

Curves & Surfaces in Computer Vision & Graphics

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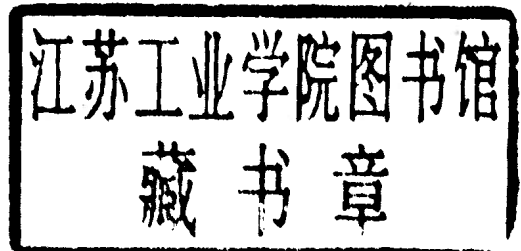
Curves and Surfaces in Computer Vision and Graphics

Leonard A. Ferrari
Rui J. P. de Figueiredo
Chairs/Editors

13-15 February 1990
Santa Clara, California

Sponsored by
SPIE—The International Society for Optical Engineering
SPSE—The Society for Imaging Science and Technology

Cooperating Organization
TAGA—Technical Association of the Graphic Arts



Published by
SPIE—The International Society for Optical Engineering
P.O. Box 10, Bellingham, Washington 98227-0010 USA



Volume 1251

SPIE (The Society of Photo-Optical Instrumentation Engineers) is a nonprofit society dedicated to advancing engineering and scientific applications of optical, electro-optical, and optoelectronic instrumentation, systems, and technology.



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Please use the following format to cite material from this book:

Author(s), "Title of Paper," *Curves and Surfaces in Computer Vision and Graphics*, Leonard A. Ferrari, Rui J. P. de Figueiredo, Editors, Proc. SPIE 1251, page numbers (1990).

Library of Congress Catalog Card No. 90-52677
ISBN 0-8194-0298-2

Published by
SPIE—The International Society for Optical Engineering
P.O. Box 10, Bellingham, Washington 98227-0010 USA
Telephone 206/676-3290 (Pacific Time) • Fax 206/647-1445

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Conference 1251, *Curves and Surfaces in Computer Vision and Graphics* was part of a three-conference program on Human Vision, Visual Processing, and Digital Display held at the SPIE/SPSE Symposium on Electronic Imaging Science and Technology, 11-16 February 1990, Santa Clara, California. The other conferences were:

Conference 1249, *Human Vision and Electronic Imaging: Models, Methods, and Applications*

Conference 1250, *Perceiving, Measuring, and Using Color*

Program Chair: **Bernice Rogowitz**, IBM/Thomas J. Watson Research Center

INTRODUCTION

Curves and surfaces play a central role in computer vision and graphics, as well as in a number of closely related and overlapping areas such as CAGD, CAM, medical imaging, scene analysis, and computer animation. In these fields, the representation and processing of curves and surfaces are often based on advanced mathematical concepts, especially those from algebraic and differential geometries, approximation theory, and computational mathematics.

The purpose of the 1990 SPIE/SPSE Conference on Curves and Surfaces in Computer Vision and Graphics was to bring together and promote interaction among leading researchers concerned with the theory and applications of curves and surfaces in the above fields. As evidenced in this proceedings, the quality and breadth of the papers presented satisfied in large measure our objectives. The insertion of this conference in the larger 1990 SPIE/SPSE Symposium on Electronic Imaging Science & Technology helped to broaden further the scope of our interaction with the fringe community.

The conference consisted of eleven consecutive sessions that took place from Tuesday, February 13, through Thursday, February 15, 1990. Sessions 1, 5, and 9 were invited sessions addressing topics of special relevance to the conference theme. In Session 1, Bézier described how mathematical representation and analysis of curves and surfaces impacted CAD/CAM in the French automotive industry, while the paper by de Figueiredo and Tagare explained the role that curves and surfaces are playing in computer vision. In Session 5, Abhyankar provided an overview of the parameterization of curves and surfaces, and Schumaker discussed a new approach to the reconstruction of three-dimensional objects from cross-sectional data, using a combination of volumetric and surface techniques. Finally, in the Invited Session 9, Ferrari, Silbermann, and Sankar described the current work at Irvine on the efficient generation, storage, and transformation of curves and surfaces, exploiting the derivative properties of smooth piecewise polynomial functions.

Two papers in Session 2, Surface Generation, presented interesting extensions to the Bézier patch formation; another discussed blending functions for interpolating a network of curves. Sessions 3 and 4, on surface fitting, contained eight papers describing various important aspects of the topic, including a testbed for comparison of parametric surface methods. Sessions 6 and 7, devoted to curves, had seven papers describing encouraging developments on the representation,

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analysis, and applications of curves. Sessions 8 and 10 covered scene analysis and consisted of seventeen papers spanning a wide range of topics, such as homotopy methods in motion estimation, global coverings in curve detection, and wavelets in surface estimation and reconstruction. These two sessions, rich in ideas and results, constituted a fine conclusion to the conference.

As chairs of the conference, we would like to express our appreciation to the members of the program committee for their help in the conference program organization; to the various authors and other participants—especially those from abroad who traveled large distances to attend the conference—for their valuable contributions both in formal presentations and informal discussions; to the SPIE/SPSE staff for their logistic and managerial support; and to Dr. Bernice E. Rogowitz for her cooperation in the overall conference organization.

Leonard A. Ferrari
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SESSION 1

General Session: Invited Papers

Chair

Leonard A. Ferrari

University of California/Irvine

CAD/CAM in the French automobile industry .

Pierre E. Bézier

Consultant

ABSTRACT

The French aircraft industry began paying attention to CAD/CAM before 1960, and the car builders soon followed suit, but their problem was somewhat different .

It can be said that, from the start, the aim was to obtain a complete description of the shape of the sheet-metal parts , i.e. car body elements, and to use it to carry information throughout the entire process, from styling to inspection of stamped and assembled parts .

One of the first and foremost conditions was to adopt a mathematical solution that could be easily understood and operated by designers, draughtsmen and machine-tool operators .

Consequently, the system should not be used to translate an already existing set of drawings, but to directly express with figures and numbers the shape previously defined, be it scantily, by small or large scale sketches and 3D mockups .

Now, numerical data are carried from R.&D. division to those of Production Engineering and tool shops; it is compulsory, too, that the system be compatible with those of subcontractors and suppliers .

To get full advantage of CAD/CAM, it is often necessary to bring important and radical change in the functioning of R. & D. and Production of the company , and sometimes to other divisions .

1. CONDITIONS FOR A SYSTEM

1.1. Need of a true definition .

In the last fifties, numerically controlled machine-tools (NCMT's) were mainly devoted to drilling and boring, sometimes to paraxial milling; but when the improvement of computer-control made it really practical, contouring could be seriously considered .

For the French car builders, the process for defining a car body and the relevant stamping tools had never been quite satisfactory, and it was felt that the use of the computer could improve it very much .

Up to that time, car body design and tooling involved some major steps, namely :

- . small scale hand-made sketches and mockups
- . full scale tentative drawings derived from offsets measured on mockups .

- . full scale "clay model", lacquered and equipped with wheels, windshield, door glasses, chrome trimmings, etc .
- . full scale drawings of the "skin" panels and inside parts such as frame , brackets, etc .
- . full scale solid master .
- . design and drawing of stamping, welding, as well as inspection equipment .
- . milling of the stamping tools by copy of replicas of portions of the master .
- . setting by hand of the stamping tools in order to remove the cuts resulting from the copy-milling .

It is visible that this was a pretty long process, but the major disadvantage was that each operator, designer, plasterer, pattern-maker or tool setter was practically free to bring some slight alteration, either to improve the aesthetical aspect of the part or to facilitate the manufacturing process. Consequently, there was no clear and undebatable definition, hence arguments, delay and expenses, not only between styling, R.& D. and production, but with subcontractors and suppliers .

1.2. Requirements for a CAD/CAM system .

To beat the existing system, which had the big advantage of a very long practice and experience, a new system should comply with some capital requirements :

- . it should reduce the total time, and specially that of producing drawings and clay-model .
- . it should be able to define the smallest part of a part .
- . definition, i.e. numerical data, should be easily transmitted, notwithstanding distance and language .
- . the mathematical principle should be easily understood by technicians -designers, draughtsmen, machine tool operators - who have a good knowledge of geometry and descriptive geometry, but have no deep acquaintance with analysis .
- . the cost of a system should be kept within reasonable limits, specially because at that time the benefits of CAD/CAM could be predicted but not yet proved .

2. CURVES AND SURFACES .

2.1. Lines and circles .

In the drawings related with mechanical parts, the only surfaces which could, at that time, be completely and accurately defined were planes, cylinders, cones, spheres and toruses .

Helicoids and gear flanks were, too, precisely described, but

they are manufactured with specially created machines, and their production was not liable to be significantly improved by numerical control, at least at that time.

A small group of scientists, in the Orsay university (near Paris) devised a software producing curves and surfaces, which used lines and circles; for other curves, and in particular for space free-form curves, the solution was to split them into a great number of small arcs of circles, but that proved time consuming and difficult to apply to space curves and, even more, to surfaces; nevertheless, systems such as Euclid (Matra) and Profiladata (of Terranti) gave some good results, but the need for a more general system finally prevailed.

2.2. Parametric curves and surfaces.

As early as 1958, Paul de Casteljau, a mathematician working for Citroën, devised a system based on parametric spaces and Bernstein's functions; it seems that, at that time, the policy of the company was to use it for translating existing drawings into numbers; besides, the company was much secretive, and the basis of the system could not be published before 1972.

Shortly after 1960, a few people at Renault began to take interest in the problem, though the general management was not much enthusiastic about it.

At that time, the normal practice was to trace cross sections, one hundred millimeters apart, on the surface of the model, and this meant that one half of the skin was divided into more than two thousands patches; hence, blending them via computing would be an enormous task; on the contrary, the project should be to start from so-called "character lines" and keep the quantity of important patches to a very strict minimum, maybe a score, plus some small patches for describing the details.

The system which looked best suited to express curves and surfaces was to use polygons and nets, since it would give the operator, designer or draughtsman, a firm control upon end conditions, i.e. points, slope, curvature and twist; slope continuity between adjacent patches and curves was easily ensured; as regards curvature, it is practically never required; most of time, the order of the functions was limited to three, but in case of need it could be raised, the practical limit being ten or twelve, but this has been, up to now, very seldom the case.

It is to be noted that it is feasible to trace a parametric curve or patch onto a bi-parametric patch.

2.3. Distortion of a triparametric space.

Moreover, a 3D object can be defined in a tri-parametric space which, initially, coincides with a Cartesian set of references, but which can be distorted by moving the knots of the treillis which plays, for a volume, the same part as a net for a patch,

so modifying the total shape of the object which has been inscribed in it . The drawback is that this is liable to increase the order of the new definition up to an order which is above the practical capacity of the computer; this obstacle can be overcome by an approximation process but , in fact, this technic is seldom used .

2.4. Splines and NURBS .

New possibilities appeared with rational functions and Non Uniform Rational B-Splines (NURBS) but , notwithstanding their advantages, polynomials and polygons are still widely used .

3. HARDWARE .

3.1. Drawing machines .

In the first sixties, some numerically controlled drawing machines were available - Gerber, U.D.M., K.V.F. as well as cathode ray tubes (CRT), but the problem was to control them with an adequate software .

3.2. Milling machines .

The prevailing opinion , at Renault, was to provide the styling department and the drawing office with special numerically controlled milling machines able to carve rapidly in a soft material, polystyrene or polyurethane foam, plaster, wood or resin, the shape of a significant portion of a car body or, even better, the entire mockup of a complete clay model .

Such a machine would be far from conventional, and Renault built half a dozen of them; they had a limited span , say $2.2 \times 1.7 \times 1.2$ meters , spindle power did not exceed 0.7 kW; the feed was rapidly raised from ten to sixty foot-per-minute; finally, plano-millers capable of directly carving a complete car have been purchased by Peugeot and Renault after 1980, to be at the disposal of the styling and drawing divisions .

In the same time, press tool shops were equipped with numerically controlled heavy milling machines to replace the copy-milling ones .

Needless to say, compatibility with the rest of the factory equipment for the computers selected for controlling the CRT's as well as the drawing and milling machines .

4. OPERATION OF CAD/CAM SYSTEMS .

At the outset, Renault's aim was to use CAD/CAM from styling to tool shop and inspection, so as to use the same definition throughout the whole process, hence reducing time and , it was hoped, cost; but time was the most important benefit to be expected .

4.1. Styling .

Evidently, the stylists could have used the system, since its geometry was simple and could be employed easily. Nevertheless, most

of them were reluctant, probably afraid that the system would reduce their freedom, and only a few of them adopted it rapidly.

The first task of the stylist is to define a few tentative solutions with the help of sketches, perspective views, as well as small mockups, hand made out of plastic wax, or carved in styro-foam blocks by a milling machine.

Then, the stylist insists on obtaining a full scale drawing in order to judge and build for himself an opinion before the 3D model is produced.

4.2. First set of drawings .

Tracing a full scale set of drawings is the task of the designers; they either start from the numerical data, if any, provided by the styling division, or from the offsets measured on a hand made small scale mockup.

As long as that work has been performed without the help of CAD/CAM, the drawings contained a lot of cross sections, then transformed into templates to help build the clay-model, and it took weeks, if not a couple of months, to obtain it; now it takes a much shorter time, since the numerical data are directly fed to the milling service, and some stylists have agreed that a CRT image is just a sketch replacing the full scale drawing, and they will wait for the completion of the clay model to judge the work of the designer, hence, the full scale drawing of the skin is sometimes omitted.

One should observe that a CRT image can include color, shading, reflection lines which give it a very realistic aspect.

4.3. Clay model .

The clay model, after carving on a numerically controlled milling machine, is covered with a colored plastic sheet and completed with wheels, windscreen, door glasses, locks, handles, bumpers, lamps, grille, chrome trimming, etc, so much that it looks like a real car.

At that moment, it can be modified at the request of people from style, sales, not forgetting top management, but when it has been finally accepted, its definition is supposed to be "frozen".

4.4. Final drawing .

4.4.1. Tracing .

The next step is to give a complete definition of each sheet metal part, i.e. skin panels plus frame, inner panels and the elements that will hold mechanical and electrical components: frame, power plant, steering mechanism, front and rear axles, transmission, the many electric motors, dashboard instruments, etc.

During that period, it is vitally important that the Production Engineering division be requested to come and check the feasibility

of the parts; sometimes, it leads to ask for a slight alteration, and the stylist's opinion must be sought in case it is related with a skin panel .

4.4.2. Computing stress etc .

When parts have been, at least provisionally, defined, it becomes possible to use the finite elements (F.E.) method to compute stress, vibration frequencies, as well as crash resilience . Of course, experiments remain absolutely necessary, but it is now sure that the results obtained by computing are proved true by the tests with very few exceptions ; this is a capital point, since it spares a good amount of time and limits the number of tests which require costly hand made prototypes .

4.5. Stamping tools engineering .

The Tool Engineering and Production division evidently derives a great profit from CAD/CAM use; the drawing office must define the surfaces on which the metal blanks are clamped in the press tool; besides, it is often compulsory that the part be tilted in order to facilitate the action of the punch (or "plunger") , and the choice of the tilting angle was a difficult and time consuming operation; with the help of the computer, a result can be obtained in less than half an hour, and it is feasible to try a few solutions and choose the best available .

4.6. Stamping tools manufacturing .

4.6.1. Pattern shop .

The definition of the stamping tools being transmitted to the pattern shop, the special milling machines carve the consumable styrofoam patterns, taking into account the amount of material to be removed during the manufacturing process .

4.6.2. Tool manufacturing .

The same data control the motion of the heavy plano-millers which manufacture the stamping tools and replaced the copy-milling machines .

The traditional practice was to mill the free-form surfaces with a large ball-end or toroid cutter, leaving large cuts that needed hand finishing with pneumatic chisel, portable grinding wheel and file, plus final checking - so called "die spotting" - against a replica of the master; needless to say, this was a long, tiring and costly operation, the accuracy of which could not be excellent, and this entailed a final adjustment of the tools in order to ensure a correct connection between parts to be assembled together .

Now, due to the accuracy of the definition and of that of the numerical control, hand setting has been replaced by a final polishing to erase the ridges which are no more than .0008 inch .