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（1991—1995）

主 编　方天祺

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前　　言

本集是《内蒙古自然科学基金项目论文集》第二册，它收录了基金资助项目于1991年至1995年间公开发表的论著。本书的编辑原则仍循前例，内容分为论文全文、论文摘要和标题三部分。全书计收论文36篇，摘要82篇，标题626条。由于种种原因，本书可能未将发表的全部论著收入集内，遗漏恐难避免，但基本上收入了这五年中内蒙古自然科学基础研究和应用基础研究的主要工作。

从本书内容可以看出，由于我区科技工作者的辛勤努力，五年来我们在自然科学研究的领域中取得了显著的成绩。基础研究有了新的进展，不少论文发表在国内外有影响的学术刊物上。一些应用基础研究成果具有良好的应用前景。同时，通过基金项目的开展也培养了许多科技人才。这表明内蒙古自然科学基金的实施对促进我区科教事业的发展，贯彻"科教兴区"的方针发挥了重要作用。

我们相信，本集的出版将为自治区各级领导和科技人员了解我区自然科学研究的概况和发展趋势，为进一步推进我区的自然科学基础研究，提供一本有益的资料。

在内蒙古科委的领导、支持和内蒙古大学科技处同仁的全力协助下，《内蒙古自然科学基金项目论文集》顺利编辑出版，谨向他们致以诚挚的感谢。

编　者

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内蒙古自然科学基金项目论文集

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主　　编　方天祺

副 主 编　刘学敏　李铁生　张治务　斯力更

编　　委　冯启元　李逢泽　张鹤龄　邵金旺

　　　　　孙慧宽　阿　牧

参编人员　周林平　康宝军　侯富英　李晓萍

　　　　　齐海春

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DIFFERENTIAL · DIFFERENCE INEQUALITIES WITH UNBOUNDED DELAYS AND THEIR APPLICATIONS

LIGENG SI and WANBIAO MA

Department of Mathematics, Inner Mongolia Normal University, Huhhot 010022, P. R. China

Key words and phrases: Differential · difference inequality, unbounded delay, neutral · type, retarded · type, uniformly stable, globally asymptotically stable, instability.

1. INTRODUCTION

IN RECENT years, the development of modern technology has resulted in the planning, designing and realization of sophisticated systems that have become increasingly large in scope and complex in structure, therefore many researchers have directed their attention to various problems that arise in connection with large · scale systems.

It is well known that in the stability anlysis of large · scale dynamical systems, the methods of vector Lyapunov functions and scalar Lyapunov functions are the main and powerful methods. But, generally, great difficulties arise in applying these methods to large · scale dynamical systems with unbounded delays. Particularly, for the stability analysis of neutral − type large · scale dynamical systems with unbounded delays, very few results have been obtained.

In this paper, firstly, we establish two very important differential · difference inequalities with unbounded delays. Secondly, by making use of these inequalities, we study the global stability and instability of several classes of large · scale dynamical systems with unbounded delays (including neutral · type), and obtain some simple and practical sufficient criteria. Our results generalize and replenish the studying of this subject.

2. DIFFERENTIAL · DIFFERENCE INEQUALITIES WITH UNBOUNDED DELAYS

Definition 1. An $n\times n$ matrix $C=(c_{ij})_{n\times n}$ is an M · matrix if $c_{ij}\leq 0$, $i\neq j, i, j=1,2.\ldots,n$, and all principal minors of C are positive.

THEOREM 1. ASSUME THAT THE FUNCTION $P(t)=\mathrm{col}\ (p_1(t),\ldots,p_n(t)):R\to R_+^n$ is continuous and satisfies the differential · difference in equality

$$\mathrm{k}\dot{p}\ (t)\leq g(t)(Ap(t)+B\overline{p}(t)) \qquad \textit{for } t\geq t_0, \tag{2.1}$$

where $\mathrm{k}=diag(\mathrm{k}_1,\ldots,\mathrm{k}_n)$, k_i are nonnegative constants; $g(t)=\mathrm{diag}(g_1(t),\cdots,g_n(t))$ is con · tinuous such that $g_i(t)\geq\sigma>0$ for $t\geq t_0$, σ is constant; $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ are constant matrices such that $a_{ij}<0$, $a_{ij}\geq 0, i\neq j$, $b_{ij}\geq 0, i,j=1,2,\ldots,n$, and that $-(A$

$+B$) is an M-matrix; $\bar{p}(t) = col(\bar{p}_1(t),\dots,\bar{p}_n(t))$.

($\bar{p}_i(t) = \sup_{-\Delta(t)\le s\le 0} p_i(t+s)$, $\Delta(t):[t_0,+\infty]\to[0,\infty)$ is continuous such that

$\Delta(t) < t$ and $t-\Delta(t)\to+\infty$ as $t\to+\infty$.

Then there exists a constant $M_1\ge 1$ such that

$$p_i(t)\le M_1\|p\| \qquad \text{for } t\ge t_0, i=1,2,\dots,n, \tag{2.2}$$

$$\lim_{t\to+\infty} p_i(t)=0, \qquad i=1,2,\dots,n, \tag{2.3}$$

where

$$\|p\| = \sum_{j=1}^{n} \sup_{(-\infty,t_0]} p_j(s) < +\infty.$$

Proof. First, we prove (2.2). If $k_i>0$, from (2.1)we have

$$p_i(t)\le k_i^{-1}g_i(t)\sum_{j=1}^{n}(a_{ij}p_j(t)+b_{ij}\bar{p}_j(t)) \qquad \text{for } t\ge t_0. \tag{2.4}$$

Substituting s for t in (2.4), then multiplying both its sides by $\exp\left(k_i^{-1}a_{ii}\int_s^t g_i(u)\,du\right)$ and integrating with respect to s from t_0 to t, we obtain

$$p_i(t)\le p_i(t_0)\exp\left(k_i^{-1}a_{ii}\int_{t_0}^{t} g_i(u)du\right)$$
$$+\sum_{j=1}^{n}\int_{t_0}^{t} k_i^{-1}g_i(s)(a_{ij}\delta_{ij}^{*}p_j(s)+b_{ij}\bar{p}_j(s))\exp\left(k_i^{-1}a_{ii}\int_s^t g_i(u)du\right)ds \quad \text{for } t\ge t_0, \tag{2.5}$$

where $\delta_{ii}^{*}=0, \delta_{ij}^{*}=1, i\ne j$.

Let

$$m_i(t)=\sup_{t_0\le s\le t} p_i(s). \qquad w_{ij}=\frac{a_{ij}}{|a_{ii}|}\delta_{ij}^{*}+\frac{b_{ij}}{|a_{ii}|}, \qquad i,j=1,2,\dots,n.$$

In view of (2.5) we have

$$p_i(t)\le \|p\| + \sum_{j=1}^{n}\int_{t_0}^{t} k_i^{-1}g_i(s)(a_{ij}\delta_{ij}^{*}m_j(t)+b_{ij}(m_j(t)+\|p\|))\exp\left(k_i^{-1}a_{ii}\int_s^t g_i(u)du\right)ds$$
$$\le \|p\| + \sum_{j=1}^{n} w_{ij}m_j(t) + M_0\|p\| \qquad \text{for } t\ge t_0, \tag{2.6}$$

where $M_0=\max_{1\le i\le n}(\Sigma_{j=1}^{n} b_{ij}/|a_{ii}|)$.

Note that the right hand of (2.6)is nondecreasing, we obtain

$$m_i(t)\le \sum_{j=1}^{n} w_{ij}m_j(t)+(1+M_0)\|p\| \qquad \text{for } t\ge t_0. \tag{2.7}$$

if $k_i=0$, from (2.1)we have

$$p_i(t)\le \sum_{j=1}^{n}\left(\frac{a_{ij}}{|a_{ii}|}\delta_{ij}^{*}p_j(t)+\frac{b_{ij}}{|a_{ii}|}\bar{p}_j(t)\right) \qquad \text{for } t\ge t_0. \tag{2.8}$$

Similar arguments apply to (2.8), we can also obtain

$$m_i(t)\le \sum_{j=1}^{n} w_{ij}m_j(t)+(1+M_0)\|p\| \qquad \text{for } t\ge t_0. \tag{2.9}$$

Since $(A+B)$ is an M-matrix, by[12,theorem 2.3]there exists a group of positive numbers $d_1,\dots,d_n$ such that $\Sigma_{i=1}^{n}(a_{ij}+b_{ij})d_id_j^{-1}<0$ for $j=1,2,\dots,n$. Namely $\Sigma_{i=1}^{n}w_{ij}d_id_j^{-1}<1$ for $j=1,2,\dots,n$.

Set

$$\alpha = \max_{1\le j\le n} \sum_{i=1}^{n} w_{ij} d_j^{-1} d_i < 1, \qquad \overline{M} = (1 + M_0) \sum_{j=1}^{n} d_j,$$

$$d_0 = \max\{d_1, \ldots, d_n\}, \qquad M_1 = \frac{\overline{M}}{d_0(1-\alpha)} \ge 1.$$

Then, from (2.7)and (2.9) we have

$$\sum_{i=1}^{n} d_i m_i(t) \le \sum_{j=1}^{n} \Big(\sum_{i=1}^{n} w_{ij} d_i d_j^{-1} \Big) d_j m_j(t) + \overline{M} \| p \|$$

$$\le \alpha \sum_{j=1}^{n} d_j m_j(t) + \overline{M} \| p \| \qquad \text{for } t \ge t_0.$$

Namely

$$\sum_{i=1}^{n} m_i(t) \le \frac{\overline{M}}{d_0(1-\alpha)} \| p \| \le M_1 \| p \| \qquad \text{for } t \ge t_0.$$

Thus

$$p_i(t) \le m_i(t) \le M_1 \| p \| \qquad \text{for } t \ge t_0, i = 1, 2, \ldots, n.$$

Secondly, let us conclude that(2.3). Since $-(A+B)$ is an M-matrix, we can choose a positive number T which is sufficiently large such that $\cdot (a_{ij} + b_{ij} + |a_{ii}|\delta_{ij} \exp(-\delta T))_{n\times n}$ is an M —matrix, where $\delta_{ij} = 1 - \delta^*_{ij}, i, j = 1, 2, \ldots, n, \delta = (\min_{1\le i\le n} |a_{ii}|)\sigma/(1 + \max_{1\le i\le n} k_i)$. Hence there exists a group of positive number $d_1, \ldots, d_n$ such that

$$\sum_{j=1}^{n} (a_{ij} + b_{ij} + |a_{ii}|\delta_{ij} \exp(-\delta T)) d_j d_i^{-1} < 0 \qquad \text{for } i = 1, 2, \ldots, n.$$

Namely,

$$\sum_{j=1}^{n} (w_{ij} + \delta_{ij} \exp(-\delta T)) d_j d_i^{-1} < 1 \qquad \text{for } i = 1, 2, \ldots, n.$$

Let

$$\alpha_0 = \max_{1\le i\le n} \sum_{j=1}^{n} (w_{ij} + \delta_{ij} \exp(-\delta T)) d_j d_i^{-1} < 1,$$

$$\overline{\lim_{t\to+\infty}} (d_i^{-1} p_i(t)) = \sigma_i \qquad \text{for } i = 1, 2, \ldots, n.$$

Obviously, $\sigma_i \ge 0$ *for* $i = 1, 2, \ldots, n$, *and* $\sigma_l = \max\{\sigma_1, \ldots, \sigma_n\} < +\infty$ We claim that $\sigma_l = 0$. If $\sigma_l > 0$, let η satisfy $0 < \eta < (1-\alpha_0)\sigma_l/(1+\alpha_0)$, by $t - \Delta(t) \to +\infty$ as $t \to +\infty$ and the properties of superior—limit we can choose a sufficiently large $T_1 \ge t_0 + T$ such that

$$d_i^{-1}(\overline{p}_i(s) < \sigma_l + \eta \qquad \text{for } t - T \le s \le t, t \ge T_1, i = 1, 2, \ldots, n, \tag{2.10}$$

$$d_l^{-1} p_l(T_1) > \sigma_l + \eta. \tag{2.11}$$

If $k_l > 0$, replacing t in (2.4) by s, then multiplying its both sides by $\exp(k_l^{-1} a_{ll} \int_s^t g_l(u) du)$ and integrating with respect to s from $t - T$ to t, we obtain

$$p_l(t) \le p_l(t-T) \exp\Big(k_l^{-1} a_{ll} \int_{t-T}^{t} g_l(u) du \Big)$$

$$+ \sum_{j=1}^{n} \int_{t-T}^{t} k_l^{-1} g_l(s) (a_{lj}\delta^*_{lj} p_j(s) + b_{lj}\overline{p}_j(s)) \exp\Big(k_l^{-1} a_{ll} \int_s^t g_l(u) du \Big) ds \tag{2.2}$$

for $t \ge t_0 + T$. Hence, from (2.20),(2.11)and(2.12),we have

$$\sigma_l - \eta < d_l^{-1} p_l(T_1) \le (\sigma_l + \eta) \exp(-\delta T) + (\sigma_l + \eta) d_l^{-1} \sum_{j=1}^{n} w_{lj} d_j$$

$$= (\sigma_l + \eta) \sum_{j=1}^{n} (w_{lj} + \delta_{lj} \exp(-\delta T)) d_j d_l^{-1}$$

$$\leq \alpha_0(\sigma_l + \eta).$$

Thus $\eta > (1-\alpha_0)\sigma_l/(1+\alpha_3)$, which contradicts the choice of η . If $k_l = 0$, we can also conclude the contradiction from (2.8), (2.10) and (2.11). These contradictions show that the above claim is true. This completes the proof of theorem 1.

Remark 1. (i) By different choosing of k in theorem 1, we can obtain some important differential · difference inequalities.

(ii) If $\Delta(t)$ in theorem 1 is bounded, by making use of the methods of [10], we can obtain the following more exact estimation,

$$\sum_{i=1}^{n} p_i(t) \leq \| p \|_{\Delta} M_0 \exp(-a(t-t_0)) \qquad \text{for } t \geq t_0,$$

where $M_0 \geq 1$ and $a > 0$ are constants, $\| p \|_{\Delta} = \sum_{i=1}^{n} \sup_{-\Delta \leq s \leq 0} p_i(t_0+s)$, Δ is the bound of $\Delta(t)$

(iii) We can obtain the following nonlinear differential · difference inequalities by making use of the similar methods.

Assume that the functions $p_j(t)(j=1,2,\ldots,n)$: $R \to R_+$ are continuous and satisfy the nonlinear inequality

$$k_i p_i(t) \leq g_1(t) f_i(p_1(t),\ldots,p_n(t),\bar{p}_1(t),\cdots,\bar{p}_n(t);$$
$$\int_0^\infty A_1(u)p_1(t-u)du,\ldots,\int_0^\infty A_n(u)p_n(t-u)du)$$

for $t \geq t_0$, $i=1,2,\ldots,n$,where

(a) k_i are nonnegative constants;

(b) $g_i(t) > 0$ are continuous for $t \geq t_0$, and $\lim_{t\to\infty}\int_{t_0}^{t} g_i(s)ds = +\infty$ for $t \geq t_0$ are uniform;

(c) $\bar{p}_i(t) = \sup_{-\Delta(t)\leq\theta\leq 0} p_i(t+\theta)$, $\Delta(t)$: $[t_0, +\infty] \to [0,\infty]$ is continuous such that $\Delta(t) < t$ and $t - \Delta(t) \to +\infty$ as $t \to +\infty$;

(d) $A_i(u) \geq 0$ are continuous for $u \geq 0$ such that $\int_0^\infty A_j(u)du < +\infty$;

(e) the functions f_i are continuous such that

$$f_i(x_1,\ldots,x_n;y_1,\ldots,y_n;z_1,\ldots,z_n) \leq f_i(\bar{x}_n,\ldots,\bar{x}_1;\bar{y}_1,\ldots,\bar{y}_n;\bar{z}_1,\ldots,\bar{z}_n)$$

for $x_i = \bar{x}_i, x_j \leq \bar{x}_j (i \neq j); y_j \leq \bar{y}_j; z_j \leq \bar{z}_j (j=1,2,\ldots,n)$;

(f) there exist a continuous function $\bar{f}(\alpha_i)$ and constants $a_i > 0$ such that $\bar{f}(0) = 0$ and

$$f_i(\alpha_1\theta,\ldots\alpha_n\theta;\alpha_1\theta,\ldots,\alpha_n\theta;\alpha_1 s_1\theta,\ldots,\alpha_n s_n\theta) \leq -\bar{f}(\theta) < 0$$

for $\theta \neq 0$ and $s_i = \int_0^\infty A_i(u)du, i=1,2,\ldots,n.$

Then there exists a constant $M \geq 1$ such that

$$P_i(t) \leq M \| p \| \qquad \text{for } t \geq t_0\ i=1,2,\ldots,n;$$
$$\lim_{t\to+\infty} p_i(t) = 0, \qquad i=1,2,\ldots,n,$$

when

$$\| p \| = \max\{\sup_{\theta\leq 0} p_s(t_0+\theta), \qquad s=1,2,\ldots,n\}.$$

THEOREM 2. If the function $p(t)=\mathrm{col}(p_1(t),\ldots,p_n(t)):R\to R_+^n$ is continuous and satisfies the differential-difference inequality

$$\dot p(t)\geq f(t)(Ap(t)+B\bar p(t))\qquad \text{for } t\geq t_0, \tag{2.13}$$

or

$$\dot p(t)\geq f(t)p_*^{1/2}(t)(Ap^{1/2}(t)+B\bar p^{1/2}(t))\qquad \text{for } t\geq t_0. \tag{2.14}$$

where $p_i(t)$ are positive and nondecreasing on $(-\infty,t_0),i=1,2,\ldots,n$;

$$f(t)=\mathrm{diag}(f_1(t),\ldots,f_n(t))$$

is positive and continuous for $t\geq t_0$ such that $\int_{t_0}^{\infty}f_i(t)\mathrm{d}t=+\infty,i=1,2,\ldots,n$;

$$p_*^{1/2}(t)=\mathrm{diag}(p_1^{1/2}(t),\ldots,p_n^{1/2}(t)),$$

$$p^{1/2}(t)=\mathrm{col}(p_1^{1/2}(t),\ldots,p_n^{1/2}(t)),$$

$$\bar p(t)=\mathrm{col}(\bar p_1(t),\ldots,\bar p_n(t)),$$

$$p^{1/2}(t)=\mathrm{col}p_1^{1/2}(t),\ldots,\bar p_n^{1/2}(t)),$$

$$\bar p^{1/2}(t)=\mathrm{col}\bar p_1^{1/2}(t),\ldots,\bar p_n^{1/2}(t)),$$

$$\bar p(t)=\sup_{-\Delta(t)\leq s\leq 0}p_i(t+s),\qquad i=1,2,\ldots,n;$$

$\Delta(t):[t_0,+\infty]\to[0,+\infty)$ is continuous such that $\Delta(t)<t$ and $t-\Delta(t)\to+\infty$ as $t\to\infty$; $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})n\times n$ constant matrices such that $a_{ii}>0,a_{ij}\leq 0|,i\neq j,b_{ij}\leq 0,i,j=1,2,\ldots,n$, and that $A+B$ is an M-matrix.

Then

$$\overline{\lim_{t\to+\infty}}(p_1(t)+p_2(t)+\ldots+p_n(t))=+\infty.$$

Proof. Assume that(2.13)hold. Since $A+B$ is an M-matrix, there exists a group of positive numbers $d_1,\ldots,d_n$ such that

$$a_i=\sum_{j=1}^{n}(a_{ij}+b_{ij})d_jd_i^{-1}>0\qquad \text{for } i=1,2,\ldots,n. \tag{2.15}$$

Set

$$y(t)=\max\{d_1^{-1}p_1(t),\ldots,d_n^{-1}p_n(t)\}\qquad \text{for } t\in R.$$

Obviously, y(t) is continuous for $t\in R$ and non-decreasing for $t\leq t_0$, we claim that $y(t)$ is nondecreasing for $t\geq t_0$. If the above claim is false, we can choose $t_1>t_0$, a sufficiently small con-stant p and integer k such $y(t)$ is nondecreasing on $(-\infty,t_1]$ and monotonically decreasing on $[t_1,t_1+p]$. furthermore

$$y(t)=d_k^{-1}p_k(t)\qquad \text{for } t_1\leq t\leq t_1+p. \tag{2.16}$$

Thus, we have

$$d_k^{-1}\dot p_k(t_1)=\dot y(t_1)\leq 0, \tag{2.17}$$

$$d_i^{-1}p_i(t)\leq y(t)\qquad \text{for } -\infty<t\leq t_1,i=1,2,\ldots,n. \tag{2.18}$$

On the other hand, by (2.13),(2.15),(2.16)and(2.18)we obtain

$$\begin{aligned} d_k^{-1}\dot p_k(t_1)&\geq d_k^{-1}(f_k(t_1))\sum_{j=1}^{n}(a_{kj}p_j(t_1)d_j^{-1}+b_{kj}\bar p_j(t_1)d_j^{-1})d_j\\ &\geq f_k(t_1)\sum_{j=1}^{n}(a_{kj}+b_{kj})d_jd_k^{-1}y(t_1)\\ &=\alpha_kf_k(t_1)y(t_1)>0. \end{aligned}$$