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无界区域中的谱方法

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Spectral Methods in Unbounded Domains

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答辩委员会对论文的评语

谱方法和拟谱方法是数值求解偏微分方程的主要工具之一， 但通常的方法只适用于有界矩形区域。因此无界区域上的谱和拟谱方法是该方向的前沿课题，不仅具有理论意义，并且有较广阔的应用前景。

本文研究无界区域上的 Laguerre 拟谱方法和 Hermite 拟谱方法以及有关的高维非线性问题的高精度算法。它涉及函数逼近论，偏微分方程理论及数值分析等多个领域。所得的结果完整且深入，工作量比较大。特别在运用 Laguerre 拟谱方法和 Hermite 拟谱方法解决无界区域上非线性 P.D.E 问题时有创新。

答辩委员会认为作者基础扎实，有较强的科研能力。本文是一篇优秀的博士论文。经过投票一致通过答辩，并建议授予博士学位。

答辩委员会表决结果

本文是一篇优秀的博士论文。经过投票，一致通过答辩，并建议授予博士学位。

答辩委员会主席：姜礼尚

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摘 要

在科学和工程(例如: 海洋工程、大气科学、矿山开采等)研究中，有许多问题的运动规律是用无界区域中的定解问题来描述的。对这类问题的求解，最简单的方法是先取定一个人工边界，然后在人工边界上加上人工边界条件，最后在相应的有界区域中用通常的方法(例如差分方法、有限元方法或者谱方法等数值方法)求解。然而，这种截断的办法必然会带来相应的误差。因此建立无界区域上的高精度算法吸引了众多数学家的关注。

本文正是在这样的背景下提出了无界区域中的谱方法。这种解法是与无界区域中的正交多项式密切相关的。我们的研究由以下几部分组成:

第二章，建立了 Hermite 多项式插值逼近。作为一个应用的例子，我们讨论了直线上的 Burgers 方程的数值解。证明了该算法是稳定的和收敛的，并具有谱精度。数值例子验证了该算法的高精度性。

第三章，建立了 Laguerre 多项式插值逼近。并将它应用到半直线上的 BBM 方程上去，证明了该算法的稳定性和收敛性。数值例子同样表明了算法的高精度性。

第四章，讨论了流函数形式的 Navier-Stokes 方程在无穷带状区域中的一类始边值问题。证明了该问题的解是存在唯一的。我们还讨论了解的正则性。这些性质是我们数值求解的理论基础。

第五章，我们给出了 Laguerre-Legendre 混合谱逼近，在 x 方向用 Laguerre 谱逼近，在 y 方向用 Legendre 谱逼近。我们还建立了流函数形式的 Navier-Stokes 方程在无穷带状区域中的一类始边值问题的谱格式，证明了该格式的稳定性和收敛性。数值例子表明算法是有效的。

第六章中，我们给出了 Laguerre-Legendre 混合拟谱逼近。并将它应用到流函数形式的 Navier-Stokes 方程在无穷带状区域中的一类始边值问题中。与相应的混合谱方法相比较，混合拟谱方法在实际计算中更有效，更节省计算时间。因为它避免了计算无穷区域中积分。当然，其理论分析更困难。

本文的主要方法与技巧对无界区域中的其他偏微分方程同样适用。

关键词 谱与拟谱逼近，无界区域，不可压缩流，非线性偏微分方程，收敛性和稳定性

Abstract

Many problems in science and engineering are set in unbounded domains. The simplest method to deal with them is to set some artificial boundaries, impose certain artificial boundary conditions and then resolve them numerically. Whereas these treatments may cause additional errors. The main purpose of this work is to develop the spectral methods associated with some orthogonal systems of polynomials in unbounded domains. We start by the Hermite polynomials interpolation approximation. As an example, we apply it to the Burgers equation on the whole line. The stability and the spectral accuracy of the proposed scheme are proved. The numerical results show the high accuracy of this approach. We derive the Laguerre interpolation approximation. The pseudospectral scheme for the BBM equation on the half line is discussed. We also give a theoretical result for the stream function form of the Navier-Stokes equation in unbounded domains. The existence, uniqueness and the regularility are studied. Its mixed Laguerre-Legendre spectral scheme and mixed Laguerre-Legendre pseudospectral scheme are also constructed. We prove the stability and convergence of the proposed schemes. The numerical results show the efficiency of these approach. The main idea and techniques used in this work are also applicable to other nonlinear partial differential equations in unbounded domains.

Key words spectral and pseudospectral approximation, unbounded domain, incompressible fluid flow, nonlinear partial differential equation, convergence and stability

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Chapter 1 Introduction

1.1 History and Motivation

Recently, more and more attentions are paid to partial differential equations in unbounded domains and their numerical simulations, see, e.g., Feng[1], Guo[2], Kweon and Kellogg[3], Maday, Pernaud-Thomas and Vandeven[4] and Givoli[5].

The simplest method for solving partial differential equations in unbounded domains numerically is to set up some artificial boundaries, impose certain artificial boundary conditions, and then resolve the corresponding approximation problems in bounded domains, by finite difference method, finite element method or spectral method. For instance, Clayton and Engquist[6], Engquist and Majda[7–8] derived a hierarchy of differential boundary conditions for the wave equations based on the pseudodifferential operator theory, which were used to eliminate the reflection of waves at artificial boundaries. So the obtained solutions approximately simulate the exact solutions in the unbounded domains. Unfortunately, the higher the order of differential boundary conditions, the worse the stability of computation.

Another way is to consider exact boundary conditions at artificial boundaries, and then resolve the corresponding problems. Hagstrom and Keller[9-10] studied the exact boundary conditions for partial differential equations in cylinders. Han and Wu[11] also found some exact boundary conditions at an artificial boundary for the Laplace equation. Moreover, a sequence of approximations to the exact boundary condition at the artificial boundary was given. The same technique was used to the linear elastic equations in unbounded domain, see [12]. Whereas the above methods depend on the considered partial differential equations. So it is difficult to apply them to general nonlinear partial differential equations.

Spectral method is a powerful tool for numerical simulation of partial differential equations. It has been successfully used for numerical simulations in many fields, such as fluid dynamics, quantum mechanics, numerical weather prediction and so on. See, e.g., [13-16]). But only the problems in bounded domains considered usually. Spectral approximation in unbounded domains have received limited attention. Canuto, Hariharan and Lustman[17] dealt with the approximation to an exterior elliptic problem in two dimensions by imposing an appropriate condition at the artificial boundary. However, it causes additional errors. A reasonable way is to approximate the problems in unbounded domains by certain mutually orthogonal systems in unbounded domains. Maday, Pernaud-Thomas and Vandeven[4], Coulaud, Funaro and Kavian[18], and Funaro[19] used the Laguerre spectral method for some linear problems. Funaro and Kavian[20] considered some algorithms by using Hermite functions. But there is only few theoretical result available on spectral approximation in unbounded domains. Recently Guo[2] developed the spectral method by using Hermite polynomials and its application to a nonlinear partial differential equation. Guo and Shen[21] derived Laguerre spectral method for some nonlinear partial differential equations. In particular, they proved the stability and the convergence of the proposed schemes. The numerical results show the high accuracy of the spectral approximations.

The main aim of this work is to develop the pseudospectral methods in unbounded domains. In fact, we need some quadratures over unbounded domains in spectral method. But we only have to calculate the value of unknown function at the interpolation points in pseudospectral methods, and so save work. In particular, it is easier to deal with the nonlinear terms in the pseudospectral methods. Therefore it is more preferable in actual calculations. To do this, we first study the Hermite interpolation approximation and the Laguerre interpolation approximation, respectively. Then we use them to construct the corresponding schemes for some partial differential equations and analyze the error. The numerical results show the efficiency of these approximations.

As is well known, most of practical problems are set up in multiple-dimensional spaces, such as an infinite tube. In this case, we need mixed approximation in unbounded domains. So we establish the mixed Laguerre-Legendre spectral approximation and the mixed Laguerre-Legendre pseudospectral approximation. In the second part of this work, We also use them to construct the spectral and pseudospectral schemes for some model problems arising in fluid dynamics, to analyze the stability and the convergence of the proposed schemes, and to present some numerical results showing the efficiency of the proposed schemes.

1.2 Outline of the Work

In chapter 2, we establish Hermite pseudospectral approximation in one and multiple-dimensional spaces. As an application, we construct the Hermite pseudospectral scheme for the Burgers equation on the whole line. Its stability and convergence are also proved. The presented numerical results

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