中国科学院工程热物理研究所所长基金项目

吴仲华论文选集

《吴仲华论文选集》编辑委员会

机械工业出版社
CHINA MACHINE PRESS

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机 械 工 业 出 版 社

本书汇集了国际著名学者吴仲华教授一生所发表的最著名的论著。其中主要内容是以他名字命名的吴氏叶轮机械三元流动理论。这个理论是半个世纪以前创立的，但现在全世界所有有关叶轮机械的工厂仍然按其思想体系进行常规的实用设计与计算。多年来，国际上研究叶轮机械流动的主要学术论文基本没有不引用吴仲华教授的论著作为主要参考文献的。读者从这些论著可以真正直接学习到吴氏叶轮机械三元流动理论的精萃与吴仲华教授的严格治学方法，学习到他如何由简入繁、化繁为简，从径向平衡开始考虑叶轮机械三元流动到把三元流动简化为物理概念清晰的两个流面流动(包括把二元的叶栅流动计算简化为两个一元的计算)，后来又利用非正交曲线座标及其分速度来适应叶轮机械的特点以提高计算的精度和速度。吴仲华教授所创建的燃气热力性质表及对气动热力学基本理论的严格论述曾对工程热物理界起了很大的影响，本书收集了他在这方面的原始著作。吴仲华教授晚年还以他的深厚学识对我国能源动力工业提出发展方向，并直接领导发展总能系统的理论与实际。本书收录了这方面的主要文献。

机械工业出版社(北京市百万庄大街22号　邮政编码100037)
责任编辑：李正民　版式设计：霍永明
封面设计：姚　毅　责任印制：付方敏
北京铭成印刷有限公司印刷·新华书店北京发行所发行
2002年10月第1版第1次印刷
787mm×1092mm 1/16·34.75印张·4插页·864千字
定价：145.00元

吴仲华教授(1917～1992)

吴仲华教授生平

吴仲华教授1917年生于上海，原籍江苏省苏州市。1941年毕业于昆明国立西南联大机械系，并留校任教。1943年底考取清华大学公费留学，进美国麻省理工学院，1947年获博士学位。1947年至1951年在美国航空顾问委员会路易士喷气推进中心任研究科学家，1951年至1954年任布鲁克林大学机械系教授。1954年冬，吴仲华教授和夫人李敏华教授取道欧洲回国，以赤诚的报国之心，投身祖国的建设事业。从1955年到1961年任清华大学动力工程系教授、副主任，并创建了燃气轮机专业，任燃气轮机教研组主任。同时，于1955年创建中国科学院动力研究室，任研究室主任。1957年，当选为中国科学院学部委员(院士)。1958年在中国科学技术大学建校时，创建了工程热物理专业。1960年，中国科学院动力研究室并入中国科学院力学研究所，吴仲华教授任副所长。1978年，创建中国工程热物理学会，担任理事长。1980年，创建中国科学院工程热物理研究所，任所长。同年创办了《工程热物理学报》，任主编。1981年，被选为中国科学院主席团执行主席。1987年，任中国科学院工程热物理研究所名誉所长。1992年9月，因患癌症医治无效在北京逝世，享年76岁。

吴仲华教授根据科学发展的规律和国民经济、国防建设的需求，在我国创建了工程热物理学科，对我国工程热物理事业的发展和科学技术的进步发挥了重大的作用。

吴仲华教授在科学技术上贡献卓著。从1949年起，他发表了一系列重要论文，提出了“径向平衡”、“通流理论”等。1952年，发表了著名论文“轴流、径流和混流式亚声速与超声速叶轮机械中三元流动的普遍理论”，创造性地建立了叶轮机械三元流动理论，得到了国际学术、工程技术界的一致公认，称其为“吴氏通用理论”，其主要方程被称为“吴氏方程”。上世纪50年代由于计算机能力的限制，工程界将早期的“径向平衡”应用于工程实践设计。随着计算机的发展，三元流动理论在国际上逐步全面地应用于先进叶轮机械的设计中，使叶轮机械的性能大幅度提高。60年代中期，他提出了使用任意非正交曲线坐标

与相应的非正交速度分量的叶轮机械三元流动基本方程组，将这理论提高到了新的高度。在这基础上，他领导研究发展了一整套亚、跨、超声速计算方法与计算机程序，已在国内广泛推广应用，并得到了实验验证，在工程实际中发挥了巨大作用。至今，叶轮机械三元流动理论仍是当代先进叶轮机械设计分析的理论基础和有力工具，在国内外航空发动机和其他叶轮机械的研制中不断发挥着重要作用。吴仲华教授在工程热力学和能源科学方面也进行了大量开创性的工作，在国内外有很大的影响。

吴仲华教授对我国能源事业和能源科学的发展倾注了大量心血。1980 年，吴仲华教授组织领导有关专家，深入调查我国的能源情况，提出了解决我国能源问题的战略构想。他亲自为中央书记处作了题为“中国的能源问题及其依靠科学技术解决的途径”的报告。吴仲华教授在提高我国能源利用水平、发展我国联合循环等总能系统的基础理论研究与实践都有重大贡献。

吴仲华教授学识渊博、学风严谨、学术上卓有成就，对祖国无限热爱。上世纪 50 年代初，他满怀爱国激情，聆听新中国代表在联合国发言的往事，一直被传为我国外交史上的一段佳话。随后，在 1954 年，他和夫人毅然放弃国外已获得的地位和优裕的生活、工作条件，冲破各种困难，藉休假的机会，取道欧洲，返回祖国，决心把自己的科学才智贡献给新中国的社会主义事业。

吴仲华教授一贯追求真理。回国后他多次受到毛主席、周总理的接见，受到极大鼓舞。就是在“文化大革命”中，自身处境极为困难的情况下，他仍对真理坚信不疑，对事业执着追求，克服重重困难，坚持不懈地从事科研工作。1980 年参加中国共产党后，更加严格要求自己。

吴仲华教授有着强烈的事业心，他服从党和国家的需要，以高度的献身精神致力于我国工程热物理学科的创建、发展、提高和广泛的国际学术交流。吴仲华教授严以律己，坚持原则，生活朴素，克己奉公。他几十年如一日，坚持不懈，艰辛开拓，精心组织，认真勤恳地工作。他那强烈的事业心感染与熏陶一大批中青年科技人员，并在国内培育了一批工程热物理的专家、学者，其中不少已成为这一学科的学术带头人。

《吴仲华论文选集》编辑委员会

编者的话

吴仲华教授是世界著名的工程热物理学家，叶轮机械三元流动通用理论的创始人。作为一名杰出的科学家，他为人类科学技术的进步，为工程热物理学科的建立和发展，为祖国的经济、文化与国防建设奉献了毕生的精力，作出了巨大的贡献。他的学术成果与创新精神、科学思想和研究方法都是影响深远的，是值得后人学习继承和发扬光大的。在三元流动通用理论发表50周年之际，我们选编、出版了这本《吴仲华论文选集》。希望本书不仅成为一份历史的纪录，而且可以作为相关学科的学生和研究人员的宝贵资料，为在当今中国弘扬科学精神，促进科技事业的发展发挥重要的作用。

吴仲华教授一生著有论文多篇，是他对科学贡献的重要组成部分。我们从中精选了具有代表性的17篇，力图在有限的篇幅中全面反映他在各个时期、各个方面的工作及对学科发展史的重大影响。

自1949年至50年代初，吴仲华教授在美国相继发表“径向平衡”、“通流理论”等多项研究成果，一步步推进了叶轮机械设计理论由简单向精确、由低级向高级的发展，并终于在此基础上产生飞跃，提出了全面反映气体运动参量的三维变化，使用流片形状任意和厚度变化的 S_1 和 S_2 两类流面进行计算的完整理论。本书的第1~4篇论文选自那个时期。第1篇的内容为“径向平衡”，它最后发展成沿 S_2 流面流动。第2篇的内容为平面叶栅的中心流线法求解，是早年唯一计算叶栅可压流的实际有效办法。第3篇就是发表了著名叶轮机械三无流动通用理论的里程碑式的文献。在推导动叶栅中相对流动方程时发现、提出的相应于绝对流动中滞止焓的重要参量，稍后在第4篇(对一篇文章讨论的短文)中被命名为(相对)滞止转子焓，rothalpy (转子焓)一词于此诞生。

第6、7两篇论文反映了吴仲华教授回国后继续从事这方面研究的情况。前者是关于中心流线法的发展，把第2篇文章的思想与方法从平面叶栅推广到任意回转面上去。后者对叶轮机械气动设计的理论和方法做了全面的阐述和讨论。

随着计算机应用的发展，吴仲华教授一系列成果的意义越来越被人们所认识。在他所开拓的领域，应用和研究非常活跃，采用他的方法及在他的影响下，一批计算结果被得出，一批先进航空发动机被设计成功。从第10篇论文可以了解到20多年间国内外这方面的进展情况。

吴仲华教授在60年代就率先将贴体正交曲线网格引入叶轮机械内部流动计算。他将张量这一数学工具与三元流动通用理论的实际完美结合，推导出了使用非正交曲线坐标和非正交速度分量的 S_1 和 S_2 流面上的基本方程，达到了令人赞叹的新高度。若干年后在国际会议上发表本书所选的第11篇论文时引起了会场的轰动。

在中国科学院工程热物理研究所，吴仲华教授亲自领导发展了一整套叶轮机械内部亚、跨、超声速流动计算方法，编制了设计问题和分析问题的计算机程序，付诸使用，并用实验测量结果加以验证。在中国举行的第2届国际空气发动机会议上，吴仲华教授首次以本书所选的第13篇论文作了大会报告。

在第9篇论文中，吴仲华教授通过对方程的严格推导，清楚地阐述了各种情况下气流中粘性力的作功问题，指出了在一些教科书和科研报告中已出现的概念上的混淆(比如"磨擦生热")、推导上的不严格等情况以及因此得出的错误结果。吴仲华教授一贯强调在工程技术科学领域也须极为重视理论的严密、基本概念的正确，此文是这一思想的体现在工程技术界产生了很大影响。

本书中最后一篇是由美国 NASA Lewis Research Center 出资、Clemson 大学邀请，吴仲华教授在身患癌症的情况下，倾注最后的心血完成的长篇专著。这位著名的工程热物理学家，大型工程科学计算的先驱者、实践者，因叶轮机械方面的卓越成就享有盛誉，其影响经久不衰。国际吸气式发动机大会在每两年一次的年会上设立了永久性的"吴仲华讲座"，以纪念吴仲华教授在这一领域的突出贡献。

第5、第8两篇论文反映了吴仲华教授为发展我国燃气轮机和冲压式喷气推进系统的循环分析和设计计算所做的应用性和前瞻性研究。研究空气和不同燃料系数的燃烧产物(燃气)的精确的热力性质，以及推进系统在超声速和高超声速膨胀和激波压缩过程中的变比热特性，为以后的工作奠定了坚实的基础。

吴仲华教授对我国能源动力事业和能源科学的发展倾注了大量心血，在能源科学技术的基础理论与实践方面都有重大贡献。20世纪80年代发表的4篇文章(本书第12、14、15、16篇)，反映了他在这方面的建树与影响。第12篇是拨乱反正后他在中央书记处为全体中央领导同志举办的一次讲座上的报告(后来还应中共中央党校邀请讲课,并在1982年中美能源与环境学术会议上以英文发表)。报告从科学技术角度提出解决我国能源问题的战略构思，对我国能源动力发展的相关决策一直起着指导作用。第14篇是他在1981年全国能源标准化工作会议上的讲话摘要，对我国能源标准与能源法规的制定提出完整的指导性意见。第15篇为1981年在国家机械工业委员会组织的讨论发展和应用燃气轮机问题的座谈会上的报告(摘要)，报告全面阐述了燃气轮机发展的趋势与重要性以及我国策略，时至今日，吴仲华教授的意见仍然具有现实指导意义。在第16篇文章中，吴仲华教授从科学基本原理出发，提出能源利用必须"按照能量品位的高低进行梯级利用"的总能系统概念，它已成为能源科学发展的主流思想，对能源利用，特别是燃气轮机的发展应用产生了巨大而深远的影响。

鉴于是历史的纪录，我们在编辑时尽量保持了论文的原貌，除了对原件中明显的笔误和印刷错误进行订正外，仅作了必要的技术处理(如全书格式、字体的统一等)。由于历史的原因，原文中采用的符号、计量单位有一些与现行标准规定的不一致，但都是本学科曾习惯采用的，且符号的含义多在文内有所说明，尚不至于影响阅读、理解，故本着尊重历史的原则，不一一加以修改了，这一点在此特别说明。

2002年6月28日

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1

APPLICATION OF RADIAL-EQUILIBRIUM CONDITION TO AXIAL-FLOW COMPRESSOR AND TURBINE DESIGN *

SUMMARY

Basic general equations governing the three-dimensional compressible flow of gas through a compressor or turbine are given in terms of total enthalpy, entropy, and velocity components of the gas. Two methods of solution are obtained for the simplified, steady axially symmetric flow; one involves the use of a number of successive planes normal to the axis of the machine and short distances apart, and the other involves only three stations for a stage in which an appropriate radial-flow path is used. Methods of calculation for the limiting cases of zero and infinite blade aspect ratios and an approximate method of calculation for finite blade aspect ratio are also given. In these methods, the blade loading and the shape of the annular passage wall may be arbitrarily specified.

The analysis shows that the radial motion of gas consists of a gradual, generally monotone component due to the taper in the passage wall, and an oscillatory component due to the radial variation of the specific mass flow at different stations along the axis of the machine specified in the design. The streamline is curved by this radial flow and a corresponding radial pressure gradient is required to maintain this curvature. The magnitude of this gradient is increased with high Mach number of gas flow and high aspect ratio of blade row. The conventional method of calculation, in which the effect of radial motion on the radial distribution of gas state is neglected, is found to be applicable only for the limiting case of zero aspect ratio.

An analysis of the equations governing the flow shows that a designer is free to prescribe a reasonable radial variation of one of the velocity components or other thermodynamic properties of the gas at any station within the blade region. The various ways of using this degree of freedom and the different types of design obtained are discussed. Numerical computations are then made for two types of compressor and one type of turbine. The results indicate that, even in the case of nontapered passage walls, appreciable radial motion occurs and the corresponding effects are of significant magnitude and should be considered in design.

INTRODUCTION

The design of a compressor or a turbine (either of which is referred to hereinafter as "a turboma-

* 原文为 NACA Report 955, 1950，合作者有 Wolfenstein，L。

chine") may be divided into two phases. The first phase concerns the type of design to be used, or the determination of the most desirable possible variations of velocity and thermodynamic properties of the gas in planes normal to the axis of the machine between successive blade rows. The second phase concerns the design of blades that will give the desired variations of velocity and other properties of gas in these planes. In the first phase, the condition of radial equilibirum (that is, the radial component of the equation of motion) must be used. The flow of gas in a turbomachine is curvilinear; it is curved not only by the whirling motion of gas, but also by the radial motion of the gas (reference 1). The equation of motion then specifies the radial pressure gradient required to provide the centripetal force to maintain the curved flow.

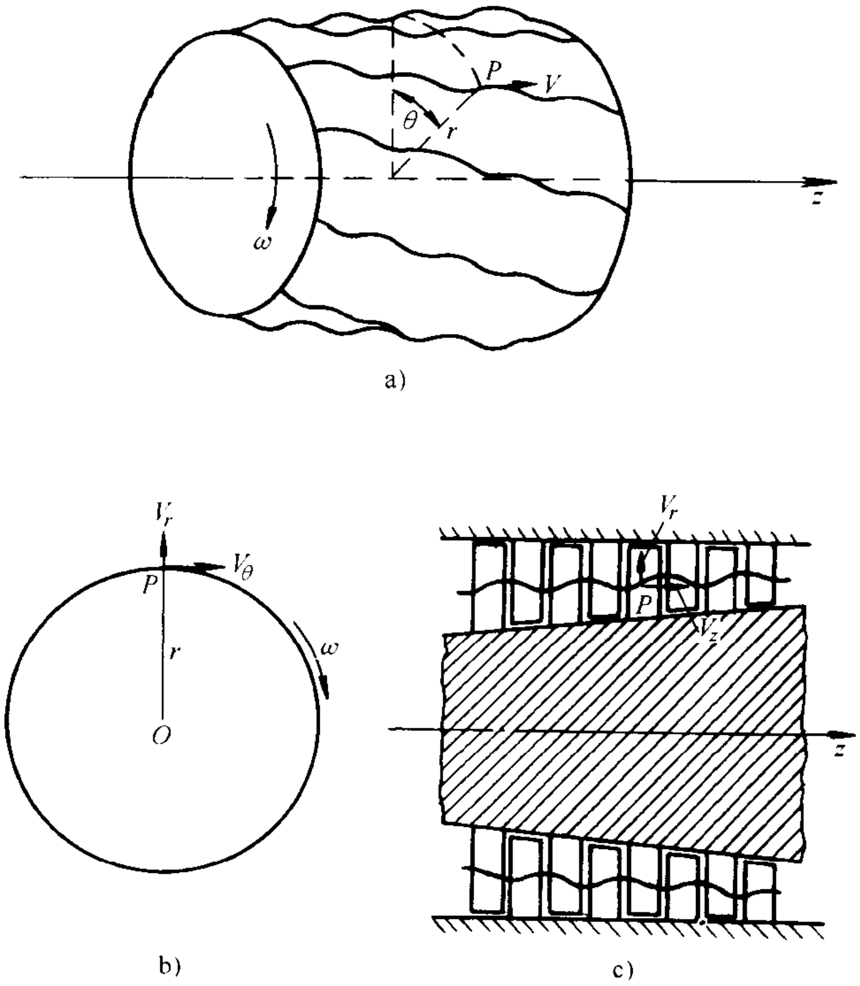

FIGURE 1 —Stream surface over four similar stages of multistage turbomachine and intersectiou of stream surface with planes normal to and containing axis of machine.

(a) Stream surface over four stages of multistage turbomachine.

(b) Intersection of stream surface with plane normal to axis.

(c) Intersection of stream surface with axial plane.

In figure 1 (a), a curved stream surface over four similar stages of a multistage turbomachine is shown and figures 1 (b)and 1 (c)show the intersections of this stream surface with planes normal to and containing the axis of the machine, respectively. The radial pressure gradient due to the whirling motion of gas is always positive; whereas that due to the radial motion of gas may be either positive or negative, depending on whether the curvature caused by the motion is inward or outward from the axis of the machine at the point of consideration. Even when the radial motion involved is small, if the gas velocity is high and the blade aspect ratio is large, the radial pressure gradient due to the radial motion is of significant magnitude compared with that due to the whirling motion of gas and should be included in the design calculation.

In the calculation of the state of gas in the normal planes far upstream and downstream of a single row of blades, where the radial motion is small, the pressure gradient is essentially due to the whirling motion alone. Experimental measurement checked well with the calculation when only the whirling motion was considered (references 1 to 3). For the general case of the gas in the normal planes between closely spaced successive blade rows, however, no satisfactory theory exists to calculate the magnitude of the radial displacement of the streamlines and its effect on the radial distribution of the state of the gas. A preliminary theoretical investigation of this problem conducted at the NACA Lewis laboratory was completed in April 1948 and is presented herein.

In the analysis, the general equations governing the three-dimensional flow of gas in turbomachines are expressed in terms of total enthalpy, entropy, and velocity components of the gas. They are developed primarily for the case of steady axially symmetric flow corresponding to the limiting case of an infinite number of blades. Two numerical methods of solution are presented; one uses a number of successive stations through the turbomachine, the other uses only three stations for a stage in which an appropriate radial-flow path is employed.

Methods of solution for the limiting cases of very small and very large blade aspect ratio are then discussed. An approximate solution of the radial displacement across a blade row having a finite aspect ratio is given for the general case in which the whirling velocity of gas is prescribed in design.

The basic equations obtained are also used to investigate the maximum compatible number of radial variations of the velocity components and other thermodynamic properties of gas that a designer is free to specify. It is found that the designer can specify only one such variation at each station along the axis of the machine within the blade region. Various ways of specifying this variation and the different types of design obtained are discussed.

The methods developed are applied to two types of compressor and one type of turbine, in order to investigate the magnitude of the radial motion and its effect on design calculations.

FORMULATION OF EQUATIONS

GENERAL BASIC EQUATIONS

The three-dimensional compressible flow of gas through a turbomachine is governed by the following set of general basic equations (references 4 to 6):

From the principle of conservation of matter, the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0 \tag{1}$$

(Symbols used in this report are defined in appendix A.) The principle of conservation of momentum is expressed by the Navier-Stokes equation as

$$\rho \frac{\mathrm{D} \boldsymbol{V}}{\mathrm{D} t} = \rho \boldsymbol{F} - \nabla p + \mu \nabla^2 \boldsymbol{V} + \frac{\mu}{3} \nabla (\nabla \cdot \boldsymbol{V}) + 2 [(\nabla \mu) \cdot \nabla] \boldsymbol{V} +$$

$$(\nabla \mu) \times (\nabla \times \boldsymbol{V}) - \frac{2}{3}(\nabla \cdot \boldsymbol{V})(\nabla \mu) \tag{2}$$

where $\boldsymbol{F}$ is the external force exerted on unit mass of gas. The principle of conservation of energy may be written as

$$\frac{\mathrm{D}u}{\mathrm{D}t} + p\frac{\mathrm{D}(\rho^{-1})}{\mathrm{D}t} = Q + \frac{\Phi}{\rho} \tag{3}$$

where u is related to T by

$$\frac{\mathrm{D}u}{\mathrm{D}t} = c_v \frac{\mathrm{D}T}{\mathrm{D}t} \tag{4}$$

when conduction only is considered, Q is given by

$$Q = \rho^{-1}\nabla \cdot (k \nabla T) \tag{5}$$

and Φ is the dissipation function given by

$$\Phi = \mu\left\{2\nabla \cdot [(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}] + (\nabla \times \boldsymbol{V})^2 - 2(\boldsymbol{V}\cdot\nabla)(\nabla \cdot \boldsymbol{V}) - \frac{2}{3}(\nabla \cdot \boldsymbol{V})^2\right\} \tag{6}$$

For the range of gas temperature and pressure usually encountered in turbomachines, p, ρ, and T are accurately related by the following equation of state:

$$p = R\rho T \tag{7}$$

Theoretically, the preceding seven equations, together with the given body force, known variations of c_v, μ, and k with temperature, and suitable boundary and initial conditions, completely determine the flow of gas through the turbomachine. It is found convenient in the present investigation, however, to base the calculation on total enthalpy and entropy, which are defined by

$$H = h + \frac{1}{2}V^2 \tag{8}$$

where

$$h = u + p\rho^{-1} \tag{9}$$

and

$$T\,\mathrm{d}s = \mathrm{d}u + p\,\mathrm{d}(\rho^{-1}) \tag{10}$$

By use of equations (8) to (10), the following forms of continuty, motion, and energy equations are obtained (appendix B):

$$\nabla \cdot \boldsymbol{V} + \frac{1}{\gamma - 1}\frac{\mathrm{D}}{\mathrm{D}t}\log_e T - \frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{s}{R}\right) = 0 \tag{1a}$$

$$\begin{aligned}\nabla H = {} & \boldsymbol{F} + T\nabla s + \boldsymbol{V} \times (\nabla \times \boldsymbol{V}) - \frac{\partial \boldsymbol{V}}{\partial t} + \frac{\mu}{\rho}\left[\nabla^2 \boldsymbol{V} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{V})\right] + \\ & \frac{1}{\rho}\left\{2[(\nabla \mu)\cdot\nabla]\boldsymbol{V} + (\nabla \mu) \times (\nabla \times \boldsymbol{V}) - \frac{2}{3}(\nabla \cdot \boldsymbol{V})(\nabla \mu)\right\}\end{aligned} \tag{2a}$$

$$\begin{aligned}\frac{\mathrm{D}H}{\mathrm{D}t} = {} & Q + \frac{\Phi}{\rho} + \frac{1}{\rho}\frac{\partial p}{\partial t} + \boldsymbol{V}\cdot\left(\boldsymbol{F} + \frac{\mu}{\rho}\left[\nabla^2 \boldsymbol{V} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{V})\right] + \right. \\ & \left.\frac{1}{\rho}\left\{2[(\nabla \mu)\cdot\nabla]\boldsymbol{V} + (\nabla \mu) \times (\nabla \times \boldsymbol{V}) - \frac{2}{3}(\nabla \cdot \boldsymbol{V})(\nabla \mu)\right\}\right)\end{aligned} \tag{3a}$$

$$\frac{\mathrm{D}s}{\mathrm{D}t} = \frac{Q}{T} + R\frac{\Phi}{p} \tag{3b}$$

Equation (1a) gives the continuity relation in terms of velocity, temperature, and entropy of gas. Equation (2a) relates the gradient of total enthalpy with body force, viscous forces, velocity, and other propertics of the gas. This vector equation gives three scalar equations in three dimensions. Equation (3a) gives the rate of change of total enthalpy of gas along a streamline in terms of rate of heat additions, rate of work done by body and viscous forces, and so forth. Equation (3b) gives the rate of change of entropy along a streamline in terms of rate of heat conduction and of dissipation of energy due to viscosity.

STEADY AXIALLY SYMMETRIC FLOW

The solution of the preceding general equations with a given set of suitable boundary and initial conditions is extremely difficult. Useful results may be obtained by considering, as first done by Lorenz in hydraulic-machine theory (references 7 and 8), the limiting case of an infinite number of infinitesimally thin blades. In this simplification, the force exerted on the gas by a blade element at any radius is considered to be uniformly distributed over the stream sheet between two neighboring blades at that radius, and is considered the body force $\boldsymbol{F}$ in the previous equations. For incompressible and frictionless flow, the value thus obtained gives an average value in the circumferential direction, provided the departure from the average value is small (reference 1). Because the number of blades is usually large, this simplification is considered to be reasonable and is also used in the present investigation. For steady inlet and exit conditions, all partial derivatives wjth respect to angular coordinate θ and time t are then equal to zero and the state of gas is a function of r and z only.

The ideal case of a nonviscous gas will be considered first. In this case, there exist two more relations defining the problem. One is the fact that blade force is normal to the surface of the blade and, consequently, to the relative velocity of gas or the relative stream surface; that is,

$$\boldsymbol{F}\cdot(\boldsymbol{V}-\boldsymbol{U})=0 \tag{11}$$

or, referring to absolute cylindrical coordinates r, θ, z and the relative angular coordinate χ,

$$F_r dr + rF_\theta d\chi + F_z dz = 0 \tag{11a}$$

The other is the condition of integrability of the blade surface,

$$\boldsymbol{F}\cdot(\nabla\times\boldsymbol{F})=0 \tag{12}$$

which in the case of axial symmetry reduces to (references 8 and 9)

$$\frac{\partial}{\partial r}\left(\frac{F_z}{rF_\theta}\right)=\frac{\partial}{\partial z}\left(\frac{F_r}{rF_\theta}\right) \tag{12a}$$

From the general equations (1a), (2a), (3a), and (3b), and equations (11) and (12a), the following equations are obtained for steady axially symmetric flow of nonviscous gas: (See appendix B.)

$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r}+\frac{\partial V_z}{\partial z}+\frac{1}{\gamma-1}\left(V_r\frac{\partial}{\partial r}\log_e T+V_z\frac{\partial}{\partial z}\log_e T\right)-\left[V_r\frac{\partial}{\partial r}\left(\frac{s}{R}\right)+V_z\frac{\partial}{\partial z}\left(\frac{s}{R}\right)\right]=0 \tag{13}$$

$$\frac{\partial H}{\partial r}=F_r+T\frac{\partial s}{\partial r}+\frac{V_\theta}{r}\frac{\partial(rV_\theta)}{\partial r}+V_z\left(\frac{\partial V_z}{\partial r}-\frac{\partial V_r}{\partial z}\right) \tag{14}$$

$$0=F_\theta-\frac{1}{r}\left[V_r\frac{\partial(rV_\theta)}{\partial r}+V_z\frac{\partial(rV_\theta)}{\partial z}\right] \tag{15}$$

$$\frac{\partial H}{\partial z}=F_z+T\frac{\partial s}{\partial z}+\frac{V_\theta}{r}\frac{\partial(rV_\theta)}{\partial z}-V_r\left(\frac{\partial V_z}{\partial r}-\frac{\partial V_r}{\partial z}\right) \tag{16}$$

$$\frac{\mathrm{D}s}{\mathrm{D}t}=\frac{Q}{T} \tag{17}$$

$$\frac{\mathrm{D}H}{\mathrm{D}t}=Q+\omega\frac{\mathrm{D}(rV_\theta)}{\mathrm{D}t} \tag{18}$$

$$\frac{\partial}{\partial r}\left(\frac{F_z}{rF_\theta}\right)=\frac{\partial}{\partial z}\left(\frac{F_r}{rF_\theta}\right) \tag{12a}$$

In the preceding equations, equation (13) is the continuity equation; equations (14), (15), and (16) are the three equations of motion in the radial, circumferential, and axial directions, respectively. Equation (17) is considered to represent the energy equation and equation (18) to represent equation (11). In these equations, Q is now the heat transfer from the blade to the gas, uniformly distributed in the circumferential direction, as is the blade force $\boldsymbol{F}$. These seven equations are considered seven independent equations that relate the eight unknown variables, which consist of three blade-force components, three velocity components, and H and s of the gas. The first three quantities determine the shape of the blade and the last five quantities completely determine the state of the gas (all other thermodynamic properties of gas, such as p, ρ, and T, can be computed from them by using equations (7) to (10)).

For compressors and turbines without blade cooling, the heat transfer between blade and gas is negligible; the entropy of gas is then constant along any streamline according to the energy equation (17). If the inlet air has a uniform value of entropy, the radial and axial derivatives of entropy in the preceding equations equal zero.

In the case of real gas, the axially symmetric simplified forms of the viscous terms in equations (1a), (2a), (3a), and (3b) can be obtained in a similar manner. These terms in the equations of motion may be neglected when compared with other terms in the same equation if the boundary layers along the passage walls are relatively thin. Because of the viscous shearing stresses in the gas adjacent to the blade, the force exerted by the blade on the gas is now slightly inclined from the direction normal to the relative velocity of gas and, consequently, equations (11) and (12) are not strictly true (the force components in the equations should be replaced by the direction cosines of the normal to the blade surface). Without using equation (11), however, equation (18) can be obtained from the equation of motion and the energy equation for steady flow with the assumption that the heat generated from the frictional work remains in each stream sheet (appendix B) and can therefore be considered as representing the energy relation in the set of equations. The entropy increase along the streamline is then computed from a consideration of the actual compression or expansion process

$$p=K\rho^n \tag{19}$$

by the formula (appendix B)

$$\frac{\mathrm{D}s}{\mathrm{D}t}=\mathrm{R}\frac{n-\gamma}{(n-1)(\gamma-1)}\left(V_r\frac{\partial}{\partial r}\log_e T+V_z\frac{\partial}{\partial z}\log_e T\right) \tag{20}$$

In equation (20), n is considered known. In a given machine, n may be directly obtained from measured pressure and temperature data. In a new bdsign, n may be obtained from the assumed polytropic efficiency used in design calculations for uncooled blades:

For compression,