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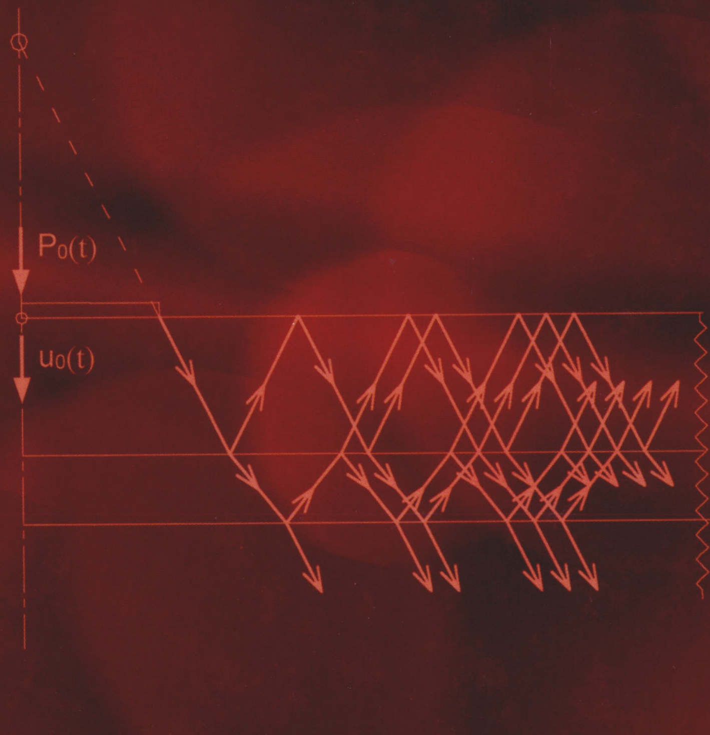
FOUNDATION VIBRATION ANALYSIS: A STRENGTH-OF-MATERIALS APPROACH



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Foreword

The exponential rise in computing capability over the last few decades has permitted the solution, in principle, of even the most complex of soil-structure interaction problems by means of detailed numerical analysis. However, there is still a gulf between the potential to analyse such problems and the practical ability to do so, either for a particular application or for research purposes. Instead, recourse is made to a range of simplifications in order to focus on that part of the total problem which is deemed to be critical. Thus foundation engineers may represent the superstructure by an idealised elastic block, or even as a uniform load applied to the ground, while structural engineers may represent the ground as a distributed bed of springs or as a rigid boundary.

Where the superstructure is modelled, it will generally be discretised not into three-dimensional continuum elements, but into a collection of one-dimensional (bar or beam) and two-dimensional (shell or plate) elements. Such elements represent what is referred to as the 'strength-of-materials' approach in this book. They make use of the dominant geometric axes evident in most structural components, together with simplifying assumptions that allow quantification of appropriate stiffness matrices for the elements. They may also incorporate more subtle rules of response that allow for local three-dimensional effects, such as edge buckling of a beam, even though the basic model is one or two dimensional.

Analysis of the ground response is less obviously amenable to treatment using simplified elements. Instead, continuum elements are almost universally adopted, although often the overall geometry of the problem is simplified to two dimensions, either in plane strain or axial symmetry. In static problems, advantage can be taken of the (commonly assumed) horizontal stratification of soil and rock using the finite layer techniques pioneered by Booker and Small, reducing the three-dimensional problem to two dimensions, although sophisticated software development is still necessary. In dynamic problems, a more typical simplification of the ground response has been the crude idealisation by lumped springs and dashpots, perhaps incorporating a plastic slider to represent the limit on bearing capacity. Such models have been used in the analysis of impact and vibration response of piles embedded in soil, but cannot easily be extended to deal with stratified soil, or with more complex foundation geometries.

The authors of the present book have set out to establish a set of 'strength-of-materials' idealisations for the geotechnical response in the analysis of foundation problems. The idealisations are akin to the bar and beam elements familiar in structural engineering, and are based on the truncated cone model developed by Meek in collaboration with Veletsos and Wolf. In his previous book (Foundation Vibration Analysis Using Simple Physical Models, Prentice Hall, 1994) Wolf extended the truncated cone model to a range of physical problems and different modes of dynamic excitation, and also documented various lumped parameter models. While the alternative solutions provided in that book have proved most useful to practising geotechnical engineers, the present book focuses on a self-contained development of the truncated cone models. This will allow practitioners to develop their own armoury of techniques for problems of interest, whether arising from structural-induced vibrations or from seismic events. Of particular merit is the inclusion of MATLAB software in the book, together with a full executable program, CONAN, which provides individuals with a starting point for custom-designed solutions.

The basic solutions are developed in the frequency domain, and thus are restricted to linear response of the soil. The building blocks are the vertical, horizontal, rotational and torsional response of a disk foundation resting on the surface of a homogeneous half-space. The power of the book rests in the detailed development of these primitive solutions, using the concept of reflection and refraction at layer interfaces, to address embedded foundations with quite arbitrary shapes in stratified soil deposits underlain by either a rigid base or an infinite medium. With well-chosen example applications (supplemented by segments of MATLAB code), the reader is shown how to apply the techniques to assess the response to free-field (seismic) ground motions or vibrations internally generated within the structure.

Treatment of the subject is comprehensive, with detailed appendices covering development of the basic solutions and their integration to address practical problems. The flow of the main text is left deliberately uncluttered in this way, and even historical documentation of the truncated cone solutions is summarised neatly in a separate appendix.

This is the sixth book by John Wolf and I am confident that it will prove as much a landmark as his previous books on dynamic soil-structure interaction. He is joined in the present book by a co-author, Andrew Deeks, whose positive influence can be seen clearly in the elegant MATLAB code and CONAN executable. The authors have taken pains to evaluate the accuracy of their approach against closed form and rigorous numerical solutions. As they point out, even at worst the errors are relatively minor in comparison with other uncertainties in the problem, particularly those associated with characterising the dynamic properties of the geotechnical medium. Just as in structural analysis there are limitations to conventional 'strength-of-materials' solutions, so there will be situations where the approaches described here may prove insufficient. However, this book establishes a powerful basis for a 'strength-of-materials' approach to dynamic foundation problems and will no doubt prove invaluable across the spectrum of practising engineers, researchers and teachers.

Mark F. Randolph
University of Western Australia

Preface

Most structural analysis is performed based on the strength-of-materials approach using bars and beams. Postulating the deformation behaviour ('plane sections remain plane'), the complicated exact three-dimensional elasticity is replaced by a simple approximate one-dimensional description that is adequate for design. The approach is very well developed, permitting complicated structural systems, such as curved skewed prestressed bridges with moving loads, to be modelled with one-dimensional bars and beams. This strength-of-materials theory is extensively taught in civil and mechanical engineering departments using the excellent textbooks available in this field.

In contrast, in geotechnical engineering, the other field of civil engineering where modelling is important, the strength-of-materials approach is not being used extensively. There are two main reasons for this. First, while in structural engineering the load bearing elements to be analysed tend to have a dominant direction determining the axes and cross-sectional properties of the bars and beams, in geotechnical engineering three-dimensional media, the soil and rock, are present. The choice of the axes and especially the cross-sectional properties (tributary section), which must be able to represent all essential features with the prescribed deformation behaviour, is thus more difficult in geotechnical engineering than in structural engineering. Second, up to quite recently, the state of development of the method was severely limited. Even just over ten years ago, only surface foundations on a homogeneous half-space representing the soil could be modelled with a strength-of-materials approach using conical bars and beams, which are called cones in the following. As the soil properties in an actual site will change with depth, this approach was only of academic interest.

This pioneering effort did, however, form the basis of important recent developments. Today, based on the same assumptions, reasonably complicated practical cases can be analysed. The site can exhibit any number of horizontal layers, permitting the modelling of a general variation of the properties with depth. Besides surface foundations, embedded foundations can be analysed. Seismic excitation can be processed without introducing any additional assumptions. Thus, the cone models can be used to model the foundation in a dynamic soil-structure-interaction analysis. Cone models work well for the low- and intermediate-frequency ranges important for machine vibrations and earthquakes, for the limit of very high frequencies as occurring for impact loads, and for the other limit, the static case. By simplifying the physics of the problem, conceptual clarity with physical insight results. In the cone models, the wave pattern is clearly postulated. The wave propagates outwards away from the disturbance spreading in the direction of propagation within the cross-section of the cone. When a discontinuity of the material properties corresponding to an interface of the soil layers is encountered, two new waves are generated: a reflected wave and a refracted wave, propagating in their own cones. When modelling with cones, the analyst feels at ease, as the same familiar concepts of strength of materials used daily in structural analysis are applied. This is in contrast to using rigorous methods, based on three-dimensional elastodynamics with a considerable mathematical complexity, which tend to intimidate practitioners and obscure physical insight. Due to the simplification of the physical problem, the mathematics of the cone models can be solved rigorously. The fundamental principles of wave propagation and dynamics are thus satisfied exactly for the cones. Closed-form solutions exist for these one-dimensional cases. This leads to simplicity in a practical application. The use of cone models does indeed lead to some loss of precision compared to applying the rigorous methods of elastodynamics. However, this is more than compensated by the many advantages mentioned above. It must also be remembered that the accuracy of any analysis will always be limited by significant uncertainties, such as in the material properties of the soil, which cannot be avoided. Summarising, the ease of use with physical insight especially, the sufficient generality and the good accuracy allow the cone models to be applied for foundation vibration and dynamic soil-structure-interaction analyses in everyday cases in a design office. It is fair to state that a balanced design using cone models leads to simplicity that is based on rationality, which is the ultimate sophistication!

Starting from scratch, the one-dimensional strength-of-materials theory for conical bars and beams, called cones, is developed and applied to practical foundation vibration problems. No prerequisites other than elementary notions of mechanics, which are taught in civil engineering departments of all universities, are required. In particular, concepts of structural dynamics are not needed to calculate the dynamic behaviour of a foundation. (To perform a dynamic soil-structure-interaction analysis the structure must also be modelled, which is, however, outside the scope of this book.) The elementary treatment is restricted to harmonic excitation (the frequency domain) in the main text, with a direct time domain analysis developed as an extension in an appendix. The transformation from the time domain to the frequency domain using a Fourier series is described in an appendix. The equations of motion of dynamic soil-structure interaction are also addressed in an appendix. As the transformation to modal coordinates, which is so powerful in structural dynamics, cannot be used for foundations because they are semi-infinite domains, wave propagation plays a key role. Wave motion in prismatic bars is introduced in an appendix, and wave propagation in one-dimensional cones is described in great detail throughout the book. Only two aspects of wave motion are actually needed: the outward propagation of waves in the initial cone away from the disturbance and the generation of the reflected and refracted waves at a material discontinuity corresponding to a soil layer interface. By tracking the reflection and refraction of each incident wave sequentially, the superimposed wave pattern up to a certain stage can be established. This yields a significant simplification in formulation and programming. A thorough evaluation of the accuracy for a wide range of actual sites is performed. A short computer program written in MATLAB forms an integral part of the book. It is introduced in stages in the various chapters of the book. A full understanding of all aspects of the code, which can easily be modified by the user, results. In addition, an executable computer program called CONAN (CONe ANalysis) with a detailed description of the input and output is provided, which can be used to analyse practical cases. A complicated machine foundation problem, a typical seismic soil-structure-interaction problem and an offshore wind turbine tower with a suction caisson foundation are analysed as examples. A dictionary translating the key technical expressions into various languages increases the international acceptance of the book.

Many of the ideas contained in this book developed gradually over the last ten years. Another book by the senior author, titled *Foundation Vibration Analysis Using Simple Physical Models*

(Prentice Hall, 1994), was written primarily to appeal to geotechnical consultants and contains a very complete description of simple physical models, where, besides cones, lumped-parameter models (spring-dashpot-mass models) and prescribed horizontal wave patterns are also derived. However, the completeness and thus redundancy of the book tend to irritate the reader. Also, significant advances have been made in the area of cone models since the publication of that book. This leads to the current book, which is self-contained, without any prerequisites, and concentrates on the method of cones, which is developed using the standard assumptions of the theory of strength of materials only. Very recent research by the authors, which streamlines the formulation, is incorporated. Following the suggestions of readers of the previous book over the years, a computer program for the analysis of practical cases is fully integrated and explained in detail. The new book is a state-of-the-art treatise regarding cone models, but can also be used as the basis for a first course in soil dynamics of geotechnical engineering (at the final year undergraduate or first year postgraduate level), and can be taught in a course in structural dynamics, as all structures have foundations that have to be analysed. As the students study bars and beams extensively in elementary structural engineering, the basis for the extension to dynamics is very solid. In addition, the book will be valuable to practising geotechnical engineers, who should only apply a computer program when they fully grasp the computational procedure it is based on. The computational procedure detailed in the book will be familiar to them, as the strength-of-materials approach is the same as used routinely in structural analysis.

The contribution of Matthias Preisig in his Diploma-thesis, which clearly demonstrates the potential of the streamlined formulation using cones, is noted. The creative research of Dr Jethro W. Meek, performed in an informal, enthusiastic and collegial atmosphere with the senior author in the beginning of the 1990s, which forms the basis of the strength-of-materials approach, is gratefully acknowledged. The authors are indebted to Professors Eduardo Kausel of MIT and John Tassoulas of the University of Texas at Austin who calculated on our request the results for comparison. Without this support a systematic evaluation of the accuracy would not have been possible. Provision of simulated strong ground motion for the 1989 Newcastle earthquake by Dr Nelson Lam of the University of Melbourne is also acknowledged with thanks. The authors are indebted to Professor Mark Randolph, Director of the Centre for Offshore Foundation Systems at the University of Western Australia, for writing the Foreword.

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Contents

1

Introduction

1.1 Statement of the problem

The following preliminary remark is appropriate. To address the goal of foundation vibration analysis, certain terms such as dynamic stiffness or effective foundation input motion are introduced. At this stage of development only a sketchy qualitative description without a clear definition is possible. The reader should not become irritated. From Chapter 2 onwards the treatment is systematic, from the bottom up and rigorous.

The objective of foundation vibration analysis is illustrated in Fig. 1.1. The response of a massless cylindrical foundation of radius r_0 embedded with depth e in a layered soil half-space is to be calculated for all degrees of freedom. The vertical wall and the horizontal base of the foundation are assumed to be rigid. As a special case a circular surface foundation can be addressed, which corresponds to $e = 0$. Horizontal layering exists with constant material properties in each layer. The jth layer with thickness d_j has shear modulus G_j, Poisson's ratio ν_j, mass density ρ_j and a hysteretic damping ratio ζ_j $(j = 1, 2, \ldots, n-1)$. The underlying homogeneous half-space is denoted with the index n. The site can also be fixed at its base (rigid underlying half-space). Linear behaviour of the site is assumed, meaning that the soil is assumed to remain *linearly elastic with hysteretic material damping* during dynamic excitation. This can be justified by noting that the allowable displacements of foundations for satisfactory operation of machines are limited to fractions of a millimetre. It should also be noted that all waves propagating towards infinity decay due to geometric spreading, resulting in soil which can be regarded as linear towards infinity. Inelastic deformations are thus ruled out.

Two types of dynamic loads, which vary with time, are considered. These consist of loads acting directly on the rigid foundation at point O (Fig. 1.1), originating from rotating machinery, for example, and excitations introduced through the soil, from seismic waves, for example. For the latter excitations only vertically propagating waves are considered, with the particle motion in either the horizontal or the vertical direction. The so-called *free-field motion*, i.e. the displacements in the virgin site before excavation, is shown schematically for these horizontal and vertical earthquakes on the left-hand side of Figs 1.2a and 1.2b respectively.

As a slight extension, any axi-symmetric foundation can be examined (Fig. 1.3). The wall does not have to be vertical, but the base must remain horizontal. The wall and the base are again assumed to be rigid.

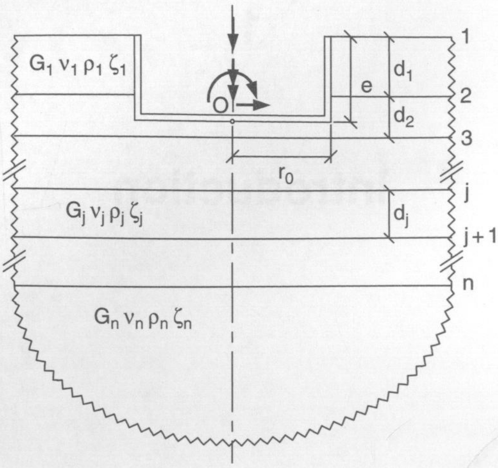

Figure 1.1 Cylindrical foundation embedded in layered soil half-space with degrees of freedom

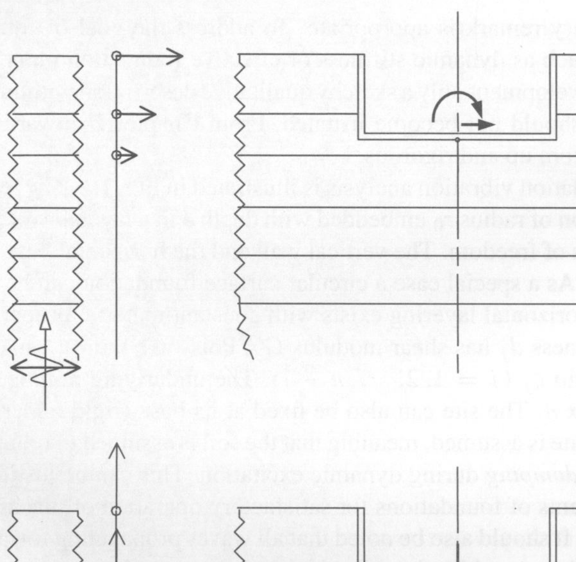

Figure 1.2 Free-field motion and effective foundation input motion for vertically propagating seismic excitation. a) Horizontal earthquake. b) Vertical earthquake

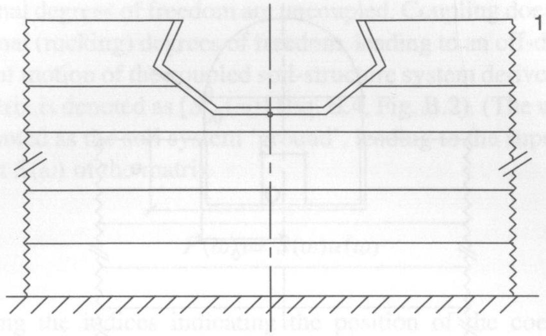

Figure 1.3 Axi-symmetric foundation embedded in soil layers fixed at base

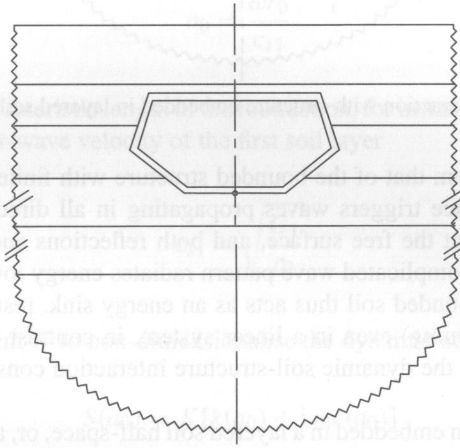

Figure 1.4 Fully-embedded foundation in layered soil half-space

Axi-symmetric inclusions can also be processed (Fig. 1.4). They are termed fully-embedded foundations, while the cases where the wall intersects the free surface are referred to as partially-embedded where a distinction is appropriate. Of course, a foundation in a full-space is always fully-embedded.

More general foundations can be transformed to axi-symmetric cases. This can be accomplished by equating a certain quantity of the general foundation to the corresponding quantity of the axi-symmetric case. For instance, when translational degrees of freedom dominate, areas in the horizontal section can be equated, while when rotational degrees of freedom dominate, moments of inertia in the horizontal section can be equated.

In many applications a structure is also present, with the structure-soil interface coinciding with the rigid wall and base of the foundation (Fig. 1.5). In this case two substructures are present, the foundation embedded in the soil and the structure. The two substructures are connected at point O forming a coupled system. This defines a *dynamic unbounded soil-structure-interaction problem*. Exterior loads can also be applied to the structure, and, as already mentioned, the dynamic excitation can be introduced through the soil (by seismic waves, for example). In such problems the responses of the structure and, to a lesser extent, of the soil are to be determined.

The coupling of the substructures enforces equilibrium and compatibility of the displacements and rotations at O. However, the dynamic behaviour of the unbounded soil, a semi-infinite domain,

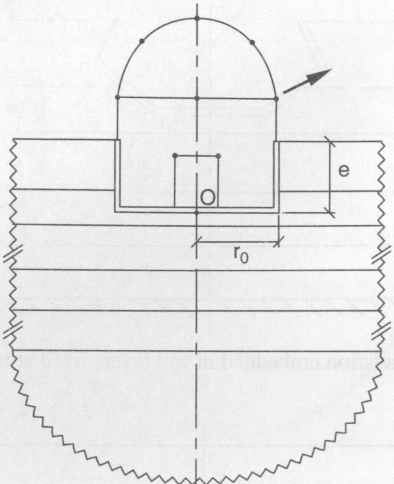

Figure 1.5 Soil-structure interaction with structure embedded in layered soil half-space

is significantly different from that of the bounded structure with finite dimensions. The motion of the structure-soil interface triggers waves propagating in all directions in the soil towards infinity. Reflections occur at the free surface, and both reflections and refractions occur at the soil layer interfaces. This complicated wave pattern radiates energy towards infinity, outside the dynamic system. The unbounded soil thus acts as an energy sink, resulting in damping (which is known as *radiation damping*) even in a linear system, in contrast to the bounded structure. The challenge in analysing the dynamic soil-structure interaction consists of modelling the soil illustrated in Fig. 1.1.

Thus, the rigid foundation embedded in a layered soil half-space, or, as it can also be described, the unbounded layered soil containing an excavation with a rigid interface, is addressed. This substructure's dynamic properties are defined on the interface with the other substructure, the structure, at point O. For seismic excitation two quantities must be determined: first, the interaction force-displacement relationship determining the contribution of the unbounded soil to the dynamic stiffness of the coupled equations; and second, the so-called effective foundation input motion arising from the seismic excitation introduced through the soil.

As the unbounded soil remains linear, the dynamic analysis can be performed in the frequency domain. As outlined in Appendices A.3 and A.4, the dynamic excitation in the time domain is expressed as the sum of a series of harmonic components (Fourier series and integral). It is thus sufficient to address a discrete harmonic excitation with a specific frequency ω, characterised by the corresponding complex amplitude, as discussed in Appendix A.1. The amplitude of the response for this harmonic excitation follows as the product of the complex frequency response function (examined in Appendix A.2) and the amplitude of the excitation.

In general, the loading applied to the structure will not be axi-symmetric. The interaction force-displacement relationship for harmonic excitation at point O (Fig. 1.1) is

$$\{P(\omega)\} = [S(\omega)]\{u(\omega)\} \tag{1.1}$$

with $\{u(\omega)\}$ denoting the amplitudes of the three displacements and three rotations at O, $\{P(\omega)\}$ the amplitudes of the three forces and three moments at O and $[S(\omega)]$ representing the *dynamic-stiffness matrix*, the complex frequency response function. As the foundation is axi-symmetric,