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In SI Units

Mechanics of Fluids

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Buoyancy and stability

'The weight of water displaced by a ship is precisely the same as the weight of that ship.'
LEONARDO

1.1. Introduction and definition of fluid mechanics

The subject of *fluid mechanics* or the *mechanics of fluids* is nowadays generally understood to cover that branch of applied science concerned with substances which cannot preserve a shape of their own. With this definition, the term *fluid* applies equally well to liquids, vapours, and gases, but excludes such semi-solids as fats and waxes. We shall concentrate on those aspects of the mechanical behaviour of fluids which are of direct interest to engineers. Hence we shall be concerned only with those fundamental principles which are essential to a rational understanding of the behaviour of fluids in civil, mechanical, aeronautical, and marine engineering installations. To do this it will be necessary to study those characteristics of a fluid which are described and measured in terms of its mechanical properties, such as density and viscosity. It will be necessary also to know the laws which govern both its equilibrium and its motion, so that we may understand how fluid forces arise. Finally, we shall study how these forces may be usefully employed in the generation, transmission and utilization of power.

1.2. Buoyancy and the principle of Archimedes

The earliest physical principle in fluid mechanics which history has preserved for us was established by Archimedes of Syracuse (287–212 B.C.). His ability as a mathematician may be judged from the fact that his estimate of π was more accurate than the 22/7 which we so frequently use as a convenient approximation. In addition, he was an inventive engineer, and designed prodigious war machines to defend Syracuse from the Roman fleet. His studies in hydrostatics, which had a strictly practical application, led him to the principle which bears his name, i.e.

Every body experiences an upthrust equal to the weight of fluid it displaces.

This principle of Archimedes implies that we may imagine replacing the body by fluid of the same kind as that surrounding it, without disturbing the latter. The upward force which the surrounding fluid exerts, is referred to as the *force of buoyancy*, and it is this force which maintains equilibrium against the weight of the imaginary volume of fluid replacing the body. The latter is, however, concentrated at the centroid of the displaced volume (Fig. 1.2(a)) so that, if equilibrium is to be preserved, the

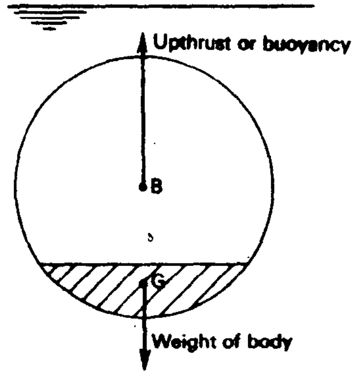

Fig. 1.2(a). *The principle of Archimedes*

upthrust must also pass through this point. **The centroid of the displaced volume is therefore referred to as the centre of buoyancy, being the point through which the buoyancy force acts.**

The centre of buoyancy (B) should not be confused with the centre of gravity (G) of the body, see Fig. 1.2(a), which depends on the weight distribution in the latter.

For example, the centre of gravity of a submersible is determined by the disposition of the weights of the various parts and gear within the hull or casing, whereas the centre of buoyancy is determined solely by the shape of the casing displacing the liquid.

It should be noted that Archimedes' Principle is equally valid for all fluids—gaseous as well as liquid. Thus, when a body is weighed a correction should strictly be made for the buoyancy due to the air. Although this is generally relatively small it is, of course, the reason why, say, a gas-filled balloon rises. Equilibrium is finally established in accordance with the buoyancy principle outlined above, i.e. the upthrust, which decreases with altitude due to the reduced density of the air, eventually becomes equal to the weight of the balloon, its contents, and that of any mooring cable.

1.3. The stability of a submerged body

A submerged body can hover in equilibrium if its weight equals its buoyancy. This condition, which is referred to as neutral buoyancy, can be approached in a submarine, e.g. by flooding tanks with water, or 'blowing' them with air, until the weight is made equal to the upthrust. As the weight may be imagined to be concentrated at G, and the buoyancy force at B (see Fig. 1.2(a)), we need to establish whether the weight will tend to hang from this virtual pivot B, or to rest upon it. Either is nominally an equilibrium position, but only the former configuration is stable. This is a perfectly general conclusion and is demonstrated in Figs. 1.3(a and b) by considering a drum with a weight fastened to its

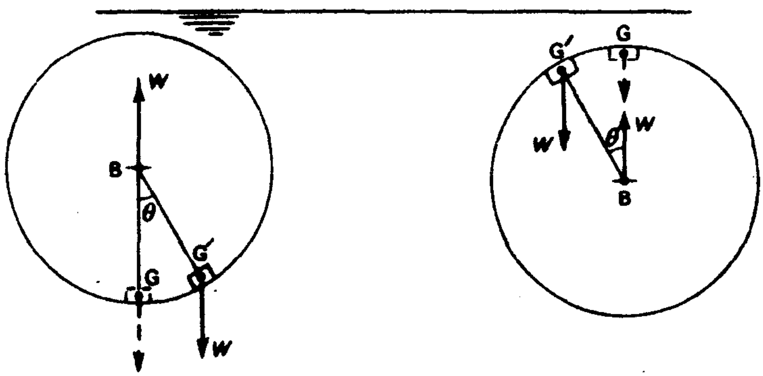

Fig. 1.3(a). *Stable* **Fig. 1.3(b).** *Unstable*

Stability of a submerged body

side as shown. The weight will naturally set itself at the lowest position, i.e. it is in stable equilibrium when G lies below B.

The test for stability is to imagine the body to be disturbed from the equilibrium position considered, G being moved to G′ in each case. In case (a) the body will experience a couple (of magnitude $W.\mathrm{BG}' \sin \theta$) tending to return it to its original configuration on being released; it is therefore said to be stable. In case (b) the body will tend to topple on being released, so that the original configuration is said to be an unstable one.

Thus, if a submerged body initially at rest be slightly displaced so that the force of buoyancy and the force of gravity acting on the body are not in the same vertical line, the body is said to be in stable or unstable equilibrium according as the resulting couple tends to bring the body back to its original position or to give it further displacement.

1.4. Density and relative density

Just as Newton's Principle of Gravitation is usually said to have been inspired by the fall of an apple from a tree in an orchard, so Archimedes'

Principle is alleged to have resulted from his absent-minded entry into a bath which was full of water. Having realized that his body was displacing a corresponding volume of water, he is said to have rushed out shouting 'Eureka!' ('I have found it!') What he had found was a way of checking the weight per unit volume (ρg) of a crown which King Hiero had ordered to be made of pure gold; he was thus able to confirm the King's suspicions concerning the honesty of the craftsman who had made it.

One most convenient way of estimating the density of a liquid (say the acid in a battery) relative to pure water is by using a hydrometer. This consists essentially of a highly stable float with a graduated stem

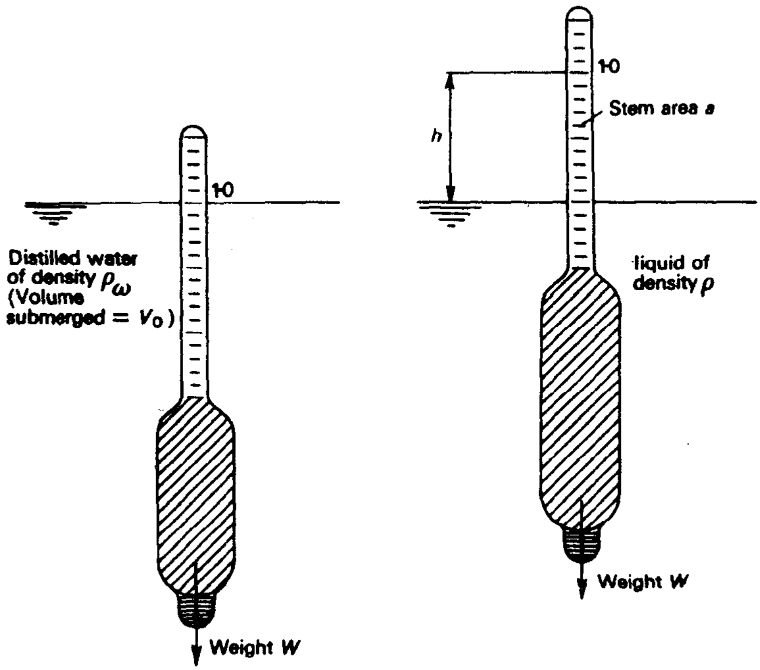

Fig. 1.4(a). *Hydrometer*

calibrated in terms of relative density as shown in Fig. 1.4(a). By Archimedes' Principle the hydrometer sets itself so that it displaces its own weight—the depth of immersion of the stem being an inverse measure of the relative density of the fluid. For accurate work a series of sensitive hydrometers is necessary, each covering only a limited range of values. For obvious reasons Customs and Excise Officers carry hydrometers when visiting breweries and distilleries.

Referring to Fig. 1.4(a), the weight W of the hydrometer remains constant. Thus, in distilled water the weight is $\rho_w g V_0$ and the position of the water level on the stem is marked 1·0 to indicate the relative density. When floated in a liquid of density ρ the weight of the hydrometer is $W = \rho g(V_0 - ah)$. Hence, $W = \rho_w g V_0 = \rho g(V_0 - ah)$

and $$h = \frac{V_0}{a}\left(1 - \frac{\rho_w}{\rho}\right)$$

from which formula the stem of the hydrometer may be graduated to show specific gravity of the liquid in which the hydrometer is floated.

EXAMPLE 1.4 (i)

What percentage of the total volume of an iceberg of density 912 kg/m³ *will extend above the surface of sea water of density* 1025 kg/m³.

Referring to Fig. 1.4(b):

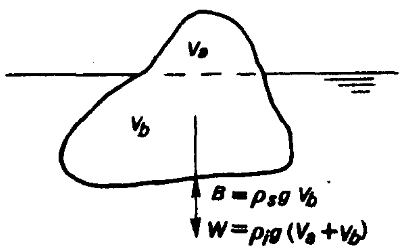

Fig. 1.4(b). *Iceberg*

$$W = B$$

$$\therefore \quad \frac{V_a}{V_b} + 1 = \frac{\rho_s}{\rho_i}$$

or $$\frac{V_a}{V_b} = \frac{1025}{912} - 1 = \frac{113}{912}$$

Hence, $$\frac{V_b}{V_a} + 1 = \frac{912}{113} + 1 = \frac{1025}{113}$$

$$= \frac{V_{total}}{V_{above}}$$

and $$\frac{V_{above}}{V_{total}} = \frac{113}{1025} \text{ or } \mathbf{11} \text{ per cent}$$

EXAMPLE 1.4 (ii)

A 'ball-cock' type of float valve is required to close when two-thirds of the volume of the spherical float is immersed in water having a density of 1 Mg/m³. *The valve has a diameter of* 12·5 mm, *and the fulcrum of the operating lever is to be* 100 mm *from the valve and* 0·45 m *from the centre of the float. Estimate the minimum diameter of the float if it is required to close the valve against a pressure of* 138 kN/m² *gauge.* $g = 9{\cdot}807$ m/s².

Referring to Fig. 1.4(c), the force required to close the valve is

$$F_A = 138 \frac{\text{kN}}{\text{m}^2} \times \frac{\pi}{4} \times 156{\cdot}3 \text{ mm}^2 = 16{\cdot}94 \text{ N}$$

Hence, the force of buoyancy required to be exerted by the float is

$$F_B = \frac{10}{45} \times 16{\cdot}94 \text{ N} = 3{\cdot}76 \text{ N}$$

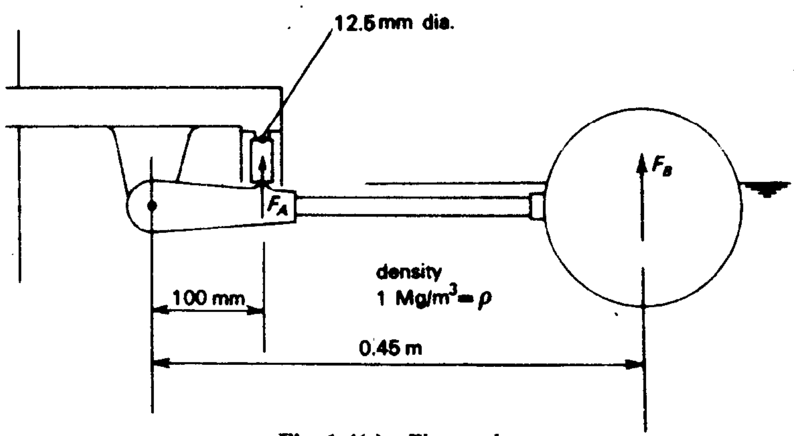

Fig. 1.4(c). *Float valve*

Also, if the volume of the sphere is denoted by V and its weight and that of the lever are neglected, then

$$F_B = \tfrac{2}{3}\rho g V = 3{\cdot}76 \text{ N}$$

i.e.

$$V = \frac{3}{2} \times \frac{3{\cdot}76 \text{ N}}{10^3 \dfrac{\text{kg}}{\text{m}^3} \times 9{\cdot}807 \dfrac{\text{m}}{\text{s}^2}\left[\dfrac{\text{Ns}^2}{\text{kg m}}\right]} = \frac{0{\cdot}577}{10^3} \text{ m}^3$$

The volume V of a sphere is related to its radius r by $V = \tfrac{4}{3}\pi r^3$, hence

$$r = \sqrt[3]{\left(\frac{3}{4} \times \frac{0{\cdot}577}{\pi} \frac{\text{m}^3}{10^3}\right)} = \frac{0{\cdot}516}{10} \text{ m}$$

and the diameter of the float is **103·2** mm.

To allow for the weight of the lever and float, and to ensure that the valve is firmly seated, a larger diameter would be used in practice.

1.5. The metacentre and metacentric height of a floating body

Archimedes' statement that a body experiences an upthrust equal to the weight of fluid it displaces is just as valid for bodies floating on the surface as for those which are submerged. It therefore embodies the principle of flotation which states that when at rest **a floating body displaces a volume of fluid equal in weight to its own.** The force of buoyancy acts, as stated in Section 1.2, through the centroid of the displaced volume, but we must note that if a floating body is heeled or pitched, its centre of buoyancy is also moved. The reason for this movement may be seen by referring to

Fig. 1.5(a) in which the heel of a ship is conveniently represented by re-drawing the water surface as RS instead of as PQ. For the ship to displace her own weight in the heeled condition, it follows that she must pivot about O, so that the buoyancy lost, owing to the heeling having rendered the weight of the wedge of fluid OPR inoperative, is balanced by the buoyancy received as a result of the displacement of the corresponding wedge of fluid OQS. If, however, the vessel has a 'flare', i.e. sloping sides, this is not quite true, but the error is generally negligible for small angles of heel.

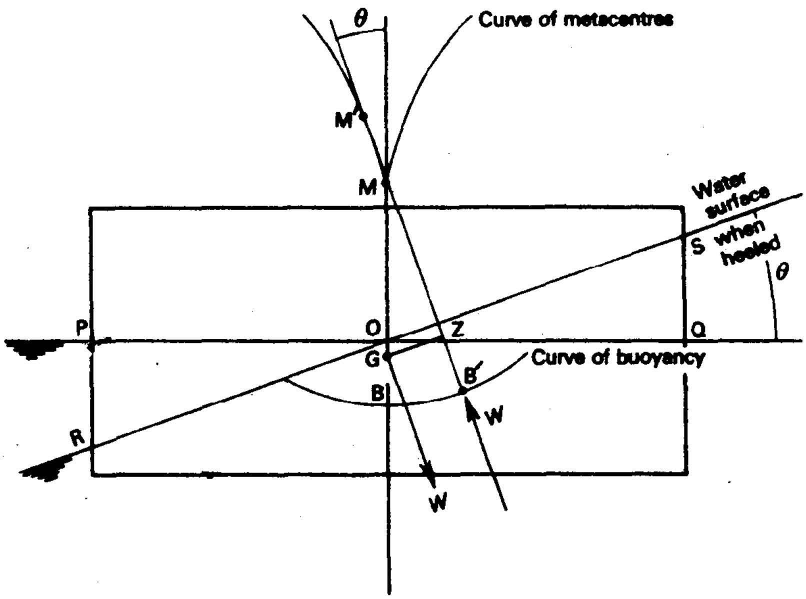

Fig. 1.5(a). *Curves of buoyancy and metacentre*

When the vessel is upright the centre of buoyancy is at B, i.e. at the centroid of the volume enclosed by the underwater shape of the vessel and the water surface PQ. When, however, the vessel is heeled, the centre of buoyancy moves to B′, the centroid of the volume enclosed by the underwater shape and the water surface RS.

The upthrust acting through B′, and the weight acting through G constitute a couple which exerts a moment equal to their magnitude, W, times the perpendicular distance, GZ, between their lines of action. The distance GZ is known as the *righting lever*, and is positive if the vessel is stable, i.e. the couple tends to restore the ship to the upright position.

The locus of the successive positions of the centre of buoyancy B′ as the angle of heel θ is increased is known as the *curve of buoyancy*. Its derivation for a particular 'displacement', i.e. weight of ship, constitutes a lengthy exercise in determining centroids and need occupy the attention of naval architects only. The shape of the curve depends exclusively on the lines of the vessel for a particular displacement. Since the centre of buoyancy is the point at which the weight of the vessel is supported, we may imagine that she rests upon that bit of tangent to the curve of buoyancy parallel to the water surface as indicated in Fig. 1.5(a).

Similarly, above the water surface, the curve to which the line of action of the upthrust remains always tangential as the ship (or water surface) is heeled, is known as the *curve of metacentres* for a particular displacement. Mathematically it follows that the curve of metacentres is the evolute of the curve of buoyancy. The cusp (M) in the former curve shown in Fig. 1.5(a) is known as the initial metacentre, and the distance GM is the initial metacentric height. It follows that **the initial metacentre, M, is the point where the line of action of the upthrust intersects the original vertical line through the centre of buoyancy, B, and the centre of gravity, G, for an infinitesimal angle of heel.**

In practice it is found that for *small* angles of heel the line of action of the upthrust passes very nearly through the initial metacentre M. Hence, so long as the angle of heel is less than say 15°, we may assume that the upthrust always acts through the fixed point M, just as the weight always acts vertically downwards through G. Under these conditions the righting moment or couple is $W \times \mathrm{GM} \times \sin\theta$, in which expression the distance GM may be considered to be a constant for the vessel. This initial value is generally implied when referring to the *transverse metacentric height* of the vessel, and is a measure of its static stiffness in roll. So long as G lies below M the righting couple will be positive, and this implies stability. In fact, **a floating body is in stable, unstable, or neutral equilibrium according as the metacentre lies above, below, or at the centre of gravity.** Obviously it is important that ships be designed such that M is normally above G for all conditions of loading, and under all circumstances of rolling.

We may note that a negative GM in the upright position is not necessarily catastrophic. Although a vessel in this condition cannot be persuaded to remain upright, she may possibly find a new stable equilibrium position by developing a 'loll' to one side or the other. A vessel with a large metacentric height is said, by naval architects, to be a 'stiff' ship which is found to be correspondingly lively, i.e. the vessel tends to roll with a predominantly greater amplitude and frequency in a rough sea. Merchant ships, especially liners, are therefore designed to have a relatively small metacentric height—say, between 0·3 and 0·6 m, but in warships sea-kindliness is sacrificed so that they have a large reserve of stability—GM varying between, say, 0·6 and 2 m according to displacement. These are the metacentric heights for 'rolling' displacements about a longitudinal axis. The metacentric heights for 'pitching' displacements about a transverse axis are, of course, much larger.

EXAMPLE 1.5 (i)

(*a*) *Outline briefly how a static stability curve may be obtained, over the whole* *range of stability of a ship, for a particular displacement and centre of gravity.*

(*b*) *Such a curve of righting levers for a vessel is given by the following ordinates at* 10 *degree intervals from the upright position:*
Righting lever in m*:* 0, 0·09, 0·50, 0·97, 1·23, 1·16, 0·79, 0·09.

(*i*) *Estimate the influence on the range of stability caused by raising the centre of gravity* 0·6 m.

(*ii*) *Discuss the behaviour of the ship with the raised centre of gravity and state what steps should be taken to correct its condition.*

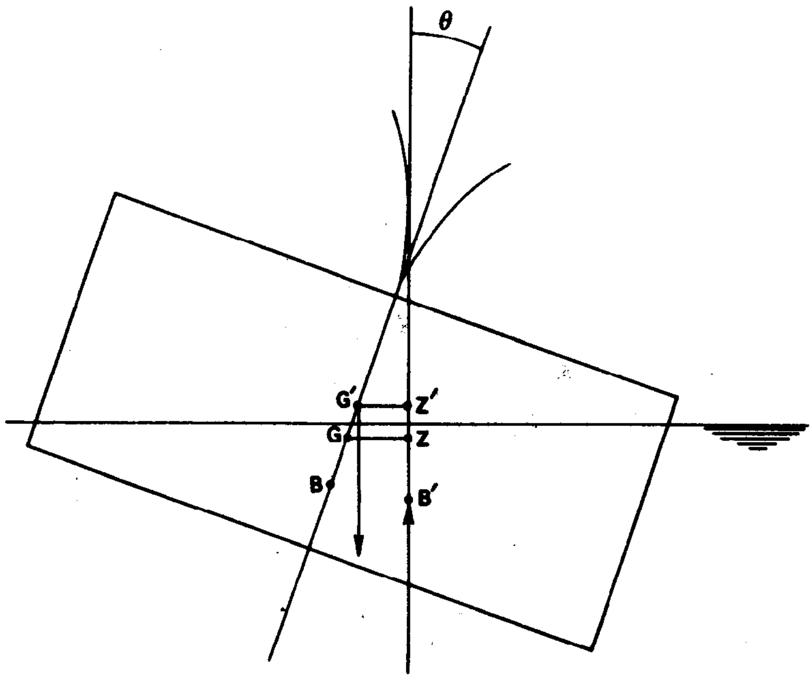

Fig. 1.5(b). *Effect of position of centre of gravity on righting lever*

(*a*) The centroid B′ of the displaced volume must first be determined from the lines of the vessel for a number of angles of heel up to the point of vanishing stability. The water surface is chosen in each case so that the displacement has the correct value.

The lever arm at which the upthrust through B′ acts about the centre of gravity G, gives the righting lever GZ, as indicated in Fig. 1.5(b). A graph showing the variation of the latter with heel is known as a static stability curve; see Fig. 1.5(c).

(*b*) *Righting Lever* (GZ)

From Fig. 1.5(c) it is seen that the effect of raising the C.G. from G to G′ is to reduce the righting lever from GZ to G′Z′. The reduction in the righting lever is thus:

$$GZ - G'Z' = GG' \sin\theta$$