at the heart of gauge invariance, which plays a dominant role in modern field theory. I have emphasized group theory, as a description of the symmetry and invariance properties that are required of any field. In addition, I have chosen to concentrate on the perturbative description of scattering as the best window into the underlying structure of the relevant fields and as the source of our most direct knowledge of the corresponding theories. These topics occupy the first half of the book.

The central formal issues in perturbative field theory concern the self-consistency of the quantum-mechanical expansion, which is the subject of the third quarter of the book. The history of quantum field theory has been driven, in large measure, by a creative tension between the demands of renormalizability and unitarity and the aesthetics of symmetry. Finally, in the fourth part, I discuss the structure of the perturbative expansion at higher orders, by which we may confront quantum field theories with precision experiments. This is a process which, despite some signal successes, is still in its infancy.

This book is intended as both text and reference. Its introductory and intermediate chapters (through Chapter 11) are directed at a reader familiar with classical and quantum mechanics, as taught at the advanced undergraduate or beginning graduate level. Familiarity at a similar level with electromagnetism and complex variable analysis are also assumed, as well as an aquaintance with the basic taxonomy of elementary particles such as electrons, neutrinos and quarks. The more advanced chapters can be used by both those familiar with the first half of the book, or with another text in field theory. I have tried to keep the discussion pedagogical and self-contained, with an emphasis on calculation. Without attempting mathematical rigor, I have tried to indicate where it may be found, as well as the nature of more advanced and formal arguments.

The material outlined below is probably more than can be conveniently presented in a year's course. As a result, it has been organized so that certain more advanced topics, although presented in their natural place in a logical progression, may be bypassed without loss of coherence. An introductory course might extend up to Section 12.4, with the omission of Sections 3.2, 3.3, 8.6, 9.5, 9.6, 10.4, 11.2–11.4 and 12.3. No essential cross references to these sections are made until Chapter 13.

The discussion is divided broadly into four parts. The first develops the methods of field theory through scalar fields. This somewhat simplified context is used to introduce the fundamental applications of group theory (Chapters 1 and 2), canonical quantization and the S-matrix (Chapter 2), the path integral and Feynman rules for diagrams and integrals (Chapter 3), and cross sections (Chapter 4). Many readers may already be familiar with some of the material in Chapter 1, especially group theory and Lorentz transformations. These topics are so central to what follows, however, that I considered it necessary to include them.

In the second part, realistic theories, with intrinsic angular momentum, are introduced and quantized, from the point of view of their space–time symmetries (Chapters 5, 6). Nonabelian fields and spontaneous symmetry breaking are introduced at the outset, on an equal footing with abelian fields. Chapter 6 includes an elementary introduction to unitary representations of the Poincaré group, and their relation to field quantization. Chapters 7 and 8 develop the Feynman rules for the components of the standard model, and give lowest-order applications to experimentally relevant cross sections. Here, representative examples are chosen from quantum electrodynamics, low-energy weak interactions and quantum chromodynamics, including a discussion of the role of ghost fields. Chapter 8 concludes with a brief introduction to the parton model, viewed as a way of interpreting cross sections in quantum chromodynamics.

As mentioned above, the order of chapters within the first two parts follows a presentation in which spin is introduced only after a relatively extensive discussion of scalar fields, up to the computation of cross sections at tree level. This approach is a matter of taste, however, and I have organized the material so that the first eight chapters may also be read in the order 1, 5, 2, 6, 3, 7, 4, 8. Realistic field theories are then discussed earlier, but it takes a little longer to get to the first cross section.

The third part deals with questions of renormalization and unitarity. The method of dimensional regularization is explained, and employed throughout. Chapter 9 also includes an introduction to various general features of Feynman integrals, including Wick rotation, time-ordered perturbation theory and perturbative unitarity. Renormalization is discussed in Chapters 10 and 11, including renormalization schemes and scales, and the renormalization group. For gauge theories, I have concentrated on the issue of unitarity. At the end of Chapter 11, the axial anomaly is used to illustrate the crucial issue of the consistency of classical symmetries at the quantum level.

Finally, in Part IV I undertake a more extensive discussion of perturbative cross sections, with an emphasis on results that extend to all orders in perturbation theory. Loop corrections in quantum electrodynamics are discussed, as well as the infrared problem at all orders. For quantum chromodynamics, I emphasize the concept of infrared safety, and its connection with renormalization, jets and the determination of the strong coupling constant. Chapter 13 treats the analytic structure of Feynman diagrams, dispersion relations and collinear divergences and presents a proof of the Kinoshita–Lee–Nauenberg theorem. Chapter 14 deals with the basis of our understanding of high-energy scattering, including factorization, evolution and the operator product expansion. Although the first two of these results are often identified with perturbative QCD, they are introduced here as general results in field theory. Finally, Chapter 15 briefly presents two issues that go beyond fixed orders in perturbation theory: bound states, and the likely asymptotic nature of the perturbative series.

In the appendices several important topics that do not fit naturally into any chapter are treated. Included are a review of the interaction picture, a derivation of symmetry factors and generating functionals and descriptions of the full standard model Lagrangian, the discrete symmetries of time reversal and charge conjugation, the Goldstone theorem and the role of the chiral anomaly in neutral pion decay. Finally, two appendices consist of *de rigeur* summaries of some useful formulas and Feynman rules.

There are two sets of references within the text. The first set is included for purposes of attribution, and often for historical interest. The second consists of references to reviews and other texts, where more discussion on a relevant topic may be found. These are marked with an 'r' before the date, as for example (r1980). The 'r' is not included in the reference list at the conclusion. In the list of references, the location of each reference is given by section in parenthesis (an 'i' indicates that the reference occurs in the introductary comments in a chapter, an 'e' that it occurs in the exercises).

The list of topics I have omitted would be longer than the list of those included. Generally, however, topics that go under the rubric of 'nonperturbative' have been slighted, including instantons and other types of vacuum structure in QCD, as well as lattice gauge theory. Loop corrections in the weak interactions have also been omitted. I have left out any discussion of supersymmetry, and other extensions of the known symmetries of the standard model. Beyond this, the great questions of gravity and of the nature of space–time remain unaddressed here, and in the standard model. In time, one may expect new theoretical structures to emerge, involving new, more unified field theories, perhaps with supersymmetry, and/or a substructure underlying the fields themselves, perhaps string theory. Whether they fall into the categories already known at this time or not, such theoretical structures will be interpreted in terms of, and their success will be measured by, the quantum field theories of today. I therefore hope that many readers will find what is included here useful in their research and their understanding.

Acknowledgements and thanks

I would like to express my appreciation to the John Simon Guggenheim Foundation for its support in the initial stages of this book, and to the Institute for Advanced Study, Princeton, for its hospitality during that period. I also owe an ongoing debt of gratitude to the Institue for Theoretical Physics at the State University of New York at Stony Brook, to its Director Chen Ning Yang, and to my other colleagues there, for making it possible to undertake this project. Their observations, questions and remarks in passing have continually given me occasion for thought, and have found echos in the pages that follow.

It is commonplace, but none the less true, that by teaching we learn, and I have been fortunate in the excellent group of students with whom I have interacted at Stony Brook. Many have helped me by reading portions of

the manuscript, and by giving suggestions. Others graduated even before the book was begun,.but all have in one way or another helped in its development. In this regard, I would like to thank in particular Lyndon Alvero, David Appel, Rahul Basu, James Botts, Claudio Coriano, Ghanashyam Date, Vittorio Delduca, Fabian Essler, Keith Kastella, Eric Laenen, Jose Labastida, Hsiang-nan Li, Ma Luo, Lorenzo Magnea, Joseph Milana, Sunil Mukhi, Annibal Ramalho, Ashoke Sen and Michael Sotiropoulos.

Another group of physicists has made this book possible through their encouragement at various crucial stages in my career. In this connection, I would like particularly to recognize Alex Dragt, Joseph Sucher and Ching-Hung Woo, who helped me, as a graduate student, develop a style of research, Shau-Jin Chang and Jeremiah Sullivan, who were so helpful in the early stages of my independent research and Thomas Applequist and Toshiro Kinoshita for generously recognizing that research at a time when it might easily have been forgotten. In addition, thanks are due especially to my collaborators in perturbative QCD, John Collins, Stephen Libby, Robert Ore, Jr, Jianwei Qiu, Dave Soper and Steven Weinberg. Both directly and indirectly, their insights have helped me find my way through the mazes that had to be traversed in the course of writing this book. Thanks also go to Jianwei Qiu for his invaluable and generous help in preparing the figures.

A book in preparation is not always left at the desk, and I thank my wife, Elise, for her patience and constant encouragement, and my daughter, Adrienne, for distracting me from it.

George Sterman

Stony Brook, NY
February 1993

Contents

PART III RENORMALIZATION

PART I

SCALAR FIELDS

1

Classical fields and symmetries

Our discussion begins with the action principle for classical fields, from which may be derived the field equations of motion. The symmetry properties of a field theory profoundly constrain its time development, through conservation laws. We introduce the Klein–Gordon Lagrangian to illustrate classical space–time and internal symmetries.

1.1 Action principle

Hamilton's principle in point mechanics

A system in classical mechanics is described by a set of generalized coordinates $\{q_i\}$, along with a Lagrangian $L(\{q_i, \dot{q}_i\})$, which depends on the q_i and their associated velocities $\{\dot{q}_i \equiv \mathrm{d}q_i/\mathrm{d}t\}$. The equations of motion for the system are the Lagrange equations,

$$\partial L/\partial q_i - (\mathrm{d}/\mathrm{d}t)(\partial L/\partial \dot{q}_i) = 0. \tag{1.1}$$

Equation (1.1) may be derived from Hamilton's principle that the motion of the system extremizes the action:

$$\delta S \equiv \delta \int_{t_1}^{t_2} \mathrm{d}t\, L(q_i(t), \dot{q}_i(t)) = 0, \tag{1.2}$$

where the variation is taken over paths $\{q_i(t)\}$ between any fixed boundary values, $\{q_i(t_1)\}$ and $\{q_i(t_2)\}$. (The derivation of eq. (1.1) from eq. (1.2) closely follows the field theory argument to be given below.)

Local field theory

In field theory, the analogue of the generalized coordinates, $\{q_i(t)\}$, is a field $\phi(\mathbf{x}, t)$, in which the discrete index i has been replaced by the continuous position vector $\mathbf{x}$. The position $\mathbf{x}$ is *not* a coordinate, but rather a parameter that labels the field coordinate ϕ at point $\mathbf{x}$ at a particular time t. There may be more than one field at each point in space, in which case the fields may carry a distinguishing subscript, as in $\phi_a(\mathbf{x}, t)$. To qualify as a mechanical system, the fields must be associated with a Lagrangian, which determines their time development. Each field describes an infinite number of coordinates, however, and we must make specific assumptions about the Lagrangian to make the system manageable.

We shall be interested in *local* field theories, in which the dynamics does not link different points in space instantaneously. It is then natural to assume that the Lagrangian may be written as an integral over another function, called the *Lagrange density*, $\mathscr{L}$,

$$L(t) = \int d^3\mathbf{x}\, \mathscr{L}(\mathbf{x}, t), \tag{1.3}$$

which depends on the set of fields and their first derivatives,

$$\mathscr{L}(\mathbf{x}, t) = \mathscr{L}(\phi_a(\mathbf{x}, t), \partial\phi_a(\mathbf{x}, t)/\partial x^\mu). \tag{1.4}$$

Conventions

In eq. (1.4) and the following, we employ the conventions

$$v^\mu = (v^0, \mathbf{v}) = g^{\mu\nu}v_\nu, \quad v_\mu = (v^0, -\mathbf{v}) = g_{\mu\nu}v^\nu, \tag{1.5a}$$

for any vector v^μ, where the metric tensor $g_{\mu\nu}(=g^{\mu\nu})$ is the diagonal matrix with nonzero elements $(1, -1, -1, -1)$. For derivatives with respect to the coordinate vector $x^\mu = (x^0, \mathbf{x})$ we use the notation (see Section 1.5),

$$\partial_\mu\phi_a \equiv \partial\phi_a/\partial x^\mu, \quad \partial^\mu\phi_a \equiv \partial\phi_a/\partial x_\mu. \tag{1.5b}$$

Here $x^0 \equiv ct$, where c is the speed of light, so that all the x^μ have dimensions of length. Generally, we shall use lower case Greek letters $(\alpha, \beta, \ldots \mu, \nu, \ldots)$ for space–time vector indices $(0, 1, 2, 3)$, and lower case italic letters $(i, j, \ldots)$ for purely spatial vector indices $(1, 2, 3)$. Finally, except where explicitly indicated, we shall use the convention that repeated indices are summed.

Lagrange equations

Since we are interested in local field theories, the fields in any region, R, of space communicate with the rest of space only through their behavior at the surface σ of R. Thus, if we specify the values of the fields everywhere in R at times t_1 and t_2, and on the surface σ for all times $t_1 < t < t_2$, we ought to have enough information to determine the fields everywhere in R for all times between t_1 and t_2. And indeed, we can derive equations of motion for any Lagrangian of the form (1.3), by demanding that the action S within R be extremal:

$$\delta S = \delta \int_{t_1}^{t_2} \mathrm{d}t \int_R \mathrm{d}^3\mathbf{x}\, \mathcal{L}(\mathbf{x}, t) = 0. \tag{1.6}$$

This variation is over all possible fields $\phi_a(\mathbf{x}, t)$ inside R,

$$\phi_a(\mathbf{x}, t) \to \phi_a(\mathbf{x}, t) + \epsilon \zeta_a(\mathbf{x}, t), \tag{1.7}$$

where ϵ is an infinitesimal parameter and where the function $\zeta_a(\mathbf{x}, t)$ satisfies

$$\zeta_a(\mathbf{x}, t_1) = \zeta_a(\mathbf{x}, t_2) = 0, \quad \zeta_a(\mathbf{y}, t) = 0, \quad \mathbf{y} \text{ on } \sigma, \tag{1.8}$$

but is otherwise arbitrary. For a fixed $\zeta_a(\mathbf{x}, t)$, the variation in the action is given by

$$\delta S = \epsilon \frac{\partial S}{\partial \epsilon} = \epsilon \int \left[\frac{\partial \mathcal{L}}{\partial \phi_a} \zeta_a + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_a)} \partial^\mu \zeta_a \right] \mathrm{d}^3\mathbf{x}\, \mathrm{d}t. \tag{1.9}$$

Next, we integrate by parts, using eq. (1.8) to eliminate end-point contributions:

$$\delta S = \epsilon \int \left\{ \frac{\partial \mathcal{L}}{\partial \phi_a} - \frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_a)} \right] \right\} \zeta_a \, \mathrm{d}^3\mathbf{x} \mathrm{d}t = 0. \tag{1.10}$$

This result must be true for *every* choice of ϵ and ζ_a, so we conclude that

$$\frac{\partial \mathcal{L}}{\partial \phi_a} - \frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_a)} \right] = 0 \tag{1.11}$$

at *every* point inside R for every time between t_1 and t_2. But eq. (1.11) must then hold at every point in space–time, since t_1, t_2 and R were chosen arbitrarily. These are the Lagrange equations for any fields $\phi_a(\mathbf{x}, t)$ that satisfy the assumptions embodied in eqs (1.3) and (1.4).

In point mechanics, the Lagrange equations are total differential equations in time, one for each coordinate. In contrast, eq. (1.11) gives one partial differential equation, involving both spatial and time derivatives, for each field ϕ_a.

1.2 Relativistic scalar fields

There are many examples in which an infinite set of total differential Lagrange equations, eq. (1.1), has a limit in a single partial differential equation of the type (1.11) (see exercise 1.1). The corresponding field theory may then be thought of as the continuum limit of a discrete system. We shall not generally take this viewpoint, however, and instead accept a continuum description as given. The inspiration for particular field theories as elementary processes has most often been found in underlying invariance principles (see Section 1.3). Thus, to begin with, we are interested in