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主编 钱伟长

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（第12集）

主编：钱伟长

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谨以此书纪念本会创建人、故董事会主席王宽诚先生

王宽诚教育基金会

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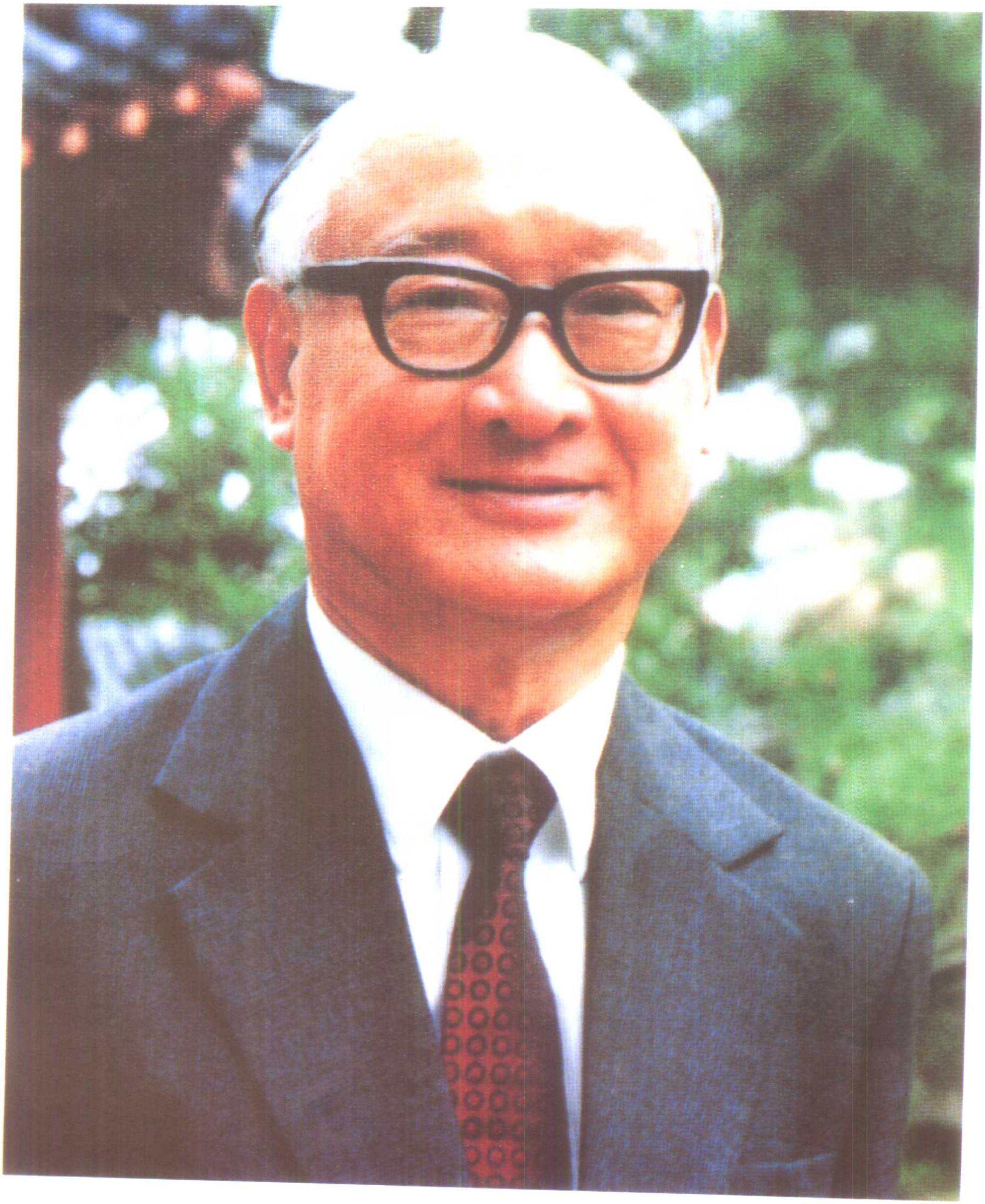

王宽诚先生

K. C. WONG (1907–1986)

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王宽诚教育基金会简介

王宽诚先生（1907～1986）为香港知名爱国人士，热心祖国教育事业，生前为故乡宁波的教育事业做出积极贡献。1985年独力捐巨资创建王宽诚教育基金会，其宗旨在于为国家培养高级科技人才，为祖国四个现代化效力。

王宽诚先生在世时聘请海内外知名学者担任基金会考选委员会和学务委员会委员，共商大计，确定采用"送出去"和"请进来"的方针，为国家培育各科专门人才，并为提高国内和港澳高等院校的教学水平，资助学术界人士互访，用以促进中外文化交流。在此方针指导下，1985、1986两年，基金会在国家教委支持下，选派学生85名前往英、美、加拿大和西德、瑞士、澳大利亚各国攻读博士学位，并计划资助国内学者赴港澳讲学，资助港澳学者到国内讲学，资助美国学者来国内讲学。正当基金会事业初具规模，蓬勃发展之时，王宽诚先生一病不起，于1986年年底逝世。这是基金会的重大损失，共事同仁，无不深切怀念，不胜惋惜。

王宽诚教育基金会在新任董事会主席张二铭先生和胡百全、林延新等董事的主持下，继承王宽诚先生为国家培育人才的遗愿，继续努力，除按计划执行外，并开发与英国学术机构合作的新项目。王宽诚教育基金会过去和现在的工作态度一贯以王宽诚先生所倡导的"公正"二字为守则，谅今后基金会亦将秉此行事，奉行不辍。借此王宽诚教育基金会《学术讲座汇编》出版之际，特简明介绍如上。王宽诚教育基金会日常工作繁重，王明远、王明勤、林延新等董事均不辞劳累，做出积极贡献。

钱伟长

一九九七年七月

前　言

王宽诚教育基金会是由已故全国政协常委、香港著名工商企业家王宽诚先生(1907～1986)出于爱国热忱，出资一亿美元于1985年在香港注册登记创立的。

1987年，基金会开设"学术讲座"项目，此项目由当时的全国政协常委、现任全国政协副主席、著名科学家、中国科学院院士、上海大学校长、王宽诚教育基金会贷款留学生考选委员会主任委员兼学务委员会主任委员钱伟长教授主持，由钱伟长教授亲自起草设立"学术讲座"的规定，资助国内学者前往香港、澳门讲学，资助美国学者和港澳学者前来国内讲学，用以促进中外学术交流，提高内地及港澳高等院校的教学质量。

本汇编收集的文章，均系各地学者在"学术讲座"活动中的讲稿。文章作者中，有年逾八旬的学术界硕彦，亦有由王宽诚教育基金会考选委员会委员推荐的学者和后起之秀。文章内容有科学技术，有历史文化，有经济专论，有文学，有宗教和中国古籍研究。本汇编涉及的学术领域颇为广泛，而每篇文章都有一定的深度和广度，分期分册以《王宽诚教育基金会学术讲座汇编》的名义出版，并无偿分送港澳和国内外部分高等院校、科研机构和图书馆，以广流传。

王宽诚教育基金会除资助"学术讲座"学者进行学术交流之外，在钱伟长教授主持的项目下，还资助由国内有关高等院校推荐的学者前往欧美亚澳参加国际学术会议，出访的学者均向所出席的会议提交论文，这些论文亦颇有水平，本汇编亦将其收入，以供参考。

王宽诚教育基金会学务委员会

凡　例

（一）编排次序

本书所收集的王宽诚教育基金会学术讲座的讲稿及由王宽诚教育基金会资助学者赴欧美亚澳参加国际学术会议的论文均按照收到文稿日期先后或文稿内容编排刊列，不分类别。

（二）分期分册出版并作简明介绍

因文稿较多，为求便于携带，有利阅读与检索，故分期分册出版，每册约 150 页至 200 页不等。为便于读者查考，每篇学术讲座的讲稿均注明作者姓名、学位、职务、讲学日期、地点、访问院校名称。国内及港澳学者到欧、美、澳及亚洲的国家和地区参加国际学术会议的论文均注明学者姓名、参加会议的名称、时间、地点和推荐的单位。上述两类文章均注明由王宽诚教育基金会资助字样。

（三）文字种类

本书为学术性文章汇编，均以学术讲座学者之讲稿原稿或参加国际学术会议学者向会议提交的论文原稿文字为准，即原讲稿或论文是中文的，即以中文刊出，原讲稿或论文是外文的，仍以外文刊出。

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王宽诚教育基金会敬赠

年　月　日

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The First Order Approximation of Non-kirchhoff-Love Theory for Elastic Circular Plate with Fixed Boundary under Uniform Surface Loading

Wei-Zang Chien☆

Abstract

Based on the approximation theory adopting non-Kirchhoff-Love assumption for three dimensional elastic plates with arbitrary shapes[1~4], the author derives a functional of generalized variation for three dimensional elastic circular plates, thereby obtains a set of differential equations and the relate boundary conditions to establish a first order approximation theory for elastic circular plate with fixed boundary and under uniform loading on one of its surface.

(I) Introduction

The axisymmetric probiem of three dimensional elastic circular plate can be treated as three dimensional axisymmetric problem of elasticity. We consider a circular plate with a uniform thickness h, and set up a circumferential coordinates (r, θ) on its middle surface with abscissa z perpendicular to it in downward direction (Fig. 1). The stress components are denoted by σ_r, σ_θ, σ_z, $\sigma_{r\theta}=\sigma_{\theta r}$, $\sigma_{zr}=\sigma_{rz}$, $\sigma_{\theta z}=\sigma_{z\theta}$ and the strain components are denoted by e_r, e_θ, e_z, $e_{r\theta}=e_{\theta r}$, $e_{zr}=e_{rz}$, $e_{\theta z}=e_{z\theta}$. In axisymmetric problems we have

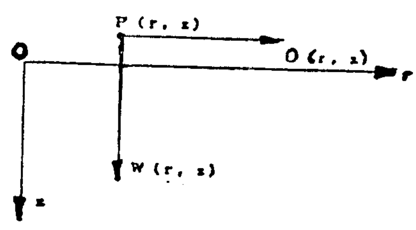

Fig. 1 The coordinates (r, z) in a circular plate and the displacement components $U(r, z)$ and $W(r, z)$ at point $P(r,z)$

☆ 作者钱伟长教授，是中国科学院院士，上海大学校长、教授，上海市应用数学和力学研究所所长。于1995年12月由王宽诚教育基金会资助，应邀在香港浸会大学讲学，此为其讲稿。

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$$\left.\begin{array}{ll}\sigma_{r\theta}=\sigma_{\theta r}=0, & \sigma_{\theta z}=\sigma_{z\theta}=0 \\ e_{r\theta}=e_{\theta r}=0, & e_{\theta z}=e_{z\theta}=0\end{array}\right\} \tag{1.1}$$

And there are only two displacement components: the radial displacement $U(r, z)$ and the deflection $W(r. z)$. The above components of stress, strain and displacement should satisfy the following equations.

(1) Strain-displacement relation:

$$\left.\begin{array}{l}e_r=\dfrac{\partial U}{\partial r},\ e_\theta=\dfrac{U}{r},\ e_z=\dfrac{\partial W}{dz}, \\ e_{rz}=\dfrac{1}{2}\left(\dfrac{\partial U}{\partial z}+\dfrac{\partial W}{\partial r}\right),\ e_{\theta z}=e_{\theta r}=0\end{array}\right\} \tag{1.2}$$

(2) Strain-stress relation:

$$\left.\begin{array}{ll}Ee_r=\sigma_r-\nu(\sigma_\theta+\sigma_z), & Ee_{rz}=(1+\nu)\sigma_{rz} \\ Ee_\theta=\sigma_\theta-\nu(\sigma_r+\sigma_z), & Ee_{\theta z}=(1+\nu)\sigma_{\theta z}=0 \\ Ee_z=\sigma_z-\nu(\sigma_r+\sigma_\theta), & Ee_{\theta r}=(1+\nu)\sigma_{\theta r}=0\end{array}\right\} \tag{1.3}$$

Stress-strain relation:

$$\left.\begin{array}{ll}\sigma_r=\dfrac{E_1}{1-\nu_1^2}[e_r+\nu_1(e_\theta+e_z)], & \sigma_{rz}=\dfrac{E_1}{1+\nu_1}e_{rz} \\ \sigma_\theta=\dfrac{E_1}{1-\nu_1^2}[e_\theta+\nu_1(e_r+e_z)], & \sigma_{\theta z}=\dfrac{E_1}{1+\nu_1}e_{\theta z}=0 \\ \sigma_z=\dfrac{E_1}{1-\nu_1^2}[e_z+\nu_1(e_r+e_\theta)], & \sigma_{\theta r}=\dfrac{E_1}{1+\nu_1}e_{\theta r}=0\end{array}\right\} \tag{1.4}$$

where E and ν are the Young's modular and Poisson ratio respectively, and E_1 and ν_1 are respectively the equivalent Young's modular and Poisson ratio in plane strain problems. They satisfy the following relations:

$$E_1=\frac{E}{1-\nu^2},\quad \nu_1=\frac{\nu}{1-\nu},\quad \frac{E}{1+\nu}=\frac{E_1}{1+\nu_1} \tag{1.5}$$

(3) Equilibrium equations of stress in axisymmtric problems:

$$\frac{1}{r}\frac{d}{dr}(r\sigma_r)-\frac{\sigma_\theta}{r}+\frac{\partial\sigma_{rz}}{\partial z}=0 \tag{1.6a}$$

$$\frac{\partial\sigma_z}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{zr})=0 \tag{1.6b}$$

where the body forces are neglected.

(4) External forces acting on the upper surface $\left(z=-\dfrac{h}{2}\right)$ and lower surface $\left(z=-\dfrac{h}{2}\right)$

$$\sigma_z=-q,\quad \sigma_{rz}=0\ \left(z=-\frac{h}{2},\ q>0 \text{ for compression}\right) \tag{1.7a}$$

$$\sigma_z=0,\quad \sigma_{rz}=0\ \left(z=+\frac{h}{2}\right) \tag{1.7b}$$

It must be pointed out that the classical theory of thin plate can not take into consideration the influence of the location of surface load, that is to say, either q exerted on the upper surface as compressional load or it exerted on the lower surface as tensional load, the solution will be the same; whereas, the theory with non-Kirchhoff-Love assumption will give different solutions for the two cases.

(5) Boundary conditions on edge surface (complete fixed)

$$U(a,\ z)=0, \qquad W(a,\ z)=0 \tag{1.8}$$

where a is the radius of the circular plate.

In this paper, we will solve the 14 partial differential equations in (1.2), (1.4) and (1.6) for the stress components σ_r, σ_θ, σ_z, σ_{rz}, $\sigma_{r\theta}=\sigma_{z\theta}=0$ and the strain components e_r, e_θ, e_z, e_{rz}, $e_{r\theta}=e_{z\theta}=0$ under the conditions (1.7) and (1.8). As $\sigma_{r\theta}$, σz_θ, $e_{r\theta}$ and $e_{z\theta}$ are identically equal to zero, there are only 10 unknowns to satisfy the 10 partial differential equations which are not identically equal to zero.

(II) The Generalized Variational Principle for Elastic Circular Plate with Fixed Boundary under Uniform Load on Its upper Surface $\left(z=-\frac{h}{2}\right)$

The generalized variational principle for an equilibrium elastic plate with fixed boundary under surface loading has been studied in the previous paper[1]. This paper will discuss the generalized variational principle for an axisymmetric equilibrium elastic circular plate with fixed boundary under uniform surface loading.

The strain energy density ε of three dimensional elastic body in axisymmetric strain field can be expressed by

$$\varepsilon=\frac{E_1}{2(1-\nu_1^2)}[(e_r+e_\theta+e_z)^2+2(1-\nu_1)(e_{rz}{}^2-e_re_z-e_re_\theta-e_ze_\theta)] \tag{2.1}$$

For stresses σ_r, σ_θ, σ_z, σ_{zr} and strains e_r, e_θ, e_z, e_{zr} which satisfy stress-strain relations (1.4), it is easy to prove that

$$\delta\varepsilon=\sigma_r\delta e_r+\sigma_\theta\delta e_\theta+\sigma_z\delta e_z+2\sigma_{rz}\delta e_{rz} \tag{2.2}$$

We shall prove that the generalized variational principle of this problem with the variables σ_r, σ_θ, σ_z, σ_{zr}, e_r, e_θ, e_z, e_{zr}, U, W may be written as

$$\delta\Pi=0 \qquad \text{(stationary condition of variation)} \tag{2.3}$$

where Π is the functional of generalized variation

$$\begin{aligned}\Pi=&\int_0^a\int_{(h)}\varepsilon 2\pi r dr dz-\int_0^a qW_-2\pi r dr-\int_{(h)}(U\sigma+W\sigma_{rz})_{r=a}2\pi a dz\\&-\int_0^a\int_{(h)}\left\{\sigma_r\left[e_r-\frac{\partial U}{\partial r}\right]+\sigma_\theta\left[e_\theta-\frac{U}{r}\right]+\sigma_z\left[e_z-\frac{\partial W}{\partial z}\right]+2\sigma_{rz}\left[e_{rz}\right.\right.\\&\left.\left.-\frac{1}{2}\left(\frac{\partial W}{\partial r}+\frac{\partial U}{\partial z}\right)\right]\right\}2\pi r dr dz\end{aligned} \tag{2.4}$$

where W_- is the vertical displacement of the upper surface, $\int_{(h)}(\ldots)dz$ is the integration carried out over the plate thickness, i.e.,

$$\int_{(h)}(\cdots\cdots)dz=\int_{-h/2}^{+h/2}(\cdots\cdots)dz \tag{2.5}$$

We shall now prove that under the condition that σ_r, σ_θ, σ_z, σ_{zr}, e_r, e_θ, e_z, e_{zr}, U,

W are all independent variables, the stationary condition of variation gives all the equations needed for this problem: (i) strain-displacement relations (1.2), (ii) stress-strain relations (1.4), (iii) equilibrium equations of stress (1.6), (iv) boundary conditions of external force on upper and lower surfaces (1.7), (v) boundary conditions at fixed edge surface (1.8).

Computing the variation of Π, we have

$$\begin{aligned}\delta\Pi = &\int_0^a\int_{(h)}\left\{\left[\frac{E_1}{1-\nu_1^2}(e_r+\nu_1 e_\theta+\nu_1 e_z)-\sigma_r\right]\delta e_r\right.\\
&+\left[\frac{E_1}{1-\nu_1^2}(e_\theta+\nu_1 e_r+\nu_1 e_z)-\sigma_\theta\right]\delta e\\
&\left.+\left[\frac{E_1}{1-\nu_1^2}(e_z+\nu_1 e_r+\nu_1 e_\theta)-\sigma_z\right]\delta e_z+2\left[\frac{E_1}{1+\nu}e_{rz}-\sigma_{rz}\right]\delta e_{rz}\right\}2\pi rdrdz\\
&-\int_0^a\int_{(h)}\left\{\left[e_r-\frac{\partial U}{\partial r}\right]\delta\sigma_r+\left[e_\theta-\frac{U}{r}\right]\delta\sigma_\theta+\left[e_z-\frac{\partial W}{\partial z}\right]\delta\sigma_z\right.\\
&\left.+\left[e_{rz}-\frac{1}{2}\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial r}\right)\right]\delta\sigma_{rz}\right\}2\pi rdrdz+\int_0^a\int_{(h)}\left\{\sigma_r\frac{\partial\delta U}{\partial r}+\sigma_\theta\frac{\delta U}{r}\right.\\
&\left.+\sigma_z\frac{\partial\delta W}{\partial z}+\sigma_{rz}\left(\frac{\partial\delta U}{\partial z}+\frac{\partial\delta W}{\partial r}\right)\right\}2\pi rdrdz-\int_0^a(q\delta W_-)2\pi rdr-\int_{(h)}[\sigma_r\delta U\\
&+\sigma_{rz}\delta W+U\delta\sigma_r+W\delta\sigma_{rz}]_{r=a}2\pi adz \qquad (2.6)\end{aligned}$$

Integrating by parts, we can prove that

$$\begin{aligned}&\int_0^a\int_{(h)}\sigma_r\frac{\partial\delta U}{\partial r}2\pi rdrdz=\int_{(h)}(\sigma_r\delta U)_{r=a}2\pi adz-\int_0^a\int_{(h)}\frac{\partial}{\partial r}(r\sigma_r)\delta U2\pi drdz\\
&\int_0^a\int_{(h)}\sigma_{rz}\frac{\partial\delta W}{\partial r}2\pi rdrdz=\int_{(h)}(\sigma_{rz}\delta W)_{r=a}2\pi adz-\int_0^a\int_{(h)}\frac{\partial}{\partial r}(r\sigma_{rz})\delta W2\pi drdz\\
&\int_0^a\int_{(h)}\sigma_z\frac{\partial\delta W}{\partial z}2\pi rdrdz=\int_0^a(\sigma_z^+\delta W_+-\sigma_z^-\delta W_-)2\pi rdr-\int_0^a\int_{(h)}\frac{\partial\sigma_z}{\partial z}\delta W2\pi rdrdz\\
&\int_0^a\int_{(h)}\sigma_{rz}\frac{\partial\delta U}{\partial z}2\pi rdrdz=\int_0^a(\sigma_{rz}^+\delta U_+-\sigma_{rz}^-\delta U_-)2\pi rdr-\int_0^a\int_{(h)}\frac{\partial}{\partial r}(\partial_{rz})\delta U2\pi rdrdz\end{aligned} \qquad (2.7)$$

Then, (2.6) may be rewritten as

$$\begin{aligned}\delta\Pi=&\int_0^a\int_{(h)}\left\{\left[\frac{E_1}{1-\nu_1^2}(e_r+\nu_1 e_\theta+\nu_1 e_z)-\sigma_r\right]\delta e_r+\left[\frac{E_1}{1-\nu_1^2}(e_\theta+\nu_1 e_r+\nu_1 e_z)-\sigma_\theta\right]\delta e_\theta\right.\\
&\left.+\left[\frac{E_1}{1-\nu_1^2}(e_z+\nu_1 e_r+\nu_1 e_\theta)-\sigma_z\right]\delta e_z+2\left[\frac{E_1}{1+\nu_1}e_{rz}-\sigma_{rz}\right]\delta e_{rz}\right\}2\pi rdrdz\\
&-\int_0^a\int_{(h)}\left\{\left[e_r-\frac{\partial U}{\partial r}\right]\delta\sigma_r+\left[e_\theta-\frac{U}{r}\right]\delta\sigma_\theta+\left[e_z-\frac{\partial W}{\partial z}\right]\delta\sigma_z\right.\\
&\left.+\left[e_{rz}-\frac{1}{2}\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial r}\right)\right]\delta\sigma_{rz}\right\}2\pi rdrdz-\int_0^a\int_{(h)}\left\{\left[\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_r)\right.\right.\\
&\left.\left.-\frac{\sigma_\theta}{r}+\frac{\partial\sigma_{rz}}{\partial z}\right]\delta U+\left[\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rz})+\frac{\partial\sigma_z}{\partial z}\right]\delta W\right\}2\pi rdrdz\\
&+\int_0^a\{\sigma_z^+\delta W_+-(q+\sigma_z^-)\delta W_-+\sigma_{rz}^+\delta U_+-\sigma_{rz}^-\delta U_-\}2\pi rdr\\
&-\int_{(h)}(U\delta\sigma_r+W\delta\sigma_{rz})_{r=a}2\pi adz \qquad (2.8)\end{aligned}$$

It should be pointed out that $\delta\sigma_r$, $\delta\sigma_\theta$, $\delta\sigma_z$, $\delta\sigma_{zr}$ at every point inside the plate and on the edge surface, δe_r, δe_θ, δe_z, δe_{zr} at every point inside the plate, δU, δW at every point inside the plate and on the upper and lower surfaces are all independent variations; therefore, the stationary condition of variation $\delta\Pi=0$ leads to equations (1.2), (1.4), (1.6), (1.7) and (1.8)Thus, we have proved that this condition $\delta\Pi=0$ represents the generalized variational principle for the problem. Indeed, this variational principle with all the constraints of variation being removed is the most generalized variational principle.

Functional Π entirely free from all the constraint conditions is not practical for it involves very complicate calculation. To keep a part of constraint conditions will simplify the problem.

Let us now keep (1.2) and (1.4) as the variational constraints applied to the variables σ_r, σ_θ, σ_z, σ_{zr}, e_r, e_θ, e_z, e_{zr}, U, and W. Then, the functional of variation defined by those variables may be written as

$$\Pi^*=\int_0^a\int_{(h)} \varepsilon 2\pi r dr dz-\int_0^a qW_- 2\pi r dr-\int_{(h)} (U\sigma_r+W\sigma_{rz})_{r=a}2\pi a dz \tag{2.9}$$

And $\delta\varepsilon$ should satisfy (2.2). If we take the variations of σ_r, σ_θ, σ_z, σ_{zr}, e_r, e_θ, e_z, e_{zr}, U and W under the constraints (1.2) and (1.4); then, the stationary condition of variation of the functional Π^* must satisfy (1.7), (1.8) and (1.9), and therefore must be the solution of the problem.

(III) The Most General Theory of Elastic Circular Plate with Non-Kirchhoff-Love Assumption ($e_z, e_{rz} \neq 0$) and Its First Order Approximation Theory

In the previous papers [1,3], it is shown that in the theory without using Kirchhoff-Love assumptions, the values of e_z and e_{rz} are not required to be zero. So, we can express them in terms of two polynomial series of z, i. e.,

$$e_z=\sum_{k=0}^{\infty} A_k z^k \tag{3.1a}$$

$$e_{rz}=\sum_{k=0}^{\infty} (S_{2k}+zS_{2k+1})\left(\frac{1}{4}h^2-z^2\right)z^{2k} \tag{3.1b}$$

where A_k, S_{2k} and S_{2k+1} are all field functions of r. It must be pointed out that e_{rz} in the above expression already satisfy condition (1.7), which requires that the shear stresses on the upper and lower surfaces vanish. The polynomial series for W can be obtained by integrating (1.2c) with respect to z. Then, substituting W into (1.2d) and integrating the result with respect to z yields the polynomial series for U. The results are as follows

$$W(r,\ z)=w(r)+\sum_{k=0}^{\infty}\frac{1}{k+1}A_k(r)z^{k+1} \tag{3.2a}$$

$$U(r,\ z)=u(r)-\frac{dw}{dr}z-\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)}\frac{dA_k}{dr}z^{k+2}+2\sum_{k=0}^{\infty}\left\{\frac{1}{2k+1}\left[\frac{1}{4}h^2-\frac{2k+1}{2k+3}z^2\right]z^{2k+1}S_{2k}+\frac{1}{2k+2}\left[\frac{1}{4}h^2-\frac{2k+2}{2k+4}z^2\right]z^{2k+2}S_{2k+1}\right\} \tag{3.2b}$$

where $u(r)$ and $w(r)$ are indeed the displacements of middle surface. The two variables and A_k, S_{2k}, S_{2k+1} are all field functions of r to be determined. The expressions for the components e_r and e_z can be derived by substituting (3.2) into (1.2). The differential equations and the relate boundary conditions which they must satisfy can be derived from the stationary conditions of variation of the functional Π^*. We must point out that the expressions for $\sigma_r, \sigma_\theta, \sigma_z, \sigma_{zr}$, e_r, e_θ, e_z, e_{zr}, U and W determined by the above method should satisfy (1.2) and (1.4); so, the ordinary differential equations and the relate boundary conditions for u, w, A_k, S_{2k}, S_{2k+1} $(k=0, 1, 2, \ldots)$ can be obtained through the stationary conditions of variation $\delta\Pi^*=0$.

To establish a reasonable approximation theory, we may take only a few terms in the polynomial expressions to make the approximate calculation. In this paper, we shall take the first two terms in the expressions of e_z and e_{rz} respectively as the bases to establish an approximation theory—the first order approximation theory.

Let us take the following approximate expressions

$$e_z=A_0+A_1 z \tag{3.3a}$$

$$e_{rz}=\left(\frac{h^2}{4}-z^2\right)(S_0+S_1 z) \tag{3.3b}$$

where A_0, A_1, S_0 and S_1 are four undetermined functions or r. Integrating e_z and e_{rz} with respect to z, we have $U(r, z)$ and $W(r, z)$ expressed by the following polynomial series of z

$$W(r, z)=w(r)+A_0 z+\frac{1}{2}A_1 z^2 \tag{3.4a}$$

$$U(r, z)=u(r)-\frac{dw}{dr}z-\frac{1}{2}\frac{dA_0}{dr}z^2-\frac{1}{6}\frac{dA_1}{dr}z^3+2\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)zS_0$$
$$+\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2 S_1 \tag{3.4b}$$

where $w(r)$ and $u(r)$ are undetermined functions. Hence, $U(r,z)$ and $W(r, z)$ are expressed by u, w, A_0, A_1, S_0 and S_1. And the strain components other than e_z and e_{rz} which are expressted by these variables in (3.3), can be written as

$$e_r=\frac{\partial U}{\partial r}=\frac{du}{dr}-\frac{d^2 w}{dr^2}z-\frac{1}{2}\frac{d^2A_0}{dr^2}z^2-\frac{1}{6}\frac{d^2A_1}{dr^2}z^3+2\left[\left(\frac{h}{2}\right)^2-\frac{1}{3}z^2\right]z\frac{dS_0}{dr}$$
$$+\left[\left(\frac{h}{2}\right)^2-\frac{1}{2}z^2\right]z^2\frac{dS_1}{dr} \tag{3.5a}$$

$$e_\theta=\frac{U}{r}=\frac{u}{r}-\frac{1}{r}\frac{dw}{dr}z-\frac{1}{2r}\frac{dA_0}{dr}z^2-\frac{1}{6r}\frac{dA_1}{dr}z^3+\frac{2}{r}\left[\left(\frac{h}{2}\right)^2-\frac{1}{3}z^2\right]zS_0$$
$$+\frac{1}{r}\left[\left(\frac{h}{2}\right)^2-\frac{1}{2}z^2\right]z^2 S_1 \tag{3.5b}$$

$$e_{r\theta}=e_{z\theta}=0 \tag{3.5c, d}$$

The corresponding components of stress in (1.4) are given by

$$\sigma_r=\frac{E_1}{1-\nu_1^2}\left\{\frac{du}{dr}+\nu_1\frac{u}{r}-\left(\frac{d^2w}{dr^2}+\nu_1\frac{1}{r}\frac{dw}{dr}\right)z+\nu_1(A_0+A_1 z)\right.$$
$$-\frac{1}{2}\left(\frac{d^2A_0}{dr^2}+\frac{\nu_1}{r}\frac{dA_0}{dr}\right)z^2-\frac{1}{6}\left(\frac{d^2A_1}{dr^2}+\frac{\nu_1}{r}\frac{dA_1}{dr}\right)z^3$$
$$\left.+2\left[\left(\frac{h}{2}\right)^2-\frac{1}{3}z^2\right]z\left(\frac{dS_0}{dr}+\frac{\nu_1 S_0}{r}\right)+\left[\left(\frac{h}{2}\right)^2-\frac{1}{2}z^2\right]z^2\left(\frac{dS_1}{dr}+\frac{\nu_1 S_1}{r}\right)\right\}$$
$$\tag{3.6a}$$

$$\sigma_\theta=\frac{E_1}{1-\nu_1^2}\left\{\frac{u}{r}+\nu_1\frac{du}{dr}-\left(\frac{1}{r}\frac{dw}{dr}+\nu_1\frac{dw^2}{dr^2}\right)z+\nu_1(A_0+A_1z)\right.$$

$$-\frac{1}{2}\left(\frac{1}{r}\frac{dA_0}{dr}+\nu_1\frac{d^2A_0}{dr^2}\right)z^2-\frac{1}{6}\left(\frac{1}{r}\frac{dA_1}{dr}+\nu_1\frac{d^2A_1}{dr^2}\right)z^3$$

$$\left.+2\left[\left(\frac{h}{2}\right)^2-\frac{1}{3}z^2\right]z\left(\frac{S_0}{r}+\nu_1\frac{dS_0}{dr}\right)+\left[\left(\frac{h}{2}\right)^2-\frac{1}{2}z^2\right]z^2\left(\frac{S_1}{r}+\nu_1\frac{dS_1}{dr}\right)\right\} \tag{3.6b}$$

$$\sigma_z=\frac{E_1}{1-\nu_1^2}\left\{\nu_1\left(\frac{du}{dr}+\frac{u}{r}\right)-\nu_1\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)z+A_0+A_1z\right.$$

$$-\frac{\nu_1}{2}\left(\frac{d^2A_0}{dr^2}+\frac{1}{r}\frac{dA_0}{dr}\right)z^2-\frac{\nu_1}{6}\left(\frac{1}{r}\frac{dA_1}{dr}+\frac{d^2A_1}{dr^2}\right)z^3$$

$$\left.+2\nu_1\left[\left(\frac{h}{2}\right)^2-\frac{1}{3}z^2\right]z\left(\frac{S_0}{r}+\frac{dS_0}{dr}\right)+\nu_1\left[\left(\frac{h}{2}\right)^2-\frac{1}{2}z^2\right]z^2\left(\frac{S_1}{r}+\frac{dS_1}{dr}\right)\right\} \tag{3.6c}$$

$$\sigma_{rz}=\frac{E_1}{1+\nu_1}\left[\left(\frac{h}{2}\right)^2-z^2\right][S_0+S_1z] \tag{3.6d}$$

Let us now simplify the expression for $\delta\Pi^*$ under the constraint conditions (1.2) and (1.4). Introducing (2.2) into (2.9) and carrying out the variational calculation, we have

$$\delta\Pi^*=\int_0^a\int_{(h)}[\sigma_r\delta e_r+\sigma_\theta\delta e_\theta+\sigma_z\delta e_z+2\sigma_{rz}\delta e_{rz}]2\pi r\,dr\,dz$$

$$-\int_0^a q\delta W_-2\pi r\,dr-\int_{(h)}[U\delta\sigma_r+W\delta\sigma_{rz}+\sigma_r\delta U+\sigma_{rz}\delta W]_{r=a}2\pi a\,dz \tag{3.7}$$

And from (1.2) we have

$$\delta e_r=\frac{\partial\delta U}{\partial r},\quad \delta e_\theta=\frac{\delta U}{r},\quad \delta e_z=\frac{\partial\delta W}{\partial z},\quad \delta e_{rz}=\frac{1}{2}\left(\frac{\partial\delta U}{\partial z}+\frac{\partial\delta W}{\partial r}\right) \tag{3.8}$$

Introducing (3.8) into (3.7) and simplifying the result through integrating by parts, it follows that

$$\delta\Pi^*=-\int_0^a\int_{(h)}\left\{\left[\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_r)+\frac{\partial\sigma_{zr}}{\partial z}-\frac{1}{r}\sigma_\theta\right]\delta U+\left[\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rz})\right.\right.$$

$$\left.\left.+\frac{\partial\sigma_z}{\partial z}\right]\delta W\right\}2\pi r\,dr\,dz+\int_0^a\{\sigma_z^+\delta W_+-(\sigma_z^-+q)\delta W_-\}2\pi r\,dr$$

$$-\int_{(h)}(U\delta\sigma_r+W\delta\sigma_{rz})_{r=a}2\pi a\,dz+\int_0^a(\sigma_{rz}^+\delta U_+-\sigma_{rz}^-\delta U_-)2\pi r\,dr \tag{3.9}$$

It should be pointed out that U and W are prescribed at $r=a$, i.e., they must satisfy (1.8); hence, the value of $(\sigma_r\delta U+\sigma_{rz}\delta W)_{r=a}$ is identically equal to zero. Next, according to (3.6d) we have $\sigma_{rz}^+=\sigma_{rz}^-=0$, it follows that the last integration in (3.9) must be identically equal to zero.

Calculating δU, δW, $\delta\sigma_r$ and $\delta\sigma_{zr}$ from (3.4) and (3.6a, b), and expressing them using δu, δw, δA_0, δA_1, δS_0, and δS_1, we can express $\delta\Pi^*$ in six independent parts: $\delta\Pi_u^*$, $\delta\Pi_w^*$, $\delta\Pi_{A0}^*$, $\delta\Pi_{A1}^*$, $\delta\Pi_{S0}^*$, $\delta\Pi_{S1}^*$; i.e.,

$$\delta\Pi^*=\delta\Pi_u^*+\delta\Pi_w^*+\delta\Pi_{A_0}^*+\delta\Pi_{A_1}^*+\delta\Pi_{S_0}^*+\delta\Pi_{S_1}^* \tag{3.10}$$

The stationary condition $\delta\Pi^*=0$ must lead to the stationary conditions for $\delta\Pi_u^*$, $\delta\Pi_w^*$, $\delta\Pi_{A0}^*$, $\delta\Pi_{A1}^*$, $\delta\Pi_{S0}^*$, $\delta\Pi_{S1}^*$; i.e,

$$\delta\Pi_u^*=0,\quad \delta\Pi_w^*=0,\quad \delta\Pi_{A_0}^*=0,\quad \delta\Pi_{A_1}^*=0,\quad \delta\Pi_{S_0}^*=0,\quad \delta\Pi_{S_1}^*=0 \tag{3.11}$$

From (3.9) we can derive $\delta\Pi_u^*$, which involves δu, as

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