ADVANCES
IN
PSYCHOLOGY

60

New Developments in Psychological Choice Modeling

Geert de Soete
Hubert Feger
Karl C. Klauer
Editors

North-Holland

NEW DEVELOPMENTS IN PSYCHOLOGICAL CHOICE MODELING

Edited by

Geert DE SOETE
University of Ghent
Belgium

Hubert FEGER
Free University of Berlin
F.R.G.

Karl C. KLAUER
Free University of Berlin
F.R.G.

NORTH-HOLLAND
AMSTERDAM · NEW YORK · OXFORD · TOKYO

ELSEVIER SCIENCE PUBLISHERS B.V.
Sara Burgerhartstraat 25
P.O. Box 211, 1000 AE Amsterdam, The Netherlands

Distributors for the United States and Canada:

ELSEVIER SCIENCE PUBLISHING COMPANY, INC.
655 Avenue of the Americas
New York, N.Y. 10010, U.S.A.

ISBN: 0 444 88057 7

Printed in The Netherlands.

ADVANCES IN PSYCHOLOGY

60

Editors:

G. E. STELMACH

P. A. VROON

NORTH-HOLLAND
AMSTERDAM · NEW YORK · OXFORD · TOKYO

NEW DEVELOPMENTS
IN
PSYCHOLOGICAL CHOICE MODELING

LIST OF CONTRIBUTORS

P. M. Bossuyt, Center for Clinical Decision Making, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands.

J. D. Carroll, AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974, U.S.A.

M. A. Croon, Psychology Department, Tilburg University, Tilburg, The Netherlands.

W. S. DeSarbo, Graduate School of Business, Marketing and Statistics Departments, University of Michigan, Ann Arbor, Michigan 48109, U.S.A.

G. De Soete, Department of Psychology, University of Ghent, Henri Dunantlaan 2, 9000 Ghent, Belgium.

H. Feger, Institute for Psychology, Free University Berlin, Habelschwerdter Allee 45, 1000 Berlin 33, FR Germany.

W. Gaul, Institute of Decision Theory and Operations Research, Faculty of Economics, P.O. Box 6380, 7500 Karlsruhe 1, FR Germany.

W. J. Heiser, Department of Data Theory, University of Leiden, Middelstegracht 4, 2312 TW Leiden, The Netherlands.

K. Jedidi, Marketing Department, Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104, U.S.A.

K. C. Klauer, Institute for Psychology, Free University Berlin, Habelschwerdter Allee 45, 1000 Berlin 33, FR Germany.

D. B. MacKay, School of Business, Indiana University, Bloomington, Indiana 47405, U.S.A.

B. Orth, Department of Psychology, University of Hamburg, Von-Melle-Park 6, 2000 Hamburg 13, FR Germany.

V. R. Rao, Johnson Graduate School of Management, Cornell University, Ithaca, New York 14853, U.S.A.

E. E. Roskam, Mathematical Psychology Group, University of Nijmegen, Montessorilaan 3, 6500 HE Nijmegen, The Netherlands.

Y. Takane, Department of Psychology, McGill University, 1205 Docteur Penfield Avenue, Montreal, PQ, Canada H3A 1B1.

R. van Blokland-Vogelesang, Department of Psychology, Free University, Van der Boechorstraat 1, Room 1B-69, P.O. Box 7161, 1007 MC Amsterdam, The Netherlands.

W. H. van Schuur, Department of Statistics and Measurement Theory, Faculty of Social Sciences, University of Groningen, Oude Boteringestraat 23, 9712 GC Groningen, The Netherlands.

J. L. Zinnes, National Analysts, 400 Market Street, Philadelphia, Pennsylvania 19106, U.S.A.

CONTENTS

INTRODUCTION

Historically, two of the most important contributions to psychological choice modeling are undoubtedly Thurstone's (1927) Law of Comparative Judgment and Coombs' (1950, 1964) unfolding theory. The framework that Thurstone's Law of Comparative Judgment provides for representing inconsistent choices is still the point of departure for much of the current work in probabilistic choice modeling. In 1987 the journal *Communication & Cognition* published a special issue on probabilistic choice models. Several of the papers in this special issue exemplify how many of the recent probabilistic choice models are still in one way of another related to Thurstone's general Law of Comparative Judgment.

An entirely different approach to modeling individual choice was offered by Coombs in his unfolding theory. Coombs' unfolding principle gave rise to many different unidimensional and multidimensional unfolding models, as illustrated in the 1988 special issue on unfolding of the German journal of social psychology *Zeitschrift für Sozialpsychologie.*

The editors of both special issues wanted to make the contributions in these issues available to a broader audience. Since the papers in the two special issues are often very much related to each other, in that some of the recent stochastic choice models are based on a geometric unfolding model or, equivalently, that some of the recent unfolding models are probabilistic, it was decided to bundle the contributions into a single edited volume. Most papers have been substantially revised since their initial publication in either *Communication & Cognition* or *Zeitschrift für Sozialpsychologie.*

The resulting volume is fairly representative of the current work in psychological choice modeling. The papers by *Heiser*, *Feger*, and *DeSarbo and Rao* concentrate on devising efficient methods for fitting deterministic unfolding models to nonmetric (*Heiser*, *Feger*) or metric (*DeSarbo & Rao*) data. In the papers by *Bossuyt and Roskam*, *Croon*, *De Soete et al.*, *Takane*, *Carroll et al.*, and *Zinnes and MacKay* new choice models are developed. Whereas *Bossuyt and Roskam* propose a new unidimensional probabilistic unfolding model, *De Soete et al.* and *Zinnes and MacKay* elaborate new multidimensional probabilistic unfolding models. *Takane* proposes a family of stochastic models where the within-subject and the between-subject inconsistency are explicitly modeled. An attempt to formulated discrete probabilistic analogs of the unfolding model is reported by *Carroll et al.*

Next come two papers that deal with the problem of assessing the validity of choice models. *Bossuyt and Roskam* discuss one approach to testing the assumptions of probabilistic models, while *Orth* explains and illustrates an axiomatization of the (deterministic) Coombsian unfolding model.

The remaining contributions of the volume contain some important applications of psychological choice modeling in the fields of political science and marketing research. *Van Blokland-Vogelesang* illustrates the use of an unfolding technique for constructing a prestige ladder, whereas *van Schuur* applies a specific unidimensional unfolding model to political science data. *DeSarbo et al.* and *Gaul* discuss probabilistic choice models and related tools that are applicable in consumer research.

As will be apparent from the various contributions in this volume, important progress has been made in psychological choice modeling in the last few years. However, many problems remain to be solved and it is our sincere hope that this volume might stimulate other researchers to work on some of these problems.

References

Coombs, C. H. (1950). Psychological scaling without a unit of measurement. *Psychological Review, 57*, 145-158.

Coombs, C. H. (1964). *A theory of data.* New York: Wiley.

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review, 34*, 273-286.

New Developments in Psychological Choice Modeling
G. De Soete, H. Feger and K. C. Klauer (eds.)

ORDER INVARIANT UNFOLDING ANALYSIS UNDER SMOOTHNESS RESTRICTIONS

Willem J. Heiser
University of Leiden, The Netherlands

Unfolding analysis is shown to have firm roots in the Thurstonian attitude scaling tradition. Next the nonmetric multidimensional approach to unfolding is described, and characterized in terms of objectives proposed for attitude scaling by Guttman. The nonmetric approach is frequently bothered by a phenomenon called degeneration, i.e., the occurrence of extremely uninformative solutions with good or even perfect fit. A new way to resolve this problem, while keeping the method order invariant, follows from the introduction of smoothness restrictions on the admissible model values. The effectiveness of requiring smoothness is illustrated with an example of political attitude scaling, and with a two-dimensional analysis of differential power attribution among children. Cross validation and resampling techniques can be used for establishing the stability of the unfolding results.

1. Introduction

Applications of the unfolding model, using any one of its associated techniques, have been remarkably scarce in social psychology, especially in view of the fact that this methodology has such a classic precursor: the *Thurstonian attitude scaling* approach (Thurstone, 1929, 1931; Thurstone & Chave, 1929; see also Thurstone, 1959). Thurstone transferred the *unimodal response model* familiar from psychophysics to the study of attitudes and opinions, more generally of *affectively loaded* responses. The *attitude score* of a subject was defined as the mean or the median scale value of the attitude statements endorsed. The selection and the allocation

This paper is a revised version of an article published in *Zeitschrift für Sozialpsychologie*, 1987, *18*, 220-235.

of scale values to the statements was usually done in a preliminary study, in which judges had to compare them with respect to their "favorability". The reader is referred to Edwards (1957) for an extensive discussion of the Thurstonian approach, including its quality criteria and various early variants. In modern terms, it can be characterized as a way to perform an *external unfolding* analysis (a name coined by Carroll, 1972), with the model of equal appearing intervals — or the method of paired comparisons — as the first stimulus scaling step, and the computation of the mean or median as a primitive method to find the *ideal point*, i.e., the location of an imaginary statement that would get maximal support from any particular subject, or group of subjects.

After the Second World War, Thurstonian attitude measurement became more and more a curiosity. The assumed possibility to obtain unique, common scale values in the first step of the judgment-endorsement procedure had always been a matter of debate. The early evidence in a variety of attitude domains, such as attitude "toward the Negro" (Hinckley, 1932), "toward a particular candidate for political office" (Beyle, 1932), "toward war" (Ferguson, 1935), and "toward one's own country" (Pintner & Forlano, 1937), seemed to be positive in the sense that very high correlations were found between sets of scale values obtained from groups of judges with widely different attitudes. However, starting with Hovland and Sherif (1952) the influential *social judgment school* (Sherif & Hovland, 1961; Sherif, Sherif, & Nebergall, 1965) cast serious doubts on the validity of trying to separate "cognitive" judgments — presumably elicited in the first step — from "affective" judgments — presumably elicited in the second step. Objections were raised against some of the standard practices, such as eliminating judges with extreme categorizing behavior. Evidence was found for meaningful and systematic *assimilation* and *contrast* effects, reflected in local distortions of the stimulus scale. In addition, the social judgment school called attention to other aspects of attitudinal responses, i.e., the range of statements strongly endorsed ("the latitude of acceptance"), the subset of statements strongly rejected ("the latitude of rejection", not necessarily consisting of statements in consecutive positions along the scale), and areas of neutrality (forming "the latitude of noncommitment" in between the regions of acceptance and rejection).

It is important to notice that, despite these criticisms and amendments, the major constituents of the Thurstonian approach remained intact. The statements were scaled in a separate judgment procedure. Attitude was conceived as a subject specific response function with respect to these scale values. Although other aspects than location of the peak were deemed important, it was still assumed — *and empirically verified* — that response strength tapers off as a function of the distance from the "own stand as an anchor point" (Sherif et al., 1965).

Meanwhile, Likert's short-cut (Likert, 1932) had become increasingly popular. It involves the reduction of the judgment to an *a priori* classification of the statements into two about equally sized classes: the favorable ones and the unfavorable ones. By adjusting the scoring direction of the responses accordingly, and by using "refinements" borrowed from test theory, the concept of a statement scale value seemed to be superfluous. Indeed, it has become common practice to ask subjects directly for their evaluations of the attitude object. Only Likert's response format survived, and statement scaling was abandoned altogether.

Guttman's (1941, 1944, 1947, 1950) contributions are much less easily summarized in a few sentences. At least three novelties that he introduced into the field of attitude measurement should be mentioned:

a. A method for finding a scale based on the *endorsement alone*;

b. Posing *reproducibility* as an explicit criterion for scale construction;

c. Scaling the *response categories*, rather than the statements themselves;

It is of some historical interest to notice that the desirability of (a), called the "response approach" by Torgerson (1958, pp. 45-48), had already been expressed at the very introduction of Thurstone's method: "Ideally, the scale should perhaps be constructed by means of voting only. It may be possible to formulate the problem so that the scale values of the statements may be extracted from the records of actual voting. If that should be possible, then the present procedure of establishing the scale values by sorting will be superseded." (Thurstone & Chave, 1929, p. 56). Guttman achieved (a) by using (b): the construction should be such that "from a person's rank alone we can reproduce his response to each of the items in a simple fashion" (Guttman, 1947, p. 249). But at the same time —

although this would not have been strictly necessary — he switched from the concept of a *statement point* (i.e., a stimulus scale value) to the idea of characterizing each statement as a set of *category points* (i.e., response alternative scale values). In addition he assumed that all category points for a single statement would ideally be ordered along the scale in their "natural" order, from "strongly disagree" via "indifferent" to "strongly agree". So in Guttman scaling each subject is characterized by a score, and each statement by some *monotonically increasing* curve, for which frequently a step function is used as a first approximation.

By contrast, and in line with the Thurstonian tradition, the unfolding technique represents each statement as a point along a scale, and each subject as some *unimodal* or *single-peaked* curve, for which frequently the location of the peak is considered to be the parameter of most interest.

The approach of this paper will be to stick to aims (a) and (b), to replace (c) with a less restrictive requirement, and to bring in again the allocation of scale values to the objects of judgments. Undoubtedly, Coombs (1950, 1964) contributed much to the conceptual development of the single-peaked response model, including coining the generic name *unfolding*. In particular, he convincingly argued that one should refrain from making strong assumptions about the measurement level of human judgments — *within*, but especially also *across* persons — and that metric information should be obtained through the study of scalability. However, his methods for actually fitting scaling models to any set of data at hand lacked the rigor of optimizing a single loss function (as the reproducibility criterion is called nowadays). The Nonmetric Multidimensional Scaling (NMDS) approach to unfolding, to be discussed in Section 2, does enjoy this property. However, it is frequently bothered by a phenomenon called *degeneration*, as shall be clarified in Section 3. Then Section 4 proposes a new approach to resolve this difficulty, based on the idea of requiring a *smooth* succession of reproduced values. Next, the method will be applied in Section 5 to some political attitude data, and to a small example concerning the perceived importance of power characteristics by different groups of children in a classroom setting. Finally, Section 6 discusses some of the diagnostics that can be used in connection with an unfolding analysis.

2. The Nonmetric Multidimensional Scaling Approach to Unfolding

The earlier formulations of the unimodal response were all one-dimensional, perhaps for reasons of simplicity, or just "another manifestation of psychologists' peculiar evaluation monomania, reducing all information to this one dimension as if people think of themselves and other objects exclusively in terms of how good or how bad they are" (McGuire, 1985, p. 242, referring to McGuire, 1984). The model can be formulated q-dimensionally right from the start, with $q = 1$ merely a special case. At our disposal is a table **P** with elements p_{ij}, each row of which corresponds to a particular subject, or group of subjects, i ($i = 1, \ldots, n$), whereas each column corresponds to a particular statement, or other piece of psychological material, j ($j = 1, \ldots, m$). **P** might contain a measure of *preference* or *response strength*, or the proportion of people in group i voting for alternative j, or any other indication of the attraction of *object j* for *source i*.

The first objective is to assign a *point* $\mathbf{y}_j$ to each object. In the one-dimensional case $\mathbf{y}_j$ is just one real-valued number that can be marked off on a line; in the two-dimensional case $\mathbf{y}_j$ is characterized by two coordinate values that can be plotted in a plane; in the q-dimensional case $\mathbf{y}_j$ is a location in a q-dimensional space (less easy to visualize and talk about, but the principles and notation remain the same). We may now view the response strength of source i as a function of the $\mathbf{y}_j$. Under the unimodal response model it is assumed that this function has a *single peak*, i.e., it decreases monotonically in all directions with respect to some central point $\mathbf{x}_i$. In addition, it is assumed that the location of the peak is specific for each source. Since response strength is maximal at the position of the central point, $\mathbf{x}_i$ is usually called the *ideal point* for source i. So the model associates objects with points, and sources with single-peaked curves or surfaces that are *shifted* with respect to each other. These shifts, or translations, are very important. Imagine, for instance, a set of unimodal curves precisely on top of each other; then any relocation of the object points along the line, although destroying the common shape, would still account for the same information. One could make the curves more skewed, double-peaked, monotonically increasing, any shape at all, by suitable reexpressions of the values against which they are plotted. But, when the curves are shifted along the object scale, the freedom of simultaneous change of shape is reduced enormously. It was Coombs (1950) who first clearly demonstrated this property of shifted single-peakedness. Similar properties of shifted monotonically increasing curves have been studied in depth by Levine (1970, 1972).

So far the description characterizes what is common to all unfolding techniques (though some are confined to the one-dimensional case). The MDS approach now proceeds as follows. Attention is restricted to those single-peaked curves and surfaces that are a decreasing function of the *distance* $d(\mathbf{x}_i,\mathbf{y}_j)$ of the object point $\mathbf{y}_j$ from the ideal point $\mathbf{x}_i$. This is almost always the ordinary Euclidean distance

$$d(\mathbf{x}_i,\mathbf{y}_j) = \left[\sum_a (x_{ia} - y_{ja})^2\right]^{1/2}, \tag{1}$$

defined here on the coordinate values x_{ia} and y_{ja} for ideal points and object points respectively, where $a = 1, \ldots, q$. A major consequence of this restriction is that the response function will always be *symmetric*. Suppose we connect all points that have equal attractivity for a given source. Such a contour line is called an *isochrest* in this context, in analogy with "isobar" and "isotherm" for lines of equal atmospheric pressure and equal temperature on a map of physical locations (Heiser & De Leeuw, 1981). In the MDS approach to unfolding the isochrests are assumed to be sets of concentric circles (or spheres, or hyperspheres, for $q > 2$) centered at the ideal point, due to their dependence on the distance function (1).

At his juncture, the set of single-peaked functions could be restricted still further, for instance by choosing the explicit model

$$\pi_{ij} = \beta_i e^{-d(\mathbf{x}_i,\mathbf{y}_j)/\alpha_i}. \tag{2}$$

Here π_{ij} denotes the predicted response strength, the decay function is of the *negative exponential* type, the parameter β_i represents the *maximum* of the function (attained when the ideal point $\mathbf{x}_i$ coincides with the object point $\mathbf{y}_j$), and the parameter α_i represents the *dispersion* or *tolerance* of source i. Both α_i and β_i are assumed to be strictly positive. Note that α_i would be a parameter of interest to workers in the tradition of the social judgment school, as it indicates the size of the latitude of acceptance relative to the latitude of rejection. From (2) it follows that the logarithm of predicted response strength is linear in the distances, and a metric