

教育部高等教育司推荐  
国外优秀信息科学与技术系列教学用书

# 算法导论

(第二版 影印版)

## INTRODUCTION TO ALGORITHMS

(Second Edition)

■ Thomas H. Cormen  
Charles E. Leiserson  
Ronald L. Rivest  
Clifford Stein



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# 前 言

20 世纪末，以计算机和通信技术为代表的信息科学和技术对世界经济、科技、军事、教育和文化等产生了深刻影响。信息科学技术的迅速普及和应用，带动了世界范围信息产业的蓬勃发展，为许多国家带来了丰厚的回报。

进入 21 世纪，尤其随着我国加入 WTO，信息产业的国际竞争将更加激烈。我国信息产业虽然在 20 世纪末取得了迅猛发展，但与发达国家相比，甚至与印度、爱尔兰等国家相比，还有很大差距。国家信息化的发展速度和信息产业的国际竞争能力，最终都将取决于信息科学技术人才的质量和数量。引进国外信息科学和技术优秀教材，在有条件的学校推动开展英语授课或双语教学，是教育部为加快培养大批高质量的信息技术人才采取的一项重要举措。

为此，教育部要求由高等教育出版社首先开展信息科学和技术教材的引进试点工作。同时提出了两点要求，一是要高水平，二是要低价格。在高等教育出版社和信息科学技术引进教材专家组的努力下，经过比较短的时间，第一批引进的 20 多种教材已经陆续出版。这套教材出版后受到了广泛的好评，其中有不少是世界信息科学技术领域著名专家、教授的经典之作和反映信息科学技术最新进展的优秀作品，代表了目前世界信息科学技术教育的一流水平，而且价格也是最优惠的，与国内同类自编教材相当。

这项教材引进工作是在教育部高等教育司和高教社的共同组织下，由国内信息科学技术领域的专家、教授广泛参与，在对大量国外教材进行多次遴选的基础上，参考了国内和国外著名大学相关专业的课程设置进行系统引进的。其中，John Wiley 公司出版的贝尔实验室信息科学研究中心副总裁 Silberschatz 教授的经典著作《操作系统概念》，是我们经过反复谈判，做了很多努力才得以引进的。William Stallings 先生曾编写了在美国深受欢迎的信息科学技术系列教材，其中有多种教材获得过美国教材和学术著作者协会颁发的计算机科学与工程教材奖，这批引进教材中就有他的两本著作。留美中国学者 Jiawei Han 先生的《数据挖掘》是该领域中具有里程碑意义的著作。由达特茅斯学院 Thomas Cormen 和麻省理工学院、哥伦比亚大学的几

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位学者共同编着的经典著作《算法导论》，在经历了 11 年的锤炼之后于 2001 年出版了第二版。目前任教于美国 Massachusetts 大学的 James Kurose 教授，曾在美国三所高校先后 10 次获得杰出教师或杰出教学奖，由他主编的《计算机网络》出版后，以其体系新颖、内容先进而倍受欢迎。在努力降低引进教材售价方面，高等教育出版社做了大量和细致的工作。这套引进的教材体现了权威性、系统性、先进性和经济性等特点。

教育部也希望国内和国外的出版商积极参与此项工作，共同促进中国信息技术教育和信息产业的发展。我们在与外商的谈判工作中，不仅要坚定不移地引进国外最优秀的教材，而且还要千方百计地将版权转让费降下来，要让引进教材的价格与国内自编教材相当，让广大教师和学生负担得起。中国的教育市场巨大，外国出版公司和国内出版社要通过扩大发行数量取得效益。

在引进教材的同时，我们还应做好消化吸收，注意学习国外先进的教学思想和教学方法，提高自编教材的水平，使我们的教学和教材在内容体系上，在理论与实践的结合上，在培养学生的动手能力上能有较大的突破和创新。

目前，教育部正在全国 35 所高校推动示范性软件学院的建设和实施，这也是加快培养信息科学技术人才的重要举措之一。示范性软件学院要立足于培养具有国际竞争力的实用性软件人才，与国外知名高校或著名企业合作办学，以国内外著名 IT 企业为实践教学基地，聘请国内外知名教授和软件专家授课，还要率先使用引进教材开展教学。

我们希望通过这些举措，能在较短的时间，为我国培养一大批高质量的信息技术人才，提高我国软件人才的国际竞争力，促进我国信息产业的快速发展，加快推动国家信息化进程，进而带动整个国民经济的跨越式发展。

教育部高等教育司

二〇〇二年三月

# Preface

This book provides a comprehensive introduction to the modern study of computer algorithms. It presents many algorithms and covers them in considerable depth, yet makes their design and analysis accessible to all levels of readers. We have tried to keep explanations elementary without sacrificing depth of coverage or mathematical rigor.

Each chapter presents an algorithm, a design technique, an application area, or a related topic. Algorithms are described in English and in a “pseudocode” designed to be readable by anyone who has done a little programming. The book contains over 230 figures illustrating how the algorithms work. Since we emphasize *efficiency* as a design criterion, we include careful analyses of the running times of all our algorithms.

The text is intended primarily for use in undergraduate or graduate courses in algorithms or data structures. Because it discusses engineering issues in algorithm design, as well as mathematical aspects, it is equally well suited for self-study by technical professionals.

In this, the second edition, we have updated the entire book. The changes range from the addition of new chapters to the rewriting of individual sentences.

## **To the teacher**

This book is designed to be both versatile and complete. You will find it useful for a variety of courses, from an undergraduate course in data structures up through a graduate course in algorithms. Because we have provided considerably more material than can fit in a typical one-term course, you should think of the book as a “buffet” or “smorgasbord” from which you can pick and choose the material that best supports the course you wish to teach.

You should find it easy to organize your course around just the chapters you need. We have made chapters relatively self-contained, so that you need not worry about an unexpected and unnecessary dependence of one chapter on another. Each chapter presents the easier material first and the more difficult material later, with

section boundaries marking natural stopping points. In an undergraduate course, you might use only the earlier sections from a chapter; in a graduate course, you might cover the entire chapter.

We have included over 920 exercises and over 140 problems. Each section ends with exercises, and each chapter ends with problems. The exercises are generally short questions that test basic mastery of the material. Some are simple self-check thought exercises, whereas others are more substantial and are suitable as assigned homework. The problems are more elaborate case studies that often introduce new material; they typically consist of several questions that lead the student through the steps required to arrive at a solution.

We have starred (★) the sections and exercises that are more suitable for graduate students than for undergraduates. A starred section is not necessarily more difficult than an unstarred one, but it may require an understanding of more advanced mathematics. Likewise, starred exercises may require an advanced background or more than average creativity.

### **To the student**

We hope that this textbook provides you with an enjoyable introduction to the field of algorithms. We have attempted to make every algorithm accessible and interesting. To help you when you encounter unfamiliar or difficult algorithms, we describe each one in a step-by-step manner. We also provide careful explanations of the mathematics needed to understand the analysis of the algorithms. If you already have some familiarity with a topic, you will find the chapters organized so that you can skim introductory sections and proceed quickly to the more advanced material.

This is a large book, and your class will probably cover only a portion of its material. We have tried, however, to make this a book that will be useful to you now as a course textbook and also later in your career as a mathematical desk reference or an engineering handbook.

What are the prerequisites for reading this book?

- You should have some programming experience. In particular, you should understand recursive procedures and simple data structures such as arrays and linked lists.
- You should have some facility with proofs by mathematical induction. A few portions of the book rely on some knowledge of elementary calculus. Beyond that, Parts I and VIII of this book teach you all the mathematical techniques you will need.

## **To the professional**

The wide range of topics in this book makes it an excellent handbook on algorithms. Because each chapter is relatively self-contained, you can focus in on the topics that most interest you.

Most of the algorithms we discuss have great practical utility. We therefore address implementation concerns and other engineering issues. We often provide practical alternatives to the few algorithms that are primarily of theoretical interest.

If you wish to implement any of the algorithms, you will find the translation of our pseudocode into your favorite programming language a fairly straightforward task. The pseudocode is designed to present each algorithm clearly and succinctly. Consequently, we do not address error-handling and other software-engineering issues that require specific assumptions about your programming environment. We attempt to present each algorithm simply and directly without allowing the idiosyncrasies of a particular programming language to obscure its essence.

## **To our colleagues**

We have supplied an extensive bibliography and pointers to the current literature. Each chapter ends with a set of “chapter notes” that give historical details and references. The chapter notes do not provide a complete reference to the whole field of algorithms, however. Though it may be hard to believe for a book of this size, many interesting algorithms could not be included due to lack of space.

Despite myriad requests from students for solutions to problems and exercises, we have chosen as a matter of policy not to supply references for problems and exercises, to remove the temptation for students to look up a solution rather than to find it themselves.

## **Changes for the second edition**

What has changed between the first and second editions of this book? Depending on how you look at it, either not much or quite a bit.

A quick look at the table of contents shows that most of the first-edition chapters and sections appear in the second edition. We removed two chapters and a handful of sections, but we have added three new chapters and four new sections apart from these new chapters. If you were to judge the scope of the changes by the table of contents, you would likely conclude that the changes were modest.

The changes go far beyond what shows up in the table of contents, however. In no particular order, here is a summary of the most significant changes for the second edition:



- Cliff Stein was added as a coauthor.
- Errors have been corrected. How many errors? Let's just say several.
- There are three new chapters:
  - Chapter 1 discusses the role of algorithms in computing.
  - Chapter 5 covers probabilistic analysis and randomized algorithms. As in the first edition, these topics appear throughout the book.
  - Chapter 29 is devoted to linear programming.
- Within chapters that were carried over from the first edition, there are new sections on the following topics:
  - perfect hashing (Section 11.5),
  - two applications of dynamic programming (Sections 15.1 and 15.5), and
  - approximation algorithms that use randomization and linear programming (Section 35.4).
- To allow more algorithms to appear earlier in the book, three of the chapters on mathematical background have been moved from Part I to the Appendix, which is Part VIII.
- There are over 40 new problems and over 185 new exercises.
- We have made explicit the use of loop invariants for proving correctness. Our first loop invariant appears in Chapter 2, and we use them a couple of dozen times throughout the book.
- Many of the probabilistic analyses have been rewritten. In particular, we use in a dozen places the technique of “indicator random variables,” which simplify probabilistic analyses, especially when random variables are dependent.
- We have expanded and updated the chapter notes and bibliography. The bibliography has grown by over 50%, and we have mentioned many new algorithmic results that have appeared subsequent to the printing of the first edition.

We have also made the following changes:

- The chapter on solving recurrences no longer contains the iteration method. Instead, in Section 4.2, we have “promoted” recursion trees to constitute a method in their own right. We have found that drawing out recursion trees is less error-prone than iterating recurrences. We do point out, however, that recursion trees are best used as a way to generate guesses that are then verified via the substitution method.

- The partitioning method used for quicksort (Section 7.1) and the expected linear-time order-statistic algorithm (Section 9.2) is different. We now use the method developed by Lomuto, which, along with indicator random variables, allows for a somewhat simpler analysis. The method from the first edition, due to Hoare, appears as a problem in Chapter 7.
- We have modified the discussion of universal hashing in Section 11.3.3 so that it integrates into the presentation of perfect hashing.
- There is a much simpler analysis of the height of a randomly built binary search tree in Section 12.4.
- The discussions on the elements of dynamic programming (Section 15.3) and the elements of greedy algorithms (Section 16.2) are significantly expanded. The exploration of the activity-selection problem, which starts off the greedy-algorithms chapter, helps to clarify the relationship between dynamic programming and greedy algorithms.
- We have replaced the proof of the running time of the disjoint-set-union data structure in Section 21.4 with a proof that uses the potential method to derive a tight bound.
- The proof of correctness of the algorithm for strongly connected components in Section 22.5 is simpler, clearer, and more direct.
- Chapter 24, on single-source shortest paths, has been reorganized to move proofs of the essential properties to their own section. The new organization allows us to focus earlier on algorithms.
- Section 34.5 contains an expanded overview of NP-completeness as well as new NP-completeness proofs for the hamiltonian-cycle and subset-sum problems.

Finally, virtually every section has been edited to correct, simplify, and clarify explanations and proofs.

### Web site

Another change from the first edition is that this book now has its own web site: <http://mitpress.mit.edu/algorithms/>. You can use the web site to report errors, obtain a list of known errors, or make suggestions; we would like to hear from you. We particularly welcome ideas for new exercises and problems, but please include solutions.

We regret that we cannot personally respond to all comments.

## Acknowledgments for the first edition

Many friends and colleagues have contributed greatly to the quality of this book. We thank all of you for your help and constructive criticisms.

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Many colleagues have used drafts of this text in courses at other schools. They have suggested numerous corrections and revisions. We particularly wish to thank Richard Beigel, Andrew Goldberg, Joan Lucas, Mark Overmars, Alan Sherman, and Diane Souvaine.

Many teaching assistants in our courses have made significant contributions to the development of this material. We especially thank Alan Baratz, Bonnie Berger, Aditi Dhagat, Burt Kaliski, Arthur Lent, Andrew Moulton, Marios Papaefthymiou, Cindy Phillips, Mark Reinhold, Phil Rogaway, Flavio Rose, Arie Rudich, Alan Sherman, Cliff Stein, Susmita Sur, Gregory Troxel, and Margaret Tuttle.

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### **Acknowledgments for the second edition**

When we asked Julie Sussman, P.P.A., to serve as a technical copyeditor for the second edition, we did not know what a good deal we were getting. In addition to copyediting the technical content, Julie enthusiastically edited our prose. It is humbling to think of how many errors Julie found in our earlier drafts, though considering how many errors she found in the first edition (after it was printed, unfortunately), it is not surprising. Moreover, Julie sacrificed her own schedule to accommodate ours—she even brought chapters with her on a trip to the Virgin Islands! Julie, we cannot thank you enough for the amazing job you did.

The work for the second edition was done while the authors were members of the Department of Computer Science at Dartmouth College and the Laboratory for Computer Science at MIT. Both were stimulating environments in which to work, and we thank our colleagues for their support.

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The second edition was produced in L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub>. Michael Downes converted the L<sup>A</sup>T<sub>E</sub>X macros from “classic” L<sup>A</sup>T<sub>E</sub>X to L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub>, and he converted the text files to use these new macros. David Jones also provided L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub> support. Figures for the second edition were produced by the authors using MacDraw Pro. As in the first edition, the index was compiled using Windex, a C program written by the authors, and the bibliography was prepared using BIB<sub>T</sub>E<sub>X</sub>. Ayorkor Mills-Tettey and Rob Leathern helped convert the figures to MacDraw Pro, and Ayorkor also checked our bibliography.

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