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主编 钱伟长

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(第3集)

主编：钱伟长

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谨以此书纪念本会创建人、故董事会主席王宽诚先生

王宽诚教育基金会

王宽诚教育基金会简介

王宽诚先生(1907～1986)为香港知名爱国人士，热心祖国教育事业，生前为故乡宁波的教育事业做出积极贡献。1985年独力捐巨资创建王宽诚教育基金会，其宗旨在于为国家培养高级科技人才，为祖国四个现代化效力。

王宽诚先生在世时聘请海内外知名学者担任基金会考选委员会和学务委员会委员，共商大计，确定采用“送出去”和“请进来”的方针，为国家培育各科专门人才，并为提高国内和港澳高等院校的教学水平，资助学术界人士互访，用以促进中外文化交流。在此方针指导下，1985、1986两年，基金会在国家教委支持下，选派学生85名前往英、美、加拿大和西德、瑞士、澳大利亚各国攻读博士学位，并计划资助国内学者赴港澳讲学，资助港澳学者到国内讲学，资助美国学者来国内讲学。正当基金会事业初具规模，蓬勃发展之时，王宽诚先生一病不起，于1986年年底逝世。这是基金会的重大损失，共事同仁，无不深切怀念，不胜惋惜。

王宽诚教育基金会在新任董事会主席张二铭先生和安子介、方善桂、胡百全、李福树等董事的主持下，继承王宽诚先生为国家培育人才的遗愿，继续努力，除按计划执行外，并开发与英国学术机构合作的新项目。王宽诚教育基金会过去和现在的工作态度一贯以王宽诚先生所倡导的“公正”二字为守则，谅今后基金会亦将秉此行事，奉行不辍。借此王宽诚教育基金会《学术讲座汇编》出版之际，特简明介绍如上。

钱　伟　长

一九八九年十二月

前　言

王宽诚教育基金会是由已故全国政协常委、香港著名工商企业家王宽诚先生（1907～1986）出于爱国热忱，出资一亿美元于1985年在香港注册登记创立的。

1987年，基金会开设“学术讲座”项目，此项目由当时的全国政协常委、现任全国政协副主席、著名科学家、中国科学院学部委员、上海工业大学校长、王宽诚教育基金会贷款留学生考选委员会主任委员兼学务委员会主任委员钱伟长教授主持，由钱伟长教授亲自起草设立“学术讲座”的规定，资助国内学者前往香港、澳门讲学，资助美国学者和港澳学者前来国内讲学，用以促进中外学术交流，提高内地及港澳高等院校的教学质量。

本汇编收集的文章，均系各地学者在“学术讲座”活动中的讲稿。文章作者中，有年逾八旬的学术界硕彦，亦有由王宽诚教育基金会考选委员会委员推荐的学者和后起之秀。文章内容有科学技术，有历史文化，有经济专论，有文学，有宗教和中国古籍研究。本汇编涉及的学术领域颇为广泛，而每篇文章都有一定的深度和广度，分期分册以《王宽诚教育基金会学术讲座汇编》的名义出版，并无偿分送港澳和国内部分高等院校、科研机构和图书馆，以广流传。

王宽诚教育基金会除资助“学术讲座”学者进行学术交流之外，在钱伟长教授主持的项目下，还资助由国内有关高等院校推荐的学者前往欧美亚澳参加国际学术会议，出访的学者均向所出席的会议提交论文，这些论文亦颇有水平，本汇编亦将其收入，以供参考。

王宽诚教育基金会学务委员会

凡　例

（一）编排次序

本书所收集的王宽诚教育基金会学术讲座的讲稿及由王宽诚教育基金会资助学者赴欧美亚澳参加国际学术会议的论文均按照收到文稿日期先后编排刊列，不分类别。

（二）分期分册出版并作简明介绍

因文稿较多，为求便于携带，有利阅读与检索，故分期分册出版，每册约 150 页至 200 页不等。为便于读者查考，每篇学术讲座的讲稿均注明作者姓名、学位、职务、讲学日期、地点、访问院校名称。国内及港澳学者到欧、美、澳及亚洲的国家和地区参加国际学术会议的论文均注明学者姓名、参加会议的名称、时间、地点和推荐的单位。上述两类文章均注明由王宽诚教育基金会资助字样。

（三）文字种类

本书为学术性文章汇编，均以学术讲座学者之讲稿原稿或参加国际学术会议学者向会议提交的论文原稿文字为准，即原讲稿或论文是中文的，即以中文刊出，原讲稿或论文是外文的，仍以外文刊出。

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第三集

目　录

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Thermal Radiation in Packed and Fluidized Beds

C. L. Tien*

1 Introduction

Many modern technologies and industrial processes utilize packed and fluidized beds of solid particles operating at temperatures high enough for thermal radiation to be a significant mechanism of heat transfer (Flamant and Arnaud, 1984; Saxena et al., 1978). Some examples are coal combustors, chemical reactors, and nuclear fuel rods. Other types of packed beds in which thermal radiation is important, even though the temperatures may not be high, are those where other modes of heat transfer have been suppressed, such as packed cryogenic microsphere insulations (Tien and Cunnington, 1973). Since most packed and fluidized beds are characterized by either high volume fractions of particles or large particles, or both, many features are unique for analyzing such systems. These features are discussed in this paper. Only gas systems are considered here since most applications of interest, such as those mentioned above, fall in the category of gas fluidized and packed beds.

Packed beds are usually characterized by densely packed particles that do not move during normal operation. The high volume fraction is generally coupled with either an absence of fluid motion or low fluid velocities. This restricts the convection contribution and renders radiation a dominant mode of energy transport. Fluidized beds, on the other hand, have lower volume fractions but higher fluid velocities. In many applications, however, the contribution of radiation to the total energy transport remains significant due to the high operating temperatures. Table 1 shows various representative values that characterize packed and fluidized beds (Ulrich, 1984; Flamant and Arnaud, 1984). A schematic depiction of the various types of packed and fluidized beds is presented in Fig. 1. A review by Haughey and Beveridge (1969) describes the structural properties of packed bed systems.

Previous studies of radiative heat transfer through packed and fluidzed beds have employed a variety of analystical and experimental techniques. Vortmeyer (1978) summarized some earlier radiation models using unit cell representations for analyzing packed beds. Such cell and layer models, in conjunction with Monte-Carlo methods, are also used by Chan and Tien (1974a) to evaluate radiative characteristics of packed beds of fixed porosity and regular structure, by Yang et al. (1983) for randomly packed beds of uniform spheres, and

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Table 1 Characteristics of gas fluidized and fixed beds

	Type of Bed	
	Fluidized	Fixed
Bed diameter (*m*)	1-10	0.3-4.0
Bed height (*m*)	0.31-5.0	0.3-30.0
Porosity	0.6-0.8	0.35-0.70
Particle diameter (*m*)	10^{-6}-10^{-2}	<0.1×(bed dia.)
Fluid velocity (*m / s*)	0.1-5.0	0.005-1.0
Pressure drop (*kPa / m*)	5-15.	0.001-1.0
Temperature (℃)		
Carbon steel	450	450
Stainless steel	750	750
Nickel alloys	1200	1200
Brick-lined	1500	1500
Overall heat transfer coefficient (J / sm^2K)	400-800	20-80
Percent of radiation in total heat transfer (750℃)	10-40	

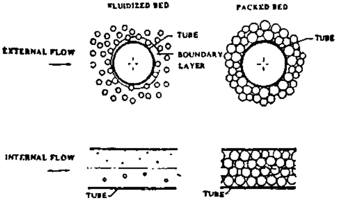

Fig. 1 Types of packed and fluidized beds

by Kudo et al. (1985) who examined different types of packing and variation in volume fraction. Brodulya and Kovensky (1983) used the cell approach by evaluating exact view factors in the unit cell by assuming the surfaces to be isothermal and diffuse. Heat transfer in fluidized beds was reviewed by Saxena et al. (1978). Glicksman and Decker (1982) examined the role of radiation and particle packing on the heat transfer from immersed surfaces

Nomenclature

b = single-particle back-scatter fraction
B = bed back-scatter fraction
c = interparticle clearance
C = cross section
C_p = specific heat at constant pressure
D = diameter of the particles
e = unit vector
E = exchange factor
f_v = solid volume fraction = $4N\pi(D/2)^3/3V$
$F(\theta)$ = form factor to account for coherent addition of intensities
$g(R)$ = radial distribution of number density, normalized by N/V
G = geometric cross-sectional area
$H(\beta) = 3(\sin\beta - \beta\cos\beta)/\beta^3$
$i = \sqrt{-1}$
I = intensity, energy / steradian / projected area
k = propagation constant = $2\pi/\lambda$; thermal conductivity
L = length of the one-dimensional medium
m = complex refractive index = $n+ik$
n = index of refraction
N = number of particles in volume V
P = radiation transmission number
q = heat flux
Q = efficiency
r = position vector
R = ratio of radial distance to particle diameter = r/D; reflectance
t = time
T = transmittance; temperature
v = velocity vector
V = volume
x,y,z = Cartesian coordinates
α = size parameter = $\pi D/\lambda$
β = $4\alpha\sin(\theta/2)$
γ = function defined by equation (22)
ϵ = m^2; emissivity
ζ = near-field complex correction factor
η = function defined by equation (26)
θ = polar angle
κ = index of absorption
λ = wavelength of incident radiation
Λ = function defined in Table 4
μ = $\cos\theta$
ν = frequency of incident radiation
Ξ = function defined in Table 4
ρ = reflectivity
σ = radiative coefficients
τ = optical path length
ϕ = azimuthal angle
Φ = scattering phase function
χ = parameter in the liquid model
ω = angular frequency = $2\pi\nu$; scattering albedo = σ_s/σ_e
Ω = solid angle

Subscripts

a = absorption
b = blackbody
d = diffraction
e = extinction
i = incident radiation
j,l = particle identification
L = length of medium
M = Mie theory for one particle
N = N particles
r = radiative
ref = reflection
s = scattering; specular
t = transmission
O = background dielectric matrix; coordinate origin
λ = wavelength

to particles in fluidized beds. Combined wall-to-fluidized bed heat transfer was studied by Flamant and Menigault (1987). Brewster and Tien (1982a) examined the issue of dependent versus independent scattering in packed and fluidized beds.

Combined radiative and conductive heat transfer in packed beds has been the subject of many studies (Chan and Tien, 1974b; Bergquam and Seban, 1971). Simultaneous radiative-convective transfer in packed and fluidized beds has also been studied (Echigo et al., 1974; Tabanfar and Modest, 1987). Experimental measurements of radiant transmission through packed and fluidized media have been reported by various researchers. The effective scattering and absorption cross sections of isothermal beds of glass, aluminum oxide, steel, and silicon carbide spheres, cylinders, and irregular grains were obtained by Chen and Churchill (1963). Similar measurements through packed and fluidized beds of glass beads were reported by Cimini and Chen (1987). Local heat transfer coefficients in large-particle fluidized beds were measured bv Goshayeshi et al. (1986) and wall-to-bed heat transfer by Flamant and Menigault (1987).

2 Theoretical Basis of Thermal Radiation in Packed / Fluidized Beds

Packed and fluidized beds are multiphase systems consisting of solid particulates and gases (liquid systems are not considered here). Thermal radiation within these beds usually is the result of emission by the hot walls and the gas-particle mixture. This radiation undergoes complex interactions with the bed primarily due to absorption and scattering processes. The three primary radiative properties that characterize the interactions of radiation with the particulate bed are the scattering coefficient, the extinction coefficient (i. e., sum of scattering and absorption coefficients), and the scattering phase function. These properties are adopted primarily due to the following considerations: *(i)* the theory of electromagnetic interaction with particles yields these values first, *(ii)* they can be directly obtained from experimental measurements, and *(iii)* other values can be inferred from these primary properties. For example, the absorption coefficient cannot be directly measured from light scattering experiments. It is obtained indirectly by measuring the scattering and extinction losses from the incident beam and evaluating the difference between the corresponding extinction and the scattering coefficients.

Computation of the transport of thermal radiation in the particulate system requires an accurate knowledge of these primary radiative characteristics. This is evident by considering the propagation of radiation within an absorbing, emitting, and scattering medium, which is governed by the equation of transfer (Kerker, 1961; Siegel and Howell,1981; Ozisik, 1973):

$$\mathbf{e}_\Omega \cdot \nabla I_\lambda(\mathbf{r}, \mathbf{e}_\Omega) = -(\sigma_{a\lambda} + \sigma_{s\lambda}) I_\lambda(\mathbf{r}, \mathbf{e}_\Omega) + \sigma_{a\lambda} I_{b\lambda}(T(\mathbf{r})) + \frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(\mathbf{r}, \mathbf{e}_{\Omega'}) \Phi(\mathbf{e}_{\Omega'} \to \mathbf{e}_\Omega) d\Omega' \quad (1)$$

where I_λ is the monochromatic radiation intensity, T the medium local temperature, $\mathbf{r}$ the position vector, $\mathbf{e}\Omega$ the unit vector in the direction of consideration, and Ω the solid angle centered around $\mathbf{e}_\Omega$. The coefficients are denoted by σ and the subscripts a, e, s refer to absorption, extinction, and scattering, respectively. The first term on the right-hand side of the equation of transfer represents the attenuation of intensity due to absorption and scattering, the second term represents the gain due to emission, and the last term is the gain due to the inscattering into the direction $\mathbf{e}_\Omega$ from all other directions. The intensity I_λ is defined as the energy per unit area per unit solid angle per unit wavelength and the scattering phase function $\Phi(\mathbf{e}_{\Omega'} - \mathbf{e}_\Omega)$ is a specification of the radiation intensity scattered from the direction $\mathbf{e}_{\Omega'}$ into the direction under consideration, normalized by the isotropic scattered radiation intensity, i.e., $\Phi(\mathbf{e}_{\Omega'} \to \mathbf{e}_\Omega) = 1$ for isotropic scattering. The different methodologies for solving equation (1) are discussed in Section 4.

The radiative coefficients are defined as the fraction of the corresponding energy loss from the propagating wave, per length of travel. The units for the coefficients σ are inverse of length, whereas the phase function is dimensionless. The radiative coefficients are functions of the optical constants of the bed materials and of the particle size, shape, and packing, where the optical constants are functions of the wavelength. The phase function is a strong function of shape and varies from the predominantly forward scattering for large particles to the semi-diffuse for small.

Thermal radiation in packed and fluidized beds is unpolarized by nature and I_λ represents the unpolarized intensity. Another important quantity, which is of greater interest than the intensity, is the heat flux. The radiative heat flux vector $\mathbf{q}_r$ is given as (Ozisik, 1973)

$$\mathbf{q}_r(\mathbf{r})=q_r\,\mathbf{e}_q=\int_\lambda\int_{4\pi} I_\lambda(\mathbf{r},\ \mathbf{e}_\Omega)\mathbf{e}_\Omega\cdot\mathbf{e}_q\,d\Omega \tag{2}$$

where $\mathbf{e}_q$ is the unit normal vector to the unit area across which the flux is being measured. Integrating equation (1) over all angles and wavelengths yields the following equation for the divergence of the radiative flux (Ozisik, 1973):

$$\nabla\cdot\mathbf{q}_r(\mathbf{r}) = 4\pi\int_\lambda \sigma_{a\lambda}I_{\lambda b}(T(\mathbf{r}))\,d\lambda-\int_\lambda \sigma_{a\lambda}\int_{4\pi} I_\lambda(\mathbf{r},\mathbf{e}_\Omega)\,d\Omega d\lambda \tag{3}$$

The energy equation accounts for radiation, convection, and conduction modes of energy transfer and can be written in the form (Siegel and Howell, 1981)

$$\rho C_p\left[\frac{\partial T}{\partial t}+(\mathbf{v}\cdot\nabla)T\right]=\nabla\cdot k\nabla\mathbf{T}+\nabla\cdot\mathbf{q}_r \tag{4}$$

where T is the temperature, v the velocity vector, ρ the density, C_p the specific heat, k the thermal conductivity, and t time. The effect of viscous dissipation has been neglected in the above equation. The energy equation obtained by combining equations (1),(3), and (4) is integro-differential and nonlinear, and cannot be simplified to a differential equation in most situations without neglecting readiative processes in the energy transfer. The full equation does not lend itself to simple closed-form solutions and direct numerical solutions require immense computational effort. Combinations of radiation with the other modes of heat transfer were first studied for cases where only radiation and conduction were present, which is characteristic of packed beds. In fluidized-bed systems, convection and radiation are the important mechanisms of energy transfer as indicated by experimental studies (Goshayeshi et al., 1986). Solutions have been obtained by incorporating simplifying approximations such as an isotropically scattering gray medium (Yener and Ozisik, 1986), and a linearly anisotropic scattering medium (Azad and Modest, 1981).

The focus of this section so far has been on the transport of radiative energy as described by the equation of transfer. The equation of transfer treats the medium as a continuum where each volume element absorbs, emits, and scatters radiation. The exact positions of the different particles in the volume are not considered; only volume-averaged values of the radiative properties are used. Other methods, which do not treat the medium as a continuum and, instead, take into account the position of particles and the boundaries between the solid and the gas ph ase, are termed the discrete models of radiative transfer. Such models usually utilize ray-tracing or view-factor techniques and are most useful for analyzing beds with large particles and high volume fractions. These models are discussed in Section 4.

3 Thermal Radiation Characteristics of Packed / Fluidized Beds

In homogeneous media such as gases, absorption and emission are the major radiative mechanisms. If the medium contains inhomogeneities, such as the particles in packed or fluidized beds, the additional mechanism of scattering is introduced. These absorption and scattering processes are governed by electromagnetic field equations and their associated boundary conditions at all interfaces. The resulting analytical problem is formidable and is usually solved by using simplifying assumptions: idealized geometry of the scatterers, independent scattering and absorption, homogeneous distribution of particles, and others discussed in the following sections.

Absorption and Scattering by a Single Particle. The absorption and scattering characteristics of a single particle are described by the solution of the electromagnetic field equations. Physically, they can be explained by the processes of reflection, refraction, and diffraction. When an electromagnetic wave strikes the particle surface, a portion of it is reflected while the remainder penetrates the particle. The beam within the particle may experience some absorption and multiple internal reflections before it escapes out of the particle in different directions, giving rise to scattering. This scattering is the contribution by refraction. The diffraction scattering process originates from the bending of the incident beams near the edge of the particle. Consequently, even a completely absorbing particle scatters radiation.

Scattering and absorption characteristics of a particle are governed by three factors: the particle shape, the particle size relative to the wavelength of the incident radiation, and the optical properties of the particle and the background medium (Tien, 1985). For the particle shape, general solutions are available for only a few common shapes such as spheres, cylinders, and spheroids (Kerker, 1961; Bohren and Huffman, 1983). The solutions are complicated even for these simple cases. The second factor is commonly expressed by a size parameter α, which is defined as $(\pi D/\lambda)$ for spheres, where D is the diameter and λ is the wavelength. The last factor is represented by the complex refractive index m defined as $(n+ik)$ where n if the index of refraction and k is the index of absorption. It should be noted that $m=(n-ik)$ is the incident wave is assumed proportional to exp $(i\omega t)$, where ω is the angular velocity and t is time. In this paper it is assumed that the proportionality is $\exp(-i\omega t)$ and hence $m=(n+ik)$. The background medium is assumed to be nonparticipating, i.e., $m=1.0$, and thus requires no further consideration.

The solution of the electromagnetic field equations yields the internal and scattered electromagnetic fields from which the corresponding extinction and scattering cross sections C_e and C_s are obtained. The cross sections are defined as the ratio of the energy loss to the incident energy flux and have the units of area. Efficiencies are defined as the dimensionless ratios of cross sections to the geometric cross-sectional area G, i.e.,

$$Q_p=C_p/G \qquad (p=a,e,s) \tag{5}$$

where $G=\pi D^2/4$ for spheres of diameter D. The phase function for spheres is defined as

$$\Phi\,(e_\Omega\to e_\Omega') = \frac{\pi^2 D^2}{C_s}\,\frac{I_s(e_\Omega\to e_\Omega')}{I_i} \tag{6}$$

where I_i is the intensity of the incident wavelength. The scattering phase functions for spherical particles of three different sizes have been plotted in Fig. 2. Figure 3 shows the absorption, extinction, and scattering efficiencies for a range of size parameters $0<\alpha<15$, $m=1.29+i0.472$.

The solutions of the electromagnetic fields are usually in the form of an infinite series (Bo-

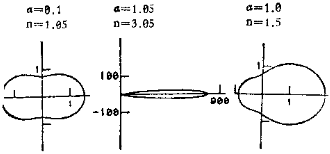

Fig. 2 Scattering phase functions for small, intermediate, and large values of α and m

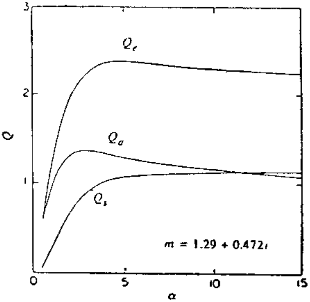

Fig. 3 Absorption, scattering, and extinction efficiencies of a spherical particle

hren and Huffman, 1983; Wiscombe, 1980) or complicated functions of α and m. However, simple expressions exist for some limiting cases that are of greatest importance for fluidized and packed bed applications. The first of these is the small particle limit, i.e., $\alpha \ll 1$, which is important for packed-bed systems such as microsphere insulations. This is called the Rayleigh limit. The second is the large particle, or geometric limit, characteristic of packed and fluidized bed combustors. The Rayleigh limit offers simple algebraic equations for radiative properties of small particles though no significant saving of computational time. In contrast to this limited numerical expediency, large computational savings are possible by using geometric scattering assumptions over the Mie theory.

For a small (Rayleigh) isolated particle of size parameter α, the extinction and scattering efficiencies corresponding to unpolarized incident radiation, obtained from the Mie theory (to terms of order α^4, $\alpha \ll 1$) are (Kerker, 1961; Bohren and Huffman, 1983)

$$Q_e(\alpha,\ \epsilon)=4\alpha \mathrm{Im}\left\{\left(\frac{\epsilon-1}{\epsilon+2}\right)\left[1+\frac{1}{15}\alpha^2\left(\frac{\epsilon-1}{\epsilon+2}\right)\frac{\epsilon^2+27\epsilon+38}{2\epsilon+3}\right]\right\}+\frac{8}{3}\alpha^4\mathrm{Re}\left\{\left(\frac{\epsilon-1}{\epsilon+2}\right)^2\right\} \quad (7)$$

$$C_s\ (\alpha,\ \epsilon)=\frac{8}{3}\alpha^4\left|\frac{\epsilon-1}{\epsilon+2}\right|^2 \quad (8)$$

where $\epsilon = m^2$. The corresponding phase function is

$$\Phi(\theta) = \frac{3}{4}(1 + \cos^2 \theta) \tag{9}$$

where θ is the polar angle between scattering and incident directions.

The simplicity of the Rayleigh-scattering approximations makes them appealing for computing radiative characteristics of small particles. Even though the limit $\alpha \ll 1$ is used to indicate the Rayleigh limit, Ku and Felske (1984) have discussed in some detail the range of parameters α and m for which the above equations can be used without causing significant error. Packed beds of microspheres and ultrafine powder, used for insulation purposes, and certain fluidized beds fall in these categories.

The geometric or large-particle limit is difficult to handle through exact solutions involving series expansions because large numbers of terms (approximately $2\alpha + 2$) are required to obtain convergence (Wiscombe, 1980). Additionally the large value of the arguments of the mathematical functions involved makes the terms in the series very difficult to evaluate. To overcome these hurdles concepts from geometric optics are introduced to analyze particles with large size parameters. Geometric optics uses the method of tracing rays as they undergo refractions and multiple reflections at the interfaces and absorption within the particle.

The total energy scattered by a large sphere may be written as the sum of diffracted, reflected, and transmitted components. Consequently the scattering efficiency is expressed as

$$Q_s = Q_d + Q_{ref} + Q_t, \qquad Q_d = 1 \tag{10}$$

where the subscripts *d*, *ref*, and *t* denote diffraction, external reflection, and transmission, respectively. For large absorbing spheres all the energy entering the sphere is eventually absorbed, yielding $Q_t = 0$. The extinction efficiency for large spheres is shown to be (Boheren and Huffman, 1983; Kerker, 1961)

$$\lim_{\alpha \to \infty} Q_e = 2 \tag{11}$$

implying $Q_a = 1 - Q_{ref} - Q_t$. The phase function for large particles is strongly fowrard scattering and must be determined by ray-tracing methods.

For large opaque spheres the efficiencies can be approximated as (Siegel and Howell, 1981)

$$Q_a = \epsilon_\lambda, \qquad Q_s = 1 - \epsilon_\lambda \tag{12}$$

where ϵ is the emissivity of the surface of the particles. The phase function is approximately isotropic, i.e., $\Phi(\theta) = 1$, if the sphere is specularly reflecting, and is

$$\Phi(\theta) = \frac{8}{3\pi}(\sin\theta - \theta \cos\theta) \tag{13}$$

if the spheres reflect diffusely. Since large particles in most packed and fluidized beds such as chemical reactors and coal combustors are diffuse, the expressions presented above represent a significant saving of computational resources compared to the complete Mie series solution.

If the diffraction component is treated as part of the propagating beam, restrictions are placed on the lower size limit for the geometric scattering to ensure that the scattered portion due to diffraction is indeed in the forward direction. The extinction efficiency is taken to be equal to unity instead of two since the diffraction portion is omitted. Similarly $Q_s = Q_{ref} + Q_t = 1 - Q_a$. If the scattered light in a cone of half angle of 5 deg is considered as part of the