

Introduction to  
**LOGICAL  
THEORY**

P. F. Strawson

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# Introduction to LOGICAL THEORY

by

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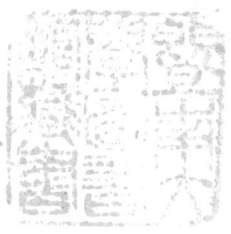
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## PREFACE

THERE are in existence many text-books and technical treatises on formal logic, and I have not sought in this book to add to their number. Many such books, excellent as they often are in their expositions of the technical and systematic aspects of logic, deal comparatively sketchily, and often rather misleadingly, with the relations between the formal systems they expound and the logical features of ordinary discourse. As a result of this omission, the true character of formal logic itself is apt to be left obscure. So this book has two complementary aims: one is to bring out some points of contrast and of contact between the behaviour of words in ordinary speech and the behaviour of symbols in a logical system; the other is to make clear, at an introductory level, the nature of formal logic itself. I have included enough of the elementary material of formal logic to provide a basis for the philosophical discussion of its nature, and to serve, if desired, as an introduction to more advanced technical treatises. Since the book is designed to be used as a general introduction to logic, I have added a concluding chapter on induction and probability.

I wish to acknowledge my great indebtedness to the many Oxford colleagues from whose discussions of the topics of this book I have profited; and among these, in particular, to Mr. H. P. Grice, from whom I have never ceased to learn about logic since he was my tutor in the subject; and to Professor Gilbert Ryle, Mr. G. A. Paul and Miss Ruby Meager, all of whom read the book either in manuscript or in proof and saved me from many inelegancies and mistakes.

P. F. S.

*Oxford,*  
*May, 1952.*

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CHAPTER I  
LOGICAL APPRAISAL

1. WHEN a man says or writes something, there are many different ways in which his performance may be judged. Among other things, we may question his truthfulness or criticize his style, we may assess the morality of what he says, or we may appraise its logic; though not all these types of assessment are appropriate to all kinds of utterance. The words 'logical' and 'illogical' are themselves among the words of logical appraisal. If you call a discourse logical, you are in some degree commending it. If you call it illogical, you are, so far, condemning it. Words and phrases which go with 'logical' are 'consistent', 'cogent', 'valid', 'it follows'; words and phrases which go with 'illogical' are 'inconsistent', 'self-contradictory', 'invalid', 'a *non sequitur*'. Part of our problem is to see what sort of appraisal these words are used for, to what kind of standards we appeal in using them. It is easy to see that these are not moral or aesthetic standards; that logical criticism is not, say, a kind of literary criticism. A slightly more difficult distinction is that between the criticism we offer when we declare a man's remarks to be untrue and the criticism we offer when we declare them to be inconsistent. In the first case we criticize his remarks on the ground that they fail to square with the facts; in the second case we criticize them on the ground that they fail to square with one another. The charge of untruth refers beyond the words and sentences the man uses to that in the world about which he talks. We deny his assertion, and, in doing so, make a counter-assertion of our own about the subject of his discourse. We contradict him. But the charge of inconsistency does not in this way refer to anything outside the statements that the man makes. We simply consider the way his statements hang together. Just from considering the sentences themselves, as they are used, we can, perhaps, see that not all the statements he makes can be true together. It is not that we contradict him, and in doing so, make a counter-assertion about the subject of his remarks;

we assert that he has contradicted himself, and, in doing this, we make no appeal to the facts and express no opinion about them. It is this kind of internal criticism that is appraisal of the logic of a piece of discourse.

#### I. INCONSISTENCY

2. Words of logical appraisal have connected meanings. To be clear about the meaning of one such word is to be clear about the meanings of the others. For example, in a proof or argument, one statement (the conclusion) is said to follow logically from, or to be logically implied by, others (the premises), if the argument is valid; and an argument is valid only if it would be inconsistent (or self-contradictory) to assert the premises while denying the conclusion; or, in other words, only if the truth of the premises is inconsistent with the falsity of the conclusion. A deductive argument is a sort of threat, which takes the form: if you accept these premises as true, then you must accept this conclusion as true as well, on pain of self-contradiction. From among the various concepts of logical appraisal, I shall select this notion of inconsistency or self-contradiction for detailed discussion. Other choices could have been made, but there are reasons, which will emerge as we go on, for making this choice.

3. What is inconsistency? It is better to approach this question indirectly, by asking a series of others. One might ask first: Why bother to avoid inconsistency? What is wrong with contradicting yourself? There is nothing morally wrong about it. It may not even be entirely pointless. Suppose a man sets out to walk to a certain place; but, when he gets half-way there, turns round and comes back again. This may not be pointless. He may, after all, have wanted only exercise. But, from the point of view of a change of position, it is as if he had never set out. And so a man who contradicts himself may have succeeded in exercising his vocal chords. But from the point of view of imparting information, of communicating facts (or falsehoods) it is as if he had never opened his mouth. He utters words, but does not say anything. Or he might be compared with a man who makes as if to give something away and then takes it back again. He arouses expectations which he does not

fulfil; and this may have been his purpose. Similarly, it may have been the purpose of a man who contradicts himself just to create puzzlement. The point is that the *standard* purpose of speech, the intention to communicate something, is frustrated by self-contradiction. Contradicting oneself is like writing something down and then erasing it, or putting a line through it. A contradiction cancels itself and leaves nothing. Consequently one cannot explain what a contradiction is just by indicating, as one might be tempted to do, a certain form of words. One might be tempted to say that a contradiction was anything of the form 'X is the case and X is not the case'. But this will not do. If someone asks you whether you were pleased by something, you may reply: 'Well, I was and I wasn't', and you will communicate perfectly well. Or there might be a convention that when one said anything of this form, the second part of the sentence was to be neglected. Then the minimum requirement for such a contradiction would be to say, first, 'X is the case and X is not the case' and, after that, 'X is not the case and X is the case'. Nevertheless, the temptation to explain a contradiction as anything of this form is, we shall see, not without point.

4. The next two questions to ask are more difficult. They are: (a) when we use these words of logical appraisal, what is it exactly that we are appraising? and (b) how does logical appraisal become possible? That is, we shall ask: what is it exactly that we declare to be inconsistent? and: what makes inconsistency possible? I have spoken of *statements* as being inconsistent with each other; and there is a temptation to think that in this context we mean by a statement the same thing as a sentence of a certain kind; or, perhaps, the meaning of such a sentence. But suppose I write on the blackboard the following two pairs of sentences: (i) 'I am under six foot tall' and 'I am over six foot tall'; (ii) 'The conductor is a bachelor' and 'The conductor is married'. In writing the sentences on the blackboard, I have, of course, not contradicted myself; for I may have written them there with a purely illustrative intention, in giving an English lesson. Someone might say: Nevertheless, the sentences in each pair *are* inconsistent with each other. But what would this mean? Would it mean that if they were

ever uttered with the intention of making a statement, an inconsistency would result? But suppose the first two sentences were uttered by different people, or by the same person at an interval of years; and that the second two sentences were uttered in different omnibuses, or in the same omnibus, but on different days. Then there would be no inconsistency. Earlier, I paraphrased 'seeing that two statements are inconsistent' as 'seeing that they cannot both be true together'. And it is clear that that of which we can say that it is true or false is also that of which we can say that it is consistent or inconsistent with another of its kind. What these examples show is that we cannot identify that which is true or false (the statement) with the sentence used in making it; for the same sentence may be used to make quite different statements, some of them true and some of them false. And this does not arise from any ambiguity in the sentence. The sentence may have a single meaning which is precisely what, as in these cases, allows it to be used to make quite different statements. So it will not do to identify the statement either with the sentence or with the meaning of the sentence. A particular statement is identified, not only by reference to the words used, but also by reference to the circumstances in which they are used, and, sometimes, to the identity of the person using them. No one would be tempted to say that the sentence 'I am over six foot tall' was inconsistent with the sentence 'You are under six foot tall'. But plainly they can be used, in certain circumstances, to make statements which are inconsistent with each other; i.e., in the case where the second sentence is addressed to the man by whom the first sentence is uttered.

It is easy to see why one is tempted to think of the sentence 'I am over six foot tall' as being inconsistent with the sentence 'I am under six foot tall'. One thinks of both sentences as being uttered, in the same breath, by the same person. In this case we should ordinarily regard that person as having contradicted himself, i.e., we should regard him as having said something and then unsaid it; and so as having said nothing. The important assumption is that the two expressions 'over six foot tall' and 'under six foot tall' are applied to the same person at the same time. Let us give the name 'incompatible predicates' to any pair of expressions the application of which

to the same person or thing at the same time results in an inconsistency. Thus we can say that one of the ways in which it is possible to say something inconsistent is by applying incompatible predicates to the same person or thing at the same time.

5. But must a language have incompatible predicates in it? And what makes predicates incompatible? I want to answer the first question by saying, not that a language must have incompatible predicates in it; only that it is very natural that it should. And I want to answer the second question by saying that it is we, the makers of language, who make predicates incompatible. One of the main purposes for which we use language is to report events and to describe things and persons. Such reports and descriptions are like answers to questions of the form: what was it like? what is it (he, she) like? We describe something, say what it is like, by applying to it words that we are also prepared to apply to other things. But not to all other things. A word that we are prepared to apply to everything without exception (such as certain words in current use in popular, and especially military, speech) would be useless for the purposes of description. For when we say what a thing is like, we not only compare it with other things, we also distinguish it from other things. (These are not two activities, but two aspects of the same activity.) Somewhere, then, a boundary must be drawn, limiting the applicability of a word used in describing things; and it is we who decide where the boundaries are to be drawn.

This metaphor of drawing boundaries is in some ways misleading. I do not mean by it that we often make conscious decisions of this kind (though we sometimes do); nor that our boundary-drawing is a quite arbitrary matter; nor that the boundaries are fixed and definite; nor that the decisions we make when we make them, are purely verbal decisions. The boundaries are more like areas of indeterminate ownership than frontier-lines. We show ourselves to be near such a boundary, and we show also its indeterminacy, when, in reply to such a question as 'Was it red?', we give such an answer as 'Well, I suppose you could call it red'. We show ourselves on the point of making a boundary-decision when, with all the facts before us, we hesitate over the application of a certain word. Does

such and such an act constitute an act of aggression or not? This case shows, too, how our decision is not a purely verbal matter; for important consequences may follow from our deciding that it is, or is not, an act of aggression. What makes our decisions, for a word already in use, non-arbitrary, is this: that our normal purpose will be defeated if the comparison implicit in the use of the word is too unnatural, if the similarity is too tenuous.

We may say: two predicates are incompatible when they lie on different sides of a boundary we have drawn: 'under six foot tall' and 'over six foot tall'; 'red' and 'orange'; 'aggressive' and 'pacific'. But this needs some explanation. Suppose you draw a closed figure on a piece of paper and then someone indicates a point on the ceiling and says: 'Does this point lie inside or outside the boundaries of the figure?' Of course, one might answer by imagining the boundaries of the figure extended in another dimension, up to the ceiling. But you might refuse to answer the question, by saying that you were drawing the boundary line only in the plane of the paper. Whatever lay outside the line in the plane of the paper was excluded from the figure. Things lying in a different plane were not excluded from it, but neither were they included in it. The figure has a certain plane of exclusiveness. And so with a word: it has a certain range of incompatibilities. 'Under six foot tall' is incompatible with 'over six foot tall'; but neither is incompatible with 'aggressive'. The last expression has a different incompatibility-range from the other two. There may sometimes be objections of a logical kind to applying expressions with different incompatibility-ranges to the same thing; but these will not be the objection that inconsistency will result from doing so.

When we apply a predicate to something, we implicitly exclude from application to that thing the predicates which lie outside the boundaries of the predicate we apply, but in the same incompatibility-range. By this I mean that if we go on to apply to the thing, in the same breath, one of the predicates which lie outside those boundaries, we shall be taken to have contradicted ourselves and said nothing. (This might be taken as a definition of 'incompatible predicates'.) But there is a qualification to be made here. Just as we might reply to the



query 'Were you pleased?' with the words 'Well, I was and I wasn't' without inconsistency, so we might apply to the same thing, in the same breath, two predicates, which would ordinarily be regarded as incompatible, without contradicting ourselves. If we do this, we invite the question 'What do you mean?'; and if we can explain what we mean, or show the point of saying what we say, then we have not contradicted ourselves. But if there is no way of doing this, we are inconsistent. Thus we might say, in answer to a question, 'He is both over six foot tall and under six foot tall', and then explain that he has a disease which makes him stoop, but that if he were cured and were able to stand upright, he would top the six-foot mark. This shows again that one cannot fully explain what self-contradiction is, just by reference to groupings of words.

6. So long as we bear this qualification in mind, we can safely speak of incompatible predicates and can safely say that, when we apply a predicate to something by way of describing it, we implicitly exclude from application to it any predicates incompatible with that which we apply. (We should be said to have contradicted anyone who had just applied any of those predicates to the thing.) When we notice that this function of exclusion is implicit in all descriptive uses of language, we should not find it surprising that language contains devices for rendering the function explicit; devices of which, in English, the word 'not' is the most prominent. There are many very different kinds of occasion on which our primary concern is with the explicit exclusion of a predicate; e.g., when we wish to contradict a previous assertion; or to correct a possible false impression; or to express the contrast between what had been expected, feared, suggested, or hoped, and the reality; sometimes, when we are answering a direct question; sometimes, when we grope towards the right description by eliminating the wrong ones. What is common to such cases is that they create a need or a motive for emphasizing a difference rather than a resemblance. It is instructive to compare the use of 'not' with the use of those words which begin with negative prefixes; like 'intolerable', 'unpretentious', 'impolite', 'non-aggressive'. These words bear their incompatibilities on their faces as surely as any