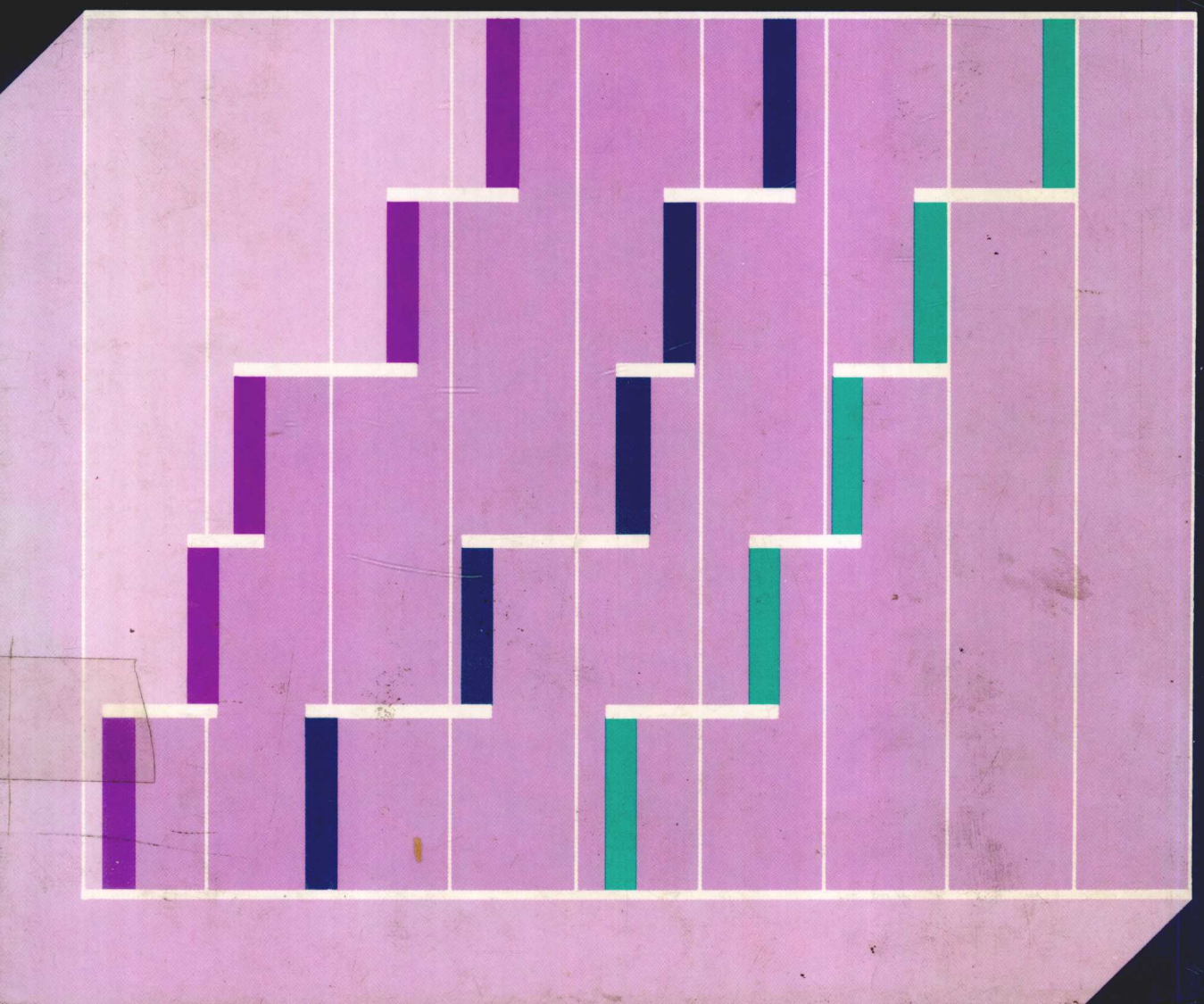
RONALD L. IMAN
W. J. CONOVER

A MODERN APPROACH TO STATISTICS



A Modern Approach To Statistics

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R. L. I.
W. J. C.

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PREFACE

To the Student

As you begin this course you may well wonder what lies in store for you. At this time in your life with limited job experience it is difficult to be familiar with situations where the application of statistical methods would be appropriate. Thus, the authors have tried to provide a variety of problem settings, examples, and exercises that will allow you to realize some of the potential uses of statistics and how statistics can be important to you in your future or present employment. Additionally, even though you master the material and gain an appreciation for the potential use of statistics while taking this course, it may be difficult at some time in the future to recall all of the details of some particular statistical procedure. Thus, a great deal of effort has gone into making this text readable and organizing the various methods in a form that would permit this book to serve as a ready reference at some time in the future when the need arises.

RONALD L. IMAN W. J. CONOVER

PREFACE

To the Instructor

In general the approach used in this book is conventional in many ways, but also unconventional in many other ways. Both authors have had extensive experience teaching and consulting, and this book reflects the authors' experiences, especially regarding the inadequacies and incompleteness of the presentation made in most basic statistics textbooks. In this vein, throughout the text the emphasis is on training the reader to go from a data-oriented situation to the proper statistical method. Most topics are introduced by means of a realistic problem setting, and then the method of solution unfolds in the subsequent discussion where the authors attempt to communicate how to analyze a real situation.

The problems, examples, and exercises further emphasize realistic settings in which these methods are useful. The problem settings and exercises are based on actual applications in the fields of biology, business, economics, psychology, sociology, education, home economics, agriculture, pharmacology, engineering, physical education, physiology, genetics, actuarial science, and others.

This book is intended as an introduction to modern statistical techniques in a one-semester course with the topics divided into lesson-size sections. Each section concludes with a set of exercises; review exercises occur at the end of each chapter. In all, the book contains approximately 540 exercises and numerous worked-out examples. Answers to selected exercises are given at the end of the text.

Each methodology in the book is presented in a self-contained format, complete with assumptions, explanation of notation, statements of hypotheses, and decision rules. This format is used for the convenience of the reader in referring back to the various statistical procedures. It is the authors' experience that most texts present methodology and assumptions in a helter-skelter fashion that makes it extremely difficult if not impossible for the reader to decide when to use each methodology. These same texts almost always fail to provide any guidance as to what to do if the assumptions are not satisfied. Thus, a question

of ethics arises. Should the assumptions be considered? Should texts make some attempt to deal with these issues? The authors do not feel that the reader should be left on his or her own, assuming that all is well if the mechanics are known, but that these issues are quite important and should be addressed.

In order to avoid these shortcomings, the sections presenting testing procedures clearly state the assumptions required for validity of the testing procedure. Many of these testing procedures rely on the underlying assumption of normality. This assumption is easily checked using graphs specifically developed for this book for use with the Lilliefors test for normality. Thus, the cumbersome chi-square goodness-of-fit test for normality is avoided, enabling the assumption of normality to be checked easily thorughout the text. For situations where the normality assumption is not satisfied, guidance toward the appropriate nonparametric test is provided. This means that the parametric and nonparametric tests are presented side-by-side throughout the text. However, the transition to the nonparametric tests is made easy by presenting them as analogues to parametric tests. This is done by applying the parametric procedure to rank transformed data. The result is a test statistic that is functionally equivalent to the usual presentation of the nonparametric test statistic. This way of presenting nonparametric tests makes them easier to learn than when the standard method is used, and it eases the burden on instructors who may not have a strong nonparametric background. This approach was described and verified in recent research papers by the authors. A tutorial paper on the subject, with appropriate references, appeared in *The American Statistician* in August 1981.

The first chapter concentrates on sampling procedures and includes a section on sample surveys reprinted with the permission of the American Statistical Association. Due to the ease with which graphs are used, it is a temptation for many instructors to skim through techniques for displaying sample data such as occur in Chapter 2. The authors hope the instructor will not yield to this temptation since most students really do not know how to display sample data, and these techniques will be used throughout the text to aid in displaying and understanding the data as well as checking the reasonableness of many decisions. This is particularly true of scatterplots and the empirical distribution function. The graph of the empirical distribution function simplifies many of the ideas that usually give students difficulty, such as working with normal probabilities, grasping the concept of power, finding sample quantiles, and performing goodness-of-fit tests.

Years of study can be devoted to the fascinating study of probability. This book is intended to teach statistics rather than probability, so in Chapter 3 only the topics in probability that are necessary to teach statistics are included.

Descriptive sample statistics such as the sample mean, standard deviation, median, mode, and proportion and the associated population parameters are given in Chapter 4. Also included in this presentation are the empirical distribution function, quantiles, the sample correlation coefficient, and the rank correlation coefficient, since these statistics are integrated into the presentation throughout the text.

The commonly used binomial and normal distributions are introduced in

Chapter 5 along with instructions on how to use their corresponding Tables A1 and A2. These tables as well as others presented in this book are all based on cumulative distributions and therefore tie in closely with the authors' repeated emphasis on empirical and cumulative distribution functions. The table of the cumulative normal distribution has the advantage over the density function approach (i.e., area under a curve) emphasized in most statistics books because students grasp its use almost immediately—something that cannot be said of the tables based on the density function, as anyone who has spent countless hours explaining how to use it knows only too well. The Lilliefors test for normality is introduced in Chapter 5. Its use requires plotting an empirical distribution function of standardized sample values in the graphs developed for this text. The Lilliefors test provides an easy and accurate test for normality and is used throughout the text to challenge the normality assumption. Sampling distributions and the Central Limit Theorem are covered in Chapter 5 as are the closely related exponential and Poisson distributions. A companion Lilliefors test for exponential distributions is given.

Point and interval estimates are considered in Chapter 6 as well as properties of estimators. In addition to the usual confidence interval for the mean of a normal population, a procedure for finding a confidence interval for the median of a population is given. This procedure may be more appropriate in nonnormal populations. Confidence intervals are also provided for the population proportion. The framework for hypothesis testing is given in Chapter 7. Hypothesis testing is demonstrated in sections covering each of the population mean, proportion, and median. Power is explained in terms of the cumulative distribution function.

The matched pairs problem is presented in Chapter 8 from the standpoint of being a good experimental design to control unwanted variation. The paired *t*-test is presented as the parametric method of analysis. The normality assumption is checked with the Lilliefors graphs and the Wilcoxon signed ranks test is given as the alternative nonparametric procedure when the normality assumption is not satisfied. The Wilcoxon signed ranks test is presented as a rank transform test. That is, the parametric paired *t*-test is applied to rank transformed differences. The result is a test statistic that is functionally equivalent to the standard presentation of the Wilcoxon signed ranks test statistic; however, the transition is quite easy for the student as all that is needed is to apply the just learned parametric paired *t*-test to the ranks. The critical values are approximated by the Student's *t*-distribution, which is quite accurate even for small sample sizes. The basis for this approach is contained in a number of research papers by the authors.

The case of two independent samples is considered in Chapter 9. Large sample techniques are given first followed by the two-sample *t*-test. Assumptions of normality and equal variance are checked. Graphs of the empirical distribution functions are presented in the examples to aid the student in understanding the two-sample problem and resultant decisions. The Wilcoxon rank sum test is presented as a rank transform test obtained from the two-sample *t*-test.

Chi-square test procedures are considered in Chapter 10, including 2×2

and $r \times c$ contingency tables, and goodness-of-fit tests. Chapter 11 reviews correlation on raw and rank transformed data prior to the study of regression techniques in Chapter 12.

Methods for linear regression are given in Chapter 12, including the usual least-squares computation procedures as well as a study of the model for linear regression and hypothesis testing. A nonparametric test for slope based on rank correlation is also given. Methods of monotone regression are presented as linear regression computations applied to rank transformed data. These methods are appropriate for either linear regression or monotonic nonlinear regression.

The completely randomized design and associated one-way analysis of variance are presented in Chapter 13. These techniques represent a generalization of the two-sample procedure of Chapter 9. Fisher's least significant difference is given as a multiple comparisons procedure; research (including that by the authors) has shown this procedure to be robust and powerful compared to other multiple comparisons techniques. The Kruskal–Wallis test is given as the rank transform analogue to the one-way analysis of variance for the completely randomized design. Computer output is utilized for the analyses in this chapter.

Depending on the emphasis desired by the instructor, a number of sections would be omitted in a 3- or 4-semester-hour course without loss of continuity. In particular, any one of the following groups of sections could be eliminated totally independent of the elimination or inclusion of any of the other groups: (1.2, 1.3, and 1.4); (4.6, 11.2, 12.5, and 12.6); 5.5; (6.4 and 7.4); and (10.1, 10.2, and 10.3).

The authors hope you will find it a refreshing experience to teach from this text and would welcome any correspondence regarding strengths or weaknesses of the book that would affect any possible future editions.

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