Lecture Notes in Physics

19

Proceedings of the Third International Conference on Numerical Methods in Fluid Mechanics

Vol. II
Problems of Fluid Mechanics

July 3-7, 1972
Universities of Paris VI and XI

Edited by Henri Cabannes and Roger Temam

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Editors' Preface

This issue of Lecture Notes in Physics is the second of two volumes constituting the Proceedings of the Third International Conference on Numerical Methods in Fluid Mechanics, which was held at the University of Paris VI, from July 3 to 7, 1972. Three general lectures and forty eight short individual communications were presented at this conference; the complete proceedings are published here. The general lectures were given by Professor A. DORODNICYN, Director of the Computing Center of the Academy of Sciences of the Soviet Union, who presented the Soviet works dealing with the solution of Navier-Stokes equations; by P. MOREL, professor at the University of Paris VI and Director at the Laboratory of Dynamical Meteorology of the National Center of scientific research (C.N.R.S.), who presented the Problems of numerical simulation of geophysical flows; by Professor R.D. RICHTMYER of the University of Colorado, U.S.A., who spoke on Methods for (generally unsteady) Flows with Shocks.

The individual communications have been separated into two groups: Fundamental Numerical Techniques and Problems of Fluid Mechanics; in each group they are published in the alphabetic order of the author, or of the first of the authors.

Volume I contains the three general lectures and the thirteen communications on Fundamental Numerical Techniques. Volume II contains the thirty five communications on Problems of Fluid Mechanics.

This Conference follows the conferences with the same topic hold at Novossibirsk, U.S.S.S. in 1969, and at Berkeley, U.S.A. in 1970 (the proceedings of which appeared in Lecture Notes in Physics, Vol. 8). The French Organizing Committee was sponsored by Commissariat à l'Energie Atomique, Electricité de France, Union des Chambres Syndicales des Industries du Pétrole, in France, and also by the Office of Naval Research and Air Force Office of Scientific Research, in the U.S.A. The Universities of Paris VI and Paris XI, and the Centre National de la Recherche Scientifique also helped the Committee in a much appreciated manner.

We wish to thank all the persons who contributed to the success of the Conference, the participants for their scientific contributions, our colleagues and younger researchers for their help in the organization and Mrs. M.T. CARTIER and Miss S. DELABEYE for their excellent secretarial work.

Finally we wish to express our appreciation to Dr. W. BEIGLBÖCK and the Springer-Verlag Company for the rapid publication of these proceedings in the series of Lecture Notes in Physics.

January 25, 1973

HENRI CABANNES
ROGER TEMAM

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Volume II

A NUMERICAL METHOD FOR HIGHLY ACCELERATED LAMINAR BOUNDARY-LAYER FLOWS

R.C. Ackerberg and J.H. Phillips

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A second-order-accurate implicit finite difference method is developed to study the boundary-layer flows that occur just upstream of a trailing edge which is attached to a free streamline. An important feature of this technique is the use of an asymptotic expansion to satisfy the boundary condition at the edge of the boundary layer while retaining a rapid algorithm for inverting the system of linear equations for each Newton iteration. The method is applied to the Kirchhoff-Rayleigh flow past a finite flat plate set perpendicular to a uniform stream. Computed velocity profiles are found to be in excellent agreement with those obtained from an asymptotic solution (Ackerberg (1970), (1971a), (1971b)) with pointwise differences being less than 1.2% over two-thirds of the profile. A detailed description of the method is given in Ackerberg and Phillips (1973).

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RELAXATION METHODS FOR TRANSONIC FLOW ABOUT WING-CYLINDER COMBINATIONS AND LIFTING SWEPT WINGS

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INTRODUCTION

It has recently been demonstrated that relaxation methods are a powerful numerical tool for obtaining steady-state solutions to the two-dimensional transonic potential equations. The basic numerical procedure, first introduced by Murman and Cole (1971), accounts for the mixed elliptic-hyperbolic character of the governing equations by using a mixed finite-difference scheme. The general procedure is to employ centered differences when the flow is locally subsonic and one-sided differences when it is locally supersonic. In this paper we extend the mixed elliptic-hyperbolic relaxation method to the transonic small disturbance equation in three dimensions. In particular, we consider transonic flow over thin lifting wings with sweep and taper and about nonlifting wing-cylinder combinations. We restrict our treatment to freestream Mach numbers less than one and to wings with subsonic trailing edges.

BASIC EQUATION AND BOUNDARY CONDITIONS

The governing equation for small disturbance transonic perturbation potential (Spreiter (1953) and the corresponding pressure coefficient can be written

$$\left(1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \phi_x\right) \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{1}$$

$$C_p = -2 \phi_x \tag{2}$$

respectively, where M_∞ is the free-stream Mach number and the ϕ is the perturbation potential divided by the free-stream velocity. In small disturbance theory the flow tangency condition at the wing surface is linearized and applied on the wing mean plane ($z = 0$) giving

$$\phi_z\Big|_{u,\ell} = \frac{df}{dx}\Big)_{u,\ell} \tag{3}$$

where $\left.\frac{df}{dx}\right|_u$ and $\left.\frac{df}{dx}\right|_\ell$ are the slopes of the upper and lower surfaces and include the effect of thickness, camber and angle of attack. In the case of a lifting wing the Kutta condition is applied, thus forcing the flow to leave all subsonic trailing edges smoothly. In the small disturbance theory the Kutta condition is satisfied by requiring that ϕ_x (pressure) be continuous across the trailing edge. In addition, provision must be made for a trailing vortex sheet downstream of the wing trailing edge. The vortex sheet is assumed to be straight and lie in the wing mean plane $z = 0$ with the conditions that ϕ_x and ϕ_z be continuous and ϕ be discontinuous through it. Due to the continuity of pressure through the vortex sheet, the jump in potential at any span station, $y = y_o$, is independent of x and is equal to the circulation about the wing section defined by

$$\Gamma(y_o) = -\oint d\phi(x, y_o, z) \tag{4}$$

for any path enclosing the wing section.

The outer flow boundary conditions for a nonlifting wing are that the perturbation velocities tend to zero with increasing distance from the wing. In the numerical method this is approximated by specifying free-stream conditions far from the wing. In the case of a lifting wing the pertrubation velocities, ϕ_y and ϕ_z, far downstream do not vanish due to the presence of the vortex sheet. At an infinite distance downstream the motion due to the vortex sheet becomes two-dimensional in the (y,z) Trefftz plane and this motion is described by the two-dimensional Laplace equation.

BASIC NUMERICAL PROCEDURE

The basic feature of the numerical method is to account for the mixed elliptic-hyperbolic nature of the governing transonic equation by central differencing the streamwise derivatives when the coefficient of ϕ_{xx} is positive and backward differencing when the coefficient is negative. Consider a three-dimensional rectangular domain and let the mesh be evenly spaced with the streamwise coordinate, $x = j\Delta x$, the spamwise coordinate, $y = k\Delta y$, and the vertical coordinate, $z = \ell\Delta z$. At each mesh point the equation type (i.e., elliptic or hyperbolic), is determined by the sign of the expression

$$V = 1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \frac{\phi_{j+1} - \phi_{j-1}}{2\Delta x} \tag{5}$$

If $V > 0$ the flow is subsonic, and the x derivatives are approximated by the centered difference

$$\left[1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \phi_x\right]\phi_{xx} = \left[1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \frac{(\phi_{j+1} - \phi_{j-1})}{2\Delta x}\right] \frac{\phi_{j+1} - 2\phi_j + \phi_{j-1}}{(\Delta x)^2} \tag{6}$$

If $V > 0$ the flow is supersonic, and the x derivatives are approximated by the backward difference

$$\left[1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \phi_x\right]\phi_{xx} = \left[1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \frac{\phi_j - \phi_{j-1}}{2\Delta x}\right] \frac{\phi_{j+1} - 2\phi_j + \phi_{j-2}}{(\Delta x)^2} \tag{7}$$

Notice that the derivative ϕ_x is also backward differenced. The y and z derivatives are replaced everywhere by the usual centered formula except at the wing root, $k = 0$, where the symmetry condition gives

$$\phi_{yy} = 2\frac{(\phi_1 - \phi_2)}{(\Delta y)^2} \tag{8}$$

and at the boundary $\ell = 1$ which is placed half a mesh spacing off the $z = 0$ plane. At these points the wing boundary condition is incorporated by writing

$$\phi_{zz} = \frac{1}{\Delta z}\left[\frac{\phi_2 - \phi_1}{\Delta z} - (\phi_z)_{z=o}\right] \tag{9}$$

Note that applying the wing boundary condition in this manner requires that the values of ϕ on the wing mean plane itself must be found by some procedure such as extrapolation. Studies made by Krupp (1971) on solutions for blunt nosed lifting airfoils have shown that in the region of the nose the best results are obtained by linear extrapolation.

The set of nonlinear algebraic equations obtained from the difference formulas are solved iteratively by a line-relaxation algorithm. Each vertical line is successively relaxed by marching toward the increasing y direction in an x = constant plane; the process is repeated for each x = constant plane in the increasing x-direction.

NONLIFTING WINGS AND WING-BODY COMBINATIONS

The numerical method can be applied to rectangular nonlifting wings in a straightforward manner. For swept and tapered wings, however, complications can occur, since in general the boundary points defining the wing shape do not fit naturally in a Cartesian grid network. A special case, which can be easily described in a Cartesian grid is that of an untapered swept wing. In this case, an equally spaced mesh may be used with $\Delta y = \Delta x/\tan \Lambda$, where Λ is the sweep angle, thus permitting the same number of chordwise mesh points on each wing section. An illustrative example is shown in Fig. 1 for a 30° sweptback wing of aspect ratio 4 and with a 6 percent (streamwise) parabolic arc section. The results calculated for $M_\infty = 0.908$ show an embedded shock wave at the wing root that weakens and becomes oblique as it proceeds outboard from the root. The calculations for this example were carried out on a 70 X 31 X 21 (xyz) grid which was evenly spaced in the (xy) plane (Δx = 5% chord and Δy = 8.66% chord) and required 140 iterations corresponding to 40 minutes of computer time on an IBM 360/67.

The transonic relaxation method has also been applied to wing-body combinations that can be represented by boundary conditions applied on combined mean planar and cylindrical control surfaces. The problem is recast into cylindrical coordinates for which Eq. (1) becomes

$$\left[1 - M_\infty^2 - (\gamma + 1) M_\infty^2 \phi_x\right]\phi_{xx} + \frac{1}{r}(r\phi_r)_r + \frac{1}{r^2}\phi_{\theta\theta} = 0 \tag{10}$$

The finite difference approximations for the derivatives in Eq. (10) are essentially the same as those that were applied to Eq. (1). The body boundary condition ($r\phi_r = R\,\mathrm{d}R/\mathrm{d}x$ where R is the body radius) is applied on the cylindrical control surface $r_{k=1}$ and can be written

$$\frac{1}{r}(r\phi_r)_r\bigg|_{k=1} = \frac{2}{r_1 \Delta r}\left[\left(\frac{r_1 + r_2}{2}\right)\left(\frac{\phi_2 - \phi_1}{\Delta r}\right) - R\frac{\mathrm{d}R}{\mathrm{d}x}\right] \tag{11}$$

while the wing boundary condition is given by Eq. (9) with Δz replaced by $r\Delta\theta$.

The results for the 30° swept wing on a straight cylinder and on a symmetrically indented cylinder based on Mach-one area-ruling are shown in Fig. 2. Note that the area-ruling eliminates the embedded shock waves on the wing. These calculations were carried out on a 77 X 30 X 23 ($xr\theta$) grid and required 200 iterations and one hour computation time on the IBM 360/67.

NUMERICAL TREATMENT OF LIFTING WINGS

In the numerical procedure for treating lifting wings, the wing is placed in the finite difference grid as shown in Fig. 3. The circulation at each span station (defined in Eq. (5)) is determined by the jump in potential at the trailing edge from the relation

$$\Gamma^{n+1} = \Gamma_k^{\,n} - \omega\left[\Gamma_k^{\,n} - \left(\phi^n_{JTE,k,o^+} - \phi^n_{JTE,k,o^-}\right)\right] \tag{12}$$

where n is the iteration count and ω is a relaxation parameter. New values of Γ are obtained at each iteration with the values of ϕ being obtained by extrapolating the values above and below the trailing edge. The continuity of pressure through the vortex sheet is maintained by holding the value of Γ, given by Eq. (12), fixed in x along the entire length of the vortex sheet and by setting $\phi_{j,k,o^+} = \phi_{j,k,o^-} + \Gamma_k$.

Difference formulas for ϕ_{zz} at the vortex sheet may be derived by noting that jumps in ϕ occur only at the vortex sheet and only odd functions may jump. Since the jump is independent of x, the solution at the vortex sheet decouples into even, ϕ^e, and odd, ϕ^o, solutions with ϕ^e satisfying Eq. (1) and ϕ^o satisfying

$$\phi^o_{yy} + \phi^o_{zz} = 0 \tag{13}$$

At the sheet itself the odd solution is given by

$$\phi^{o}(x,y,o\pm) = \pm 1/2\Gamma(y) \tag{14}$$

Therefore, ϕ_{zz} at the sheet can be written

$$\phi_{zz}\Big|_{o\pm} = \phi^{e}_{zz}\Big|_{o} \mp 1/2\Gamma_{yy} \tag{15}$$

The difference approximation for Eq. (15) is applied at points $(j,k,o-)$ (see Fig. 3) and is given by

$$\phi_{zz}\Big|_{j,k,o-} = \frac{4}{(\Delta z)^2}\left[\left(\phi_{j,k,1} - \Gamma_k\right) - 2\phi_{j,k,o-} + \phi_{j,k,-1}\right] + \frac{1}{2(\Delta y)^2}\left(\Gamma_{k+1} - 2\Gamma_k + \Gamma_{k-1}\right) \tag{16}$$

The difference formula for points $(j,k,1)$ is the usual centered difference with $\phi_{j,k,o+}$ replaced by $\phi_{j,k,o-} + \Gamma_k$. The wing tip and edge of the vortex sheet are placed midway between grid points, thus avoiding differencing at the tip singularity. The required value of potential just outboard of the tip is found by interpolation.

The infinity boundary conditions far from the wing and vortex sheet are given at some finite distance by an approximate analytical expression for the far field solution (see Klunker (1971)). The dominant term in the expression is due to lift and is proportional to the circulation integrated over the wing. The conditions at the downstream boundary, i.e., Trefftz plane, are found by relaxing Eq. (13) with boundary condition, Eq. (14), along with the rest of the flow field.

DIFFERENCING SCHEMES FOR SWEPT AND TAPERED WINGS

We now consider the application of the relaxation method to lifting wings with swept and tapered planforms. Experience with calculations about two-dimensional lifting airfoils has shown that very small mesh spacing (less than 1% chord) is required in the nose region, particularly for blunt leading edges. Satisfying this requirement with an even spaced mesh would require a prohibitive number of mesh points. An alternate approach is to use a coordinate transformation and map any swept or tapered planform into a rectangle. Such a transformation, valid for wings with finite tip chords, is given by

$$\xi(x,y) = \frac{x - x_{LE}(y)}{c(y)} \qquad \eta = y \qquad z = z \tag{17}$$

where $x_{LE}(y)$ is the value of x at the leading edge and $c(y)$ is the ratio of the local chord to the root chord. The governing small disturbance equation can be rewritten in terms of the new independent varialbes ξ,η,z in the form

$$\left\{\left[1 - M_\infty^2 - (\gamma+1)\frac{M_\infty^2}{U_\infty}\,\phi_\xi\,\frac{1}{c}\right]\frac{1}{c^2} + \xi_y{}^2\right\}\phi_{\xi\xi} + 2\xi_y\,\phi_{\xi\eta} + \xi_{yy}\phi_\xi + \phi_{\eta\eta} + \phi_{zz} = 0 \tag{18}$$

and the pressure coefficient becomes

$$C_p = -\frac{2}{c}\,\phi_\xi \tag{19}$$

The transformation given by Eq. (17) shears x to remove the sweep and stretches x to remove the taper. The effects of sweep and taper on the boundary conditions are thereby removed from the boundary conditions themselves and incorporated into the governing Eq. (18). In the region outboard of the tip the same rate of stretching and shearing is used unless $c(y)$ becomes much less than one. In this case $c(y)$ is set equal to a constant, $c(y_p)$, for values of y between some point $y_p > 1$ and the far field boundary.

Treatment of the wing root boundary condition at $\eta = 0$ in the transformed coordinate system requires special consideration since the derivatives of ϕ with respect to η are discontinuous there. The condition of symmetry leads to the relation

$$\phi_{xy})_{y=o} = \phi_{\xi\eta} + \xi_{xy}\phi_{\xi} + \xi_y\phi_{\xi\xi} = 0 \tag{20}$$

which can be used to eliminate $\phi_{\xi\eta}$ from Eq. (18). Since $c(y)=1$ at the root section, the governing equation then reduces to

$$\left[1 - M_\infty^2 - (\gamma+1) M_\infty^2 \phi_\xi - \xi_y^2\right] \phi_{\xi\xi} + (\xi_{yy} - 2\xi_y\xi_{xy})\phi_\xi + \phi_{\eta\eta} + \phi_{zz} = 0 \tag{21}$$

The $\phi_{\eta\eta}$ term in Eq. (21) is replaced by expressing ϕ at point 2 (see Fig. 4) in a Taylor series about point 1 and using the symmetry condition. The final form of the equation to be relaxed at the root boundary becomes

$$\left[1 - M_\infty^2 - (\gamma+1) M_\infty^2 \phi_\xi - \zeta^2\right] \phi_{\xi\xi} + \left[\xi_{yy} + \left(\frac{2}{\Delta\eta} - 2\xi_{xy}\right)\zeta\right]\phi_\xi + \frac{2(\phi_2 - \phi_1)}{(\Delta\eta)^2} + \phi_{zz} = 0 \tag{22}$$

with the ξ and z derivatives replaced by the already mentioned difference formulas and where

$$\zeta = \frac{n\Delta\xi}{\Delta\eta} + \xi_y$$

At each point η is picked such that $|\zeta|$ is made as small as the mesh will allow. For a wing with no taper ($\xi_{yy} = \xi_{xy} = 0$) a zero value of ζ would reduce Eq. (22) to the untransformed equation.

In subsonic regions centered difference formulas applied to Eq. (18) give essentially the same solution as that found by applying centered formulas to Eq. (1). Unfortunately, however, a difficulty arises in differencing Eq. (18) in supersonic regions. It occurs if the initiation of backward differencing in the ξ direction commences when M, the local Mach number, becomes supersonic; that is when

$$M \equiv M_\infty \left[1 + \frac{(\gamma+1)}{c}\phi_\xi\right]^{1/2} > 1 \tag{23a}$$

In such a case the coefficient of $\phi_{\xi\xi}$ is still positive since it contains the term ξ_y^2, and the calculations do not converge. Furthermore, in this case the numerical domain of dependence can not include the analytical domain of dependence traced out by the local characteristics. This is illustrated in Fig. 5.

This difficulty can be overcome by substituting for the condition of backward differencing, the requirement that

$$M > (1 + c^2\xi_y^2)^{1/2} \tag{23b}$$

which amounts to the condition that the coefficient of $\phi_{\xi\xi}$ changes sign, or alternatively, that the component of local Mach number normal to the local sweep angle becomes supersonic. It should be emphasized that the criterion given by Eq. (23b) is successful for supercritical flow fields only if local Mach numbers are sufficiently large to ensure backward differencing at shock waves. For example, application of this method at $M_\infty = 0.908$ to the 30° swept wing shown in Fig. 1 produced no detectable shock wave because at no point did the local Mach numbers satisfy condition (23b), although they did, of course, satisfy condition (23a). The method appears to give satisfactory results, however, for wings with moderate sweep angles in flows with sufficiently high local Mach numbers, examples of which are given below.

It should be pointed out that the above difficulty can be alleviated, and the ability to capture weak oblique shocks by means of Eq. (18) can be improved if a skewing technique, similar to that constructed for the root section in Eq. (22), is used in the supersonic region. The object is to find a computational molecule in the supersonic region which is aligned as closely as possible to the (xy) coordinates. Such a scheme applied to the 30° swept wing at $M_\infty = 0.908$ with the angle of the skewed computational molecule differing from the (xy) molecule by less than four degrees gave the same results as those shown if Fig. 1.

Subcritical ($M_\infty = 0.752$) and supercritical ($M_\infty = 0.853$) results obtained using the transformation method (with root skewing only) are shown in Figs. 6 and 7 for flow about a lifting swept wing at two degrees angle of attack. The constant chord, 23.75° sweptback wing with a Lockheed C141 airfoil section (11.4% thick streamwise) was tested in the NASA Ames 11-Foot Transonic Wind Tunnel by Cahill and Stanewsky (1969). The results for $M_\infty = 0.752$ are compared in Fig. 6 with both the experimental results and those obtained by the subsonic panel method of Saaris and Rubbert (1972). The present results agree will with those obtained by the panel method but both numerical methods show more lift than the experiment. The present method also shows more lift than the experiment at $M_\infty = 0.853$ (see Fig. 7), as well as a shock location aft of the experimental one. It should be mentioned that inviscid solutions generally give more lift than the experiment when compared at the same geometric angle of attack. The principal cause is that viscous effects at the trailing edge (apparently a separation and formulation of a thin turbulent wake) decrease the circulation, thereby causing the loss in lift. The associated decrease in expansion also causes the experimental shock to occur further upstream. This is not to be confused with shock induced separation which, it is believed, does not occur in the experimental data shown.

These numerical solutions were obtained using an unevenly space $(\xi\eta,z)$ grid of 68 X30 X 49 points and 7 hours of computation time for both solutions on the IBM 360/67 computer. Convergence was established when the lift changed less than 0.02 percent per iteration. The three relaxation parameters required in the method were set at 1.4 in subsonic regions, 0.7 in supersonic regions, and 1.0 for the circulation equation. Experimentation with the circulation relaxation parameter indicated that the value of one was the best choice. The use of higher values caused oscillations to occur.

CONCLUSIONS

A mixed elliptic-hyperbolic relaxation method has been applied to the study of a nonlinear small perturbation equation modeling steady, three-dimensional transonic flow. Certain nonlifting wing-body combinations were computed without difficulty, and a numerical procedure for treating lifting wings without bodies was presented. In an effort to simplify the treatment of swept and tapered wings with blunt leading edges, a coordinate transformation has been introduced to map the wing planform into a rectangle. Certain difficulties introduced by this transformation were explored, and results found from its use under valid circumstances were presented and compared with experiment.

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