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Several Problems in Applied Nonlinear Dynamical Systems

应用非线性动力系统中的几个问题

作者: 黄德斌
专业: 计算数学
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Several Problems in Applied Nonlinear Dynamical Systems

Candidate: Huang Debin
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本论文经答辩委员会全体委员审查，确认符合上海大学博士学位论文质量要求。

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答辩委员会对论文的评语

目前对高维非线性动力系统的研究是相当困难而又受到广泛关注的，黄德斌的论文选择这样一个相当前沿、跨学科性并有重要应用背景的研究方向作为毕业课题无疑是很有意义的。论文取得如下成果:

1. 得到了 ABC 流存在不变环面及混沌流线的条件，该结果证实了此模型提出的动机，同时为否定过去的一个拟遍历性猜测提供了实例。

2. 非常简明地证明了光滑 Hamiltonian 系统中低维环面的保持性，较好地推广了 KAM 定理的适用性。

3. 分别从吸引子和同宿混沌角度为非自治 sine-Gordon 方程的截断提供了合理性并合理解释了已有的数值结果。

4. 提出了一种求发展方程精确解的新方法并得到了 sine-Gordon 方程的一类有趣解。

以上结果大部分已在 *Nonlinearity*, *Phys. Lett. A* 等著名 SCI 杂志上发表，表明这些成果相当具有理论应用价值和创造性。同时也充分反映了作者已掌握了广泛坚实的基础理论知识、系统深入的专门知识及前沿发展的动态，表明了作者已具有很强的独立从事科研创新的工作能力。

答辩委员会表决结果

经答辩委员会表决，一致同意通过该生的博士论文答辩，并建议授予其博士学位。

答辩委员会主席：朱德明

1999年10月29日

Abstract

In this thesis, the following several problems in applied nonlinear dynamical systems are studied.

1. The persistence of lower dimensional invariant hyperbolic tori and their stable and unstable manifolds for the smooth perturbed Hamiltonian systems is shown by using the theory of normally hyperbolic invariant manifold and a finitely smooth version of the KAM theorem. Our proof is much more succinct and clear than the previous proof on the similar problem of analytical Hamiltonian systems.

2. Some dynamics in the famous ABC (Arnold-Beltrami-Childress) flow are considered. By a new like-KAM theorem and the high dimensional Melnikov method, the conditions of existence of its invariant tori and chaotic streamlines are obtained. These results negate directly a universal guess put forward by Poincare, Birkhoff, et al, and assert Arnold's original motivation for introducing this model.

3. A problem on numerics and dynamics is discussed. Particularly, the damped and driven periodically sine-Gordon equation is considered, and the existence and convergence of attractors under its spectral approximation are proved. This result supplies reasonability for the previous simple Galerkin modal truncation of this equation from the viewpoint of dynamics.

4. By applying the singular perturbation theory, an approximated ODE (ordinary differential equation), which is obtained by restricting the damped driven sine-Gordon equation to its GAIM (generalized asymptotic inertial manifold), is studied qualitatively. And some

homoclinic orbits and pulse orbits which explain directly chaotic jumping behavior observed in the previous numerical experiments for this equation are found.

5. By the AKNS system and introducing the wave function for the integrable equations, a method to find new exact solutions from known stationary solutions of the evolution equations is proposed. Especially, a family of interesting new exact solutions of the integrable sine-Gordon equation are obtained by a similar method and these solutions contain some like-kink, like-anti-kink and like-soliton solutions, which are very significant for the further dynamics study.

Key words invariant torus, homoclinic orbit, chaos, attractors, exact solution

摘 要

在这篇论文中，我们研究了应用非线性动力系统中的以下几个问题。

1. 利用法向双曲不变流形理论和 KAM 定理的一个有限光滑性推广，我们证明了光滑扰动 Hamilton 系统中低维不变双曲胎面及其稳定和不稳定流形的保持性。与以前关于解析 Hamilton 系统中类似问题的证明相比较，我们的证明更加简洁明了。

2. 我们考虑了著名的 ABC (Arnold-Beltrami-Childress) 流的一些动力学性质。通过一个新的 KAM 型定理和高维 Melnikov 方法，我们得到了此流中存在不变胎面和混沌流线的条件。这些结果直接否定了一个由 Poincare, Birkhoff 等人提出的遍历性猜测，同时也确证了 Arnold 引入此模型的原始动机。

3. 我们考虑了关于数值计算与动力学的一个问题。特别地，我们考虑了周期强迫阻尼 sine-Gordon 方程，并证明了在其谱逼近下吸引子的存在性和收敛性。此结果从动力学角度为以前对此方程的简单 Galerkin 模态截断提供了合理性。

4. 利用奇异扰动理论，我们定性地研究了一个经限制强迫阻尼 sine-Gordon 方程到其广义近似惯性流形上的逼近常微分方程。我们找到了一些同宿轨道和脉冲轨道，这些轨道的存在直接解释了以前在此方程的数值模拟中观察到的混沌跳跃行为。

5. 通过可积系统的 AKNS 系统和引入波函数，我们提出了一种从发展方程的已知定态解求新的精确解的方法。特别地，用类似的方法我们找到了可积 sine-Gordon 方程的一簇重要的新的精确解。这些解包含了一些类似于扭结、反扭结和孤立子的解，它们对进一步的动力学研究是非常有意义的。

关键词 不变胎面，同宿轨道，混沌，吸引子，精确解

Preface

The study of nonlinear dynamics is a fascinating subject which is at the very heart of the understanding of many important problems of the natural science. Two of the oldest and most notable classes of problems in applied nonlinear dynamical systems are the problems of celestial mechanics, especially N-body problem in the solar system, and the problems of turbulence in fluids. The first class problems are of finite dimensions, the latter class have infinite dimensions. Both phenomena have attracted the interest of scientists for a long time and lead to the formation and development of complicated dynamical systems.

There are two main reasons resulting in the rather difficulty in the study of nonlinear dynamics. One is nonlinearity. In contrast to linear systems, the evolution of nonlinear systems obeys complicated laws that, in general, cannot be arrived at by pure intuition or by elementary calculations. For a dynamical system starting from a particular initial state, it is not easy to predict if the system will evolve towards rest or towards a simple stationary state, or if it will go through a sequence bifurcations leading to periodic states or to quasi-periodic states or even to fully chaotic states. Thus, due to nonlinearity we do not know a priori towards which state a given system may evolve and we do not know when significant changes of the state may occur. The other reason is the number of dimension (high and even infinite). The study of turbulence in finite-dimensional systems suggests that the level of complexity of the phenomena increases with the level of complexity of the system, especially the dimension of phase space in the systems, i.e. the number of freedom of the systems. However, this is an inevitable problem resulting from recent developments in science and technology, such as chemical dynamics, biological dynamics, plasma physics and lasers, nonlinear optics, combustion, mathematical economy, robotics, etc. These reasons also determine that the current studying focus of nonlinear science is on dynamics of those nonlinear complicated systems with high dimensions and even infinite dimensions.

Although the study of nonlinear dynamics is rather difficult, there have been new ideas and new mathematical tools such as the work of S. Smale on attractors, the mechanism proposed by D. Ruelle and F. Takens for the explanation of turbulence, the KAM (Kolmogorov-Arnold-Moser) theory, period-doubling bifurcation to chaos for mappings of the interval [0,1], the existence of complicated attractors of dissapative dynamical systems, the newest method of anti-integrability limit proposed recently by S. Aubry [1-4] to study the complexity, and of course, some reduction methods [5], etc. Particularly, with the increase in computing, more insight into behavior of dynamical systems and into the description of chaotic behavior is obtained.

However, with the need of the understanding of new complicated phenomena appearing in new areas of science and technology such as pattern competition and spatio-temporal chaos, etc., the present methods are still not enough. For example, because of the complexity and sensitivity of certain variations, the evolution of a nonlinear complicated system cannot be predicted by mere computations, be it analytical or numerical. They can not offer a satisfactory solution even if they produce a feasible one; nonlinear phenomena are global and there is a need for a more geometrical view of the phenomena which could provide the proper guidelines for the computations. The limits of the computational methods have been pointed out by Poincare in his classic work on differential equations. He showed the need to marry analytical and geometrical methods in the dynamical theory, and although he was concerned with asymptotic analytical methods, this also applies to numerical methods since the difficulty is inherent in the problem. Therefore, in order to understand the nonlinear phenomena in applied problems very well, we have to utilize sufficiently all kinds of tools, and even to construct new mathematical methods.

However, up to now we haven't found a more systematic method to solve thoroughly the nonlinear complicated problems, even for a given applied nonlinear dynamical system, which is just as what M. C. Cross and P. C. Hohenberg asserted in Ref. [6]. They declared that with the need of understanding the mechanism of some complicated phenomena such as pattern formation, pattern competition and spatio-temporal chaos, especially in order to study complexity science, more and more methods need to be combined and to be proposed, but they also pointed out there are four applicable methods in operation at present: numerical simulation, qualitative analysis, perturbation method and finding new exact solutions.

Just on the basis of the background on nonlinear dynamics research, we hope to investigate some applied dynamics from as many aspects as possible in this thesis.

Now let us describe the content of this thesis in its chronological order. We divide the thesis into two parts. The first part is to introduce and study some dynamics such as invariant torus and homoclinic chaos in finite-dimensional dynamical systems with multi-freedom. It consists of four chapters, i.e. Chapter 1, Chapter 2, Chapter 3 and Chapter 4. The second part contains three chapters, i.e., Chapter 5, Chapter 6 and Chapter 7, which is to study nonlinear evolution equations, i.e., so-called infinite dimensional dynamical systems from several aspects such as numerics and dynamics, nonlinear dynamics theory, and soliton theory, etc. Particularly, we are focused on the study of the sine-Gordon equation.

In the first chapter, we introduce a high dimensional generalization of Melnikov method — an analytical and applicable method to detect homoclinic orbits in a given dynamical system.

In Chapter 2, we discuss the persistence of lower dimensional invariant tori for smooth Hamiltonian systems, it is very important in celestial physics. However, the previous researches were focused only on the analytical Hamiltonian systems and the rapidly convergent Newton iteration KAM proof with high technique was used. By using the normally hyperbolic invariant manifold theory and a smooth version of the KAM theorem, we show directly the persistence of some lower dimensional invariant hyperbolic tori and their stable and unstable manifolds in the smooth perturbed Hamiltonian systems under certain conditions of nonresonance and nondegeneracy. In comparison to the previous proof of similar problems in the analytical Hamiltonian systems, our proof is very succinct and clear.

In Chapter 3, we study a notable model to describe motion of fluid particles, the ABC flow first introduced by Arnold. By applying a new like-KAM theorem and the method introduced in Chapter 1, we obtain the analytical conditions of existence of invariant tori and chaotic streamlines in the ABC flow. These results negate directly a universal guess proposed by Poincare, Birkhoff, etc, and imply that a simple Eulerian represention has a complicated Lagrangian structure such as turbulence, which is Arnold's original motivation for introducing the ABC flow.

In Chapter 4, we introduce a new work, it is also how to detect homoclinic orbits in a given dynamical system with resonance. Since the method referred in Chapter 1 is invalid for such problems, we have to use the geometric singular perturbation theory to deal with the problems.

In Chapter 5, we discuss a problem on numerics and dynamics. As mentioned above, power numerical experiments play an important role in the study of nonlinear dynamics at present, but mere numerical computations are not enough for understanding nonlinear problem, for example, the determination of the numerical schemes which will accurately reflect the dynamics of the original nonlinear problems. Particularly, as an example, we take the damped and driven periodically sine-Gordon equation and prove the existence and convergence of attractors for dissapative systems under its spectral discretization. This result just supplies reasonability for the simple Galerkin modal truncation of the sine-Gordon equation from the view point of dynamics, because many former researchers showed that the simple low modal truncation system of the damped driven sine-Gordon equation reflects the main dynamics of the original PDE by numerical experiments.

In Chapter 6, by the infinite dimensional dynamical systems theory, we study qualitatively an approximated ODE on the GAIM of the damped and driven sine-Gordon equation. We obtain the conditions of existence of orbits homoclinic to a resonance band and show the existence of pulse orbits under certain parametric values by applying the singular perturbation method introduced in Chapter 4. As compared with the previous works that are focused on the simple low modal truncation systems of the damped driven sine-Gordon equation, our results are more beautiful, especially our results explain directly mechanism causing chaotic jumping behavior observed in the past numerical simulations for the damped driven sine-Gordon equation under the same parametric values.

In Chapter 7, the problem to find exact solutions for nonlinear evolution equations will be considered. It is well known that it is very difficult to study analytically the concrete dynamics in nonlinear evolution equations, such as finding the homoclinic orbits causing