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ABAMPERE. The **cgs electromagnetic unit** of current. It is that current which, when flowing in straight parallel wires 1 cm apart in free space, will produce a force of 2 dynes per cm length on each wire. One abampere is ten absolute amperes. (See **electromagnetic units.**)

ABBE-MAXWELL THEOREM. An axially symmetrical optical system is perfect if and only if it is plane-perfect for two different object planes perpendicular to the axis.

ABBE NUMBER. In correcting an optical system for first order chromatic aberration the Abbe number or reciprocal mean dispersion of a medium $\nu = (n_D - 1)(n_F - n_C)$ is used as a discrete substitute for $dn/d\lambda$. The indices of refraction n_D, n_F, n_C are for light of the wavelength of the mean of two yellow sodium lines, D (5892.9A), the blue hydrogen line, F (4861.327A), and the mean of two red hydrogen lines, C (6562.8A), respectively. For ordinary crown glass ν is approximately 60.

ABBE SINE CONDITION. An identity involving the second partial derivatives of the characteristic function of an optical system leads to an important relation that must be satisfied if a surface element containing a point source is to be imaged accurately into a corresponding surface element at the image point. This is Abbe's sine condition

$$ny \sin \theta = -n'y' \sin \theta'$$

where n, n' are indices of refraction, y, y' are distances from the optical axis, and θ, θ' are the angles light rays make with the optical axis in object, image space, respectively. If a system is corrected for spherical aberration and has an infinite exit pupil, then fulfillment of the sine condition is equivalent to freedom from coma.

The Abbe sine condition is equivalent to the extended sine relationship

$$ny \sin \theta + n'y' \sin \theta' = yy'\phi,$$

where ϕ is the power of the system, when the approximation $yy'\phi \cong 0$ is valid.

ABCOULOMB. The **electromagnetic unit** (emu) of charge. One abcoulomb is the amount of charge that passes when a steady current of one **abampere** flows for one second.

ABEL EQUATION. A mass point moves along a smooth curve in a vertical plane and under the influence of gravity alone. Given the time, t, required for the particle to fall from a point, x, to the lowest point on the curve as a function of x, what is the equation of the curve? The problem leads to a **Volterra integral equation** of the first kind

$$f(x) = \int_0^x \frac{\phi(t)dt}{\sqrt{2g(x - t)}},$$

which is a particular case of an Abel equation, where g is the acceleration of gravity.

A more general case of the Abel equation is

$$f(x) = \int_0^x (x - y)^{-\alpha}\phi(y)dy$$

where $f(x)$ is continuously differentiable for $x \geq 0$ and $0 < \alpha < 1$.

A first-order **differential equation**

$$y' = f_0(x) + f_1(x)y + f_2(x)y^2 + f_3(x)y^3$$

is also known as an Abel equation. When the $f_i(x)$ are given explicitly, the equation can often be converted into one of simpler type and solved in terms of elementary functions. In the general case the solution involves **elliptic functions.**

ABELIAN GROUP. A commutative group, namely such that $AB = BA$ where A, B are any two elements contained in it. A simple example is the **cyclic group** of order n.

ABEL IDENTITY. The sum of n terms of a sequence $a_1, a_2, a_3, \cdots$, where $a_1 = b_1c_1$, $a_2 = b_2c_2$, etc., can obviously be written in the form

$$a_1 + a_2 + \cdots + a_n = b_1s_1 + b_2(s_2 - s_1) + \cdots + b_n(s_n - s_{n-1})$$

where $s_i = c_1 + c_2 + \cdots + c_i$. Then this sum can be at once rearranged as

$$a_1 + a_2 + \cdots + a_n = s_1(b_1 - b_2) + s_2(b_2 - b_3) + \cdots + s_{n-1}(b_{n-1} - b_n) + s_nb_n,$$

which is the Abel identity. (Cf. **Abel inequality.**)

ABEL INEQUALITY. Let $k_1, k_2, \cdots$ be a non-increasing sequence of positive numbers. Denote by A the greatest of the sums $|a_1|$, $|a_1 + a_2|$, $|a_1 + a_2 + a_3|$, $\cdots$, then the Abel inequality, which is easily deduced from the **Abel identity,** states that

$$|a_1k_1 + a_2k_2 + \cdots + a_nk_n| \leqq Ak_1.$$

This inequality is useful for proving certain tests of convergence.

ABEL TEST FOR CONVERGENCE. If $a_1 + a_2 + \cdots$ is convergent and $u_1, u_2, \cdots$ is a bounded monotonic sequence, then $a_1u_1 + a_2u_2 + \cdots$ converges.

ABEL THEOREM ON POWER SERIES. If $a_0 + a_1z + a_2z^2 + \cdots$ is a power series with unity for its radius of convergence and with convergent sum of coefficients $a_0 + a_1 + a_2 + \cdots$, then the Abel theorem states that $\lim_{z\to 1}(a_0 + a_1z + a_2z^2 + \cdots)$ is equal to the sum $a_0 + a_1 + a_2 + \cdots$; that is, the power series is continuous up to and including the end point.

ABERRATIONAL CONSTANT. The aberrational constant is given by

$$k = \frac{2\pi a \operatorname{cosec} 1''}{cT\sqrt{1 - e^2}}$$

in which a is the semi-major axis of the earth's orbit, c is the velocity of light, T is the sidereal year expressed in mean solar seconds, and e is the eccentricity of the earth's orbit. The value of the aberrational constant is about 20".47.

ABERRATION OF LIGHT, ASTRONOMICAL (CORRECTIONS FOR). The apparent change in the direction of a celestial object due to the component of the earth's motion perpendicular to the line of sight and the finite speed of light is known as the *aberrational correction.*

Calling θ the true direction and θ' the displaced direction (displacement due to aberration) there results

$$\sin(\theta - \theta') = \frac{v}{c}\sin\theta'$$

in which v is the velocity of the earth and c is the speed of light. Since v/c is a very small quantity one can write the above equation as $\theta - \theta' = \frac{v}{c}\sin\theta' \operatorname{cosec} 1''$ which may be further simplified to $\theta - \theta' = k\sin\theta'$ in which $k = \frac{v}{c}$ cosec 1'' and k is known in astronomy as the **aberrational constant.** Since k does not exceed 20".5, then $\theta - \theta' = k\sin\theta$.

Since the aberrational displacements are in the plane of the ecliptic the aberrational corrections to celestial latitude B and celestial longitude λ are given by

$$\Delta\lambda = -k\sec B\cos(\lambda_0 - \lambda)$$

$$\Delta B = -k\sin B\sin(\lambda_0 - \lambda)$$

in which λ_0 is the longitude of the sun.

The aberrational corrections to the equatorial coordinates of right ascension and declination are much more complicated since the plane of the equator is inclined to the plane of the ecliptic in which the aberrational displacement takes place. Let

$$C = -k\cos\epsilon\cos\lambda_0$$

$$c = \tfrac{1}{15}\cos\alpha\sec\delta$$

$$d = \tfrac{1}{15}\sin\alpha\sec\delta$$

$$D = -k\sin\lambda_0$$

$$c' = \tan\epsilon\cos\delta - \sin\alpha\sin\delta$$

$$d' = \cos\alpha\sin\delta$$

in which ϵ is the obliquity of the ecliptic, α and δ are the true right ascension and declination, α' and δ' are the displaced right ascension and declination.

The values of log C and log D vary throughout the year due to the position of the sun. They are tabulated as the Besselian star numbers in almanacs for each day of the year.

Finally there results

$$\alpha' = \alpha + Cc + Dd$$

$$\delta' = \delta + Cc' + Dd'.$$

ABERRATIONS OF AN OPTICAL SYSTEM. Let a point (x,y) in the object plane of an optical system be imaged by the point (x',y') in the image plane. In the approximation of Gaussian optics,

$$x' = mx, \quad y' = my$$

where m is the magnification. The aberrations of the system are the deviations,

$$x = x' - mx, \quad y = y' - my;$$

which are decomposed in several ways into constituent aberration functions (see **Seidel aberrations** and **Nijboer-Zernike aberration functions**) and various methods have been developed for the calculation of the constituent aberrations (see the **Schwarzschild-Kohlschütter formulas**).

ABERRATIONS OF LENSES, SEIDEL THEORY OF. See **Seidel aberrations.**

ABSCISSA. The horizontal coordinate of a point in a two-dimensional system, commonly rectangular Cartesian, and usually designated by x. Together with the **ordinate** it locates the position of the point in a plane.

ABSOLUTE ACCELERATION. The time rate of change of **velocity** with respect to axes fixed in space or to an **inertial frame** is a vector called absolute acceleration.

ABSOLUTE ACTIVITY. A quantity defined by the equation

$$\lambda = \exp\left(\frac{\mu}{kT}\right)$$

where k is the Boltzmann constant, T, the absolute temperature and μ, the **thermal potential.** In systems undergoing chemical reactions, this formula takes the following form:

$$\lambda_i = \exp\left(\frac{\mu_i}{kT}\right),$$

where λ_i is the absolute activity of component i, and μ_i is the chemical potential of component i.

ABSOLUTE ANGULAR MOMENTUM. The **angular momentum** as measured in an **absolute coordinate system,** hence the vector product of the position vector of a particle by the **absolute momentum** of the particle.

In the atmosphere the absolute angular momentum M per unit mass of air is equal to the sum of the angular momentum relative to the earth and the angular momentum due to the rotation of the earth:

$$M = ua \cos\phi + \Omega a^2 \cos^2\phi,$$

where a is the radius of the earth, u the relative eastward speed, ϕ the latitude, and Ω the angular speed of the earth. (See **angular momentum balance, conservation of angular momentum.**)

ABSOLUTE CONTINUITY. Let the function $f(x)$ be defined on the closed interval $[a,b]$ and let l denote the total length of a finite or countable number of nonoverlapping intervals (a_i,b_i) in $[a,b]$. If the limit, as l approaches zero, of the sum $\Sigma|f(b_i) - f(a_i)|$ is zero, then $f(x)$ is said to be absolutely continuous. An absolutely continuous function is *a fortiori* continuous.

ABSOLUTE CONVERGENCE. See **convergence.**

ABSOLUTE COORDINATE SYSTEM. (Or absolute reference frame.) In meteorology, that **inertial coordinate system** which has its origin on the axis of the earth and is fixed with respect to the stars. Thus, any mechanical quantities in meteorology defined with respect to this frame take into account the movement of the earth. (See **Coriolis force, absolute vorticity.**)

ABSOLUTE DERIVATIVE OF TENSOR. See **intrinsic derivative of tensor field.**

ABSOLUTE DIFFERENTIAL CALCULUS. Theory of the differentiation of **tensor fields.** Also called *tensor calculus, Ricci calculus.*

ABSOLUTE DISPLACEMENT. The change in **position** measured with respect to coordinate axes fixed in space or to an **inertial frame** is a vector called the absolute displacement.

ABSOLUTE HUMIDITY. (1) (Also called vapor concentration, vapor density.) In a system of moist air, the ratio of the mass of **water vapor** present to the volume occupied by the mixture; that is, the density of the water vapor component.

Absolute humidity is usually expressed in grams of water vapor per cubic meter or, in engineering practice, in grains per cubic foot. (Cf. **mixing ratio, specific humidity, relative humidity, dew point.**)

(2) As occasionally used in air conditioning practice, the number of grains of water vapor per pound of moist air, which is dimensionally identical with the **specific humidity.**

ABSOLUTE INVARIANT. See **invariant.**

ABSOLUTE ISOHYPSE. A line that has the properties of both constant pressure and constant height above mean sea level. Therefore, it can be any **contour line** on a constant-pressure chart, or any **isobar** on a constant-height chart.

ABSOLUTE MAGNITUDE (STELLAR). The apparent stellar magnitude of a celestial object depends both upon the intrinsic brightness of the object and also upon the distance of the object from the earth. The absolute magnitude, either visual or photographic, is the brightness, expressed on the stellar magnitude scale, that it would have if the object were at a distance of 10 parsecs (stellar parallax 0″.1). If we call M the absolute magnitude, mg the apparent magnitude and ϕ'' the parallax in seconds of the object

$$M = mg + 5 + 5 \log \phi''.$$

The sun has an apparent magnitude of −26.72 and a parallax of 206265″. The absolute magnitude of the sun is 4.85. Antares has an apparent magnitude of 1.22 and a parallax of 0″.009. The absolute magnitude of Antares is found to be −4.0. Hence it follows that actually Antares is nearly 4000 times as bright as the sun.

ABSOLUTE MOMENTUM. (Often called absolute linear momentum.) The (linear) **momentum** of a particle as measured in an **absolute coordinate** system; hence in meteorology, the sum of the (vector) momentum of the particle relative to the earth and the (vector) momentum of the particle due to the earth's rotation.

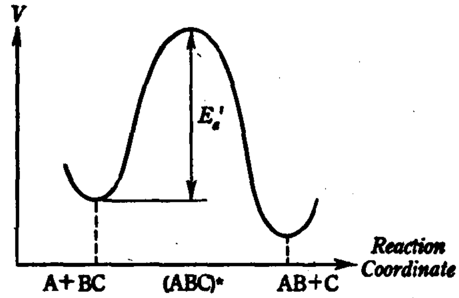
ABSOLUTE REACTION RATE THEORY. A reformulation of the **collision theory of chemical kinetics** due to Wigner, Eyring and others makes possible the expression of the rate constant in a more general form. Similar considerations may be applied to a great many of other rate processes (see **Eyring theory of transport processes**). Because the purpose of this theory is to calculate the rate in terms of molecular quantities alone, it is often called the absolute reaction rate theory.

Consider the reaction

$$A + BC \rightarrow AB + C. \qquad (1)$$

To simplify the discussion, assume that A, B and C always remain in a straight line. The course of the reaction may then be followed by noting the values of the two interatomic distances r_{AB} and r_{BC}. At the beginning of the reaction r_{AB} is large and r_{BC} is small while at the end of the reaction r_{AB} is small and r_{BC} is large.

Let us introduce the potential energy surface. The representative point of the system moves on this surface along the so-called *reaction coordinate.* The potential energy along the reaction coordinate is represented schematically in the figure. The maximum of the



Potential energy along the *reaction coordinate.*

curve corresponds to a situation where three atoms are very close to one another. Moreover this point is a *maximum* along the reaction coordinate but a *minimum* for the direction normal to the reaction coordinate. Indeed the most probable path is the path involving the minimum potential energy in going from the initial to the final state.

Therefore the point considered corresponds to a *saddle point* of the energy surface. It is called the **activated complex.**

One may now assume that the reaction rate is the product of the following three factors: (1) the average number of activated complexes; (2) the characteristic frequency of the activated complex (that is, the inverse of its lifetime); (3) the *transmission coefficient,* K, which is the probability that a chemical reaction takes place after the system has reached the activated state.

Moreover the number of activated complexes is calculated by the equilibrium assumption. (See **equilibrium theory of chemical reactions.**)

Using this description of the reaction process one derives the following expression for the reaction constant

$$k = K \frac{\phi_{\neq}(T)}{\phi_A(T)\phi_{BC}(T)} \frac{kT}{h} \exp\left(-\frac{E^{\neq}}{kT}\right). \qquad (2)$$

Here the ϕ terms are the **partition functions** $f(T,V)$, the volume factor being removed

$$f = V\phi. \qquad (3)$$

ϕ_τ corresponds to the activated complex, the degree of freedom associated with the reaction coordinate being removed; k is **Boltzmann's constant,** h is **Planck's constant,** E^x is the energy associated with the activated complex, or **activation energy** of the reaction.

This expression may also be written in the thermodynamic form

$$\mathrm{k} = \mathrm{K}\frac{kT}{h}\exp\left(-\frac{\Delta G^\ddagger}{kT}\right)$$

$$= \mathrm{K}\frac{kT}{h}\exp\left[-(\Delta H^\ddagger - T\Delta S^\ddagger)/kT\right] \quad (4)$$

where $\Delta G^\ddagger$ is a suitable **free energy** of activation and $\Delta H^\ddagger$, $\Delta S^\ddagger$ the corresponding **enthalpy** and **entropy** of activation.

ABSOLUTE SCALAR. See **scalar, absolute.**

ABSOLUTE TEMPERATURE. See **temperature.**

ABSOLUTE TENSOR (TENSOR FIELD). **Tensor (tensor field)** of weight zero. Often called tensor (tensor field) when context admits no confusion.

ABSOLUTE VALUE. See **complex number.**

ABSOLUTE VELOCITY. The time rate of change of **position** measured with respect to **absolute coordinate system** (i.e., a system with coordinate axes fixed in space or to an **inertial frame**) is a vector called the absolute velocity. For example, (1) in a turbine, the average flow velocity measured with respect to the turbine casing. (See **velocity diagram.**) (2) In meteorology, the (vector) sum of the velocity of a fluid parcel relative to the earth and the velocity of the parcel due to the earth's rotation. The east-west component is the only one affected:

$$u_a = u + \Omega a \cos\phi;$$

where u and u_a are the relative and absolute eastward speeds, Ω the angular speed of the earth's rotation, a the radius of the earth, and ϕ the latitude of the parcel.

ABSOLUTE VISCOSITY. (Also called dynamic viscosity.) The property of a fluid which determines the shearing stresses which arise in it during motion. In pure shear flow, the shearing stress τ is proportional to the transverse velocity gradient du/dy,

$$\tau = \mu\frac{du}{dy} \text{ (Newton's law of fluid friction)}$$

where μ is the absolute viscosity. In a general field of flow, u_1, u_2, u_3, of a homogeneous, Newtonian, incompressible fluid, the shearing stresses are proportional to the respective rates of change of strain (Stokes' law). The symmetric stress tensor t_{ij} is assumed to be a linear function of the rate of strain tensor e_{ij}. Taking into account that in a fluid at rest the stress is an isotropic tensor, we put

$$t_{ij} = -p\delta_{ij} + \lambda\delta_{ij}e_{kk} + 2\mu e_{ij}$$

where δ_{ij} is the Kronecker delta. Since $t_{ij} = 0$ for $e_{ij} = 0$, we have $t_{ii} = -3p$ and $3\lambda + 2\mu = 0$. Consequently

$$t_{ij} = -p\delta_{ij} - \tfrac{2}{3}\mu\delta_{ij}e_{kk} + 2\mu e_{ij}$$

when p is now the hydrostatic pressure. The scalar μ is defined as the absolute viscosity. It is a function of the thermodynamic state of the fluid and is independent of the velocity field.

The ratio

$$\nu = \frac{\mu}{\rho}$$

of absolute viscosity to density is called *kinematic viscosity.*

ABSOLUTE VORTICITY. (1) The **vorticity** of a fluid particle determined with respect to an **absolute coordinate system.**

(2) The vertical component η of the absolute vorticity (as defined above) given by the sum of the vertical component of the vorticity with respect to the earth (the **relative vorticity**) ζ and the vorticity of the earth (equal to the **Coriolis parameter**) f:

$$\eta = \zeta + f.$$

ABSOLUTE ZERO. See **temperature.**

ABSOLUTE ZERO UNATTAINABILITY. See **thermodynamics, third law of.**

ABSORBANCE. The common logarithm of the **absorptance.** It may be applied to the total radiation, the visible radiation or to a particular part of the spectrum (spectral absorbance).

ABSORPTANCE. The ratio of the luminous flux absorbed by the body to the flux it receives. (See **spectral absorptance; internal**

absorptance; absorptivity; extinction coefficient.)

ABSORPTION. (1) The process whereby the total number of particles emerging from a body of matter is reduced relative to the number entering, as a result of interaction of the particles with the body. (2) The process whereby the kinetic energy of a particle is reduced while traversing a body of matter. This loss of kinetic energy of corpuscular radiation is also referred to as moderation, slowing, or stopping. (3) The process whereby some or all of the energy of sound waves or electromagnetic radiations is transferred to the substance on which they are incident or which they traverse. (4) The process of "attraction into the mass" of one substance by another so that the absorbed substance disappears physically.

ABSORPTION COEFFICIENT. (1) In the most general use of the term absorption coefficient, applied to electromagnetic radiation and atomic and sub-atomic particles, it is a measure of the rate of decrease in intensity of a beam of photons or particles in its passage through a particular substance. (See **absorption coefficient for light.**) (2) In the case of sound, the absorption coefficient (which is also called the acoustical absorptivity) is defined as the fraction of the incident sound energy absorbed by a surface or medium, the surface being considered part of an infinite area. (See also **acoustic attenuation coefficient.**) (3) For the absorption of one substance or phase in another, as in the absorption of a gas in a liquid, the absorption coefficient is the volume of gas dissolved by a specified volume of solvent; thus a widely-used coefficient is the quantity α in the expression $\alpha = V_0/Vp$, where V_0 is the volume of gas reduced to standard conditions, V is the volume of liquid and p is the partial pressure of the gas.

ABSORPTION COEFFICIENT, AMPLITUDE. See **acoustic attenuation coefficient.**

ABSORPTION COEFFICIENT FOR LIGHT. The absorption coefficient κ_ν measures the loss in intensity suffered by a beam of light of intensity I_ν when passing through an absorbing layer of thickness ds. κ_ν depends on the physical properties of the absorbing substance, on the frequency ν, and, sometimes, on the direction of flow of radiation. The decrease in intensity is therefore

$$dI_\nu/ds = -\kappa_\nu I_\nu. \quad (1)$$

It is sometimes useful to introduce the *mass absorption coefficient*

$$\kappa_{\nu,M} = \kappa_\nu/\rho \quad (2)$$

or the *atomic absorption coefficient*

$$\kappa_{\nu,at} = \kappa_\nu/n, \quad (3)$$

where ρ stands for the density of the absorbing substance and n for the number of absorbing atoms per cubic centimeter. The loss of intensity is then

$$dI_\nu/ds = -\kappa_{\nu,M}\rho I_\nu \quad (4)$$

or

$$dI_\nu/ds = -\kappa_{\nu,at}nI_\nu. \quad (5)$$

Another usage in optics is to express the absorption coefficient in terms of the wavelength rather than the frequency (as above). In this notation the absorption coefficient is written simply as $\alpha_\lambda = -dI/I$. Then by integration (see **absorption coefficient, integrated**) one arrives at the common form of **Bouguer's law of absorption,** $I = I_0 \exp(-\alpha x)$. In traversing perpendicularly a thin layer of absorbing material of thickness x, the amplitude of vibration of light is damped by the factor $\exp(-2\pi mx/\lambda)$ where m is the *absorption index* or *absorption constant.* In consequence the intensity is damped by $\exp(-4\pi mx/\lambda)$ and so the absorption coefficient is $\alpha = 4m/\lambda$ in terms of the absorption index. (For solutions, see **Beer's law.**)

ABSORPTION COEFFICIENT, INTEGRATED. The total intensity of an absorption feature (line or unresolved band) is given by

$$I_{abs} = \int (I_\nu^0 - I_\nu)d\nu = I_\nu^0 \Delta s \int \kappa_\nu d\nu.$$

Here I_ν^0 and I_ν stand for the initial and final intensities, κ_ν for the **absorption coefficient** at frequency ν, and Δs for the thickness of the absorbing layer. The integral

$$\int \kappa_\nu d\nu$$

of the absorption coefficient κ_ν over the frequency range is called the integrated absorption coefficient.

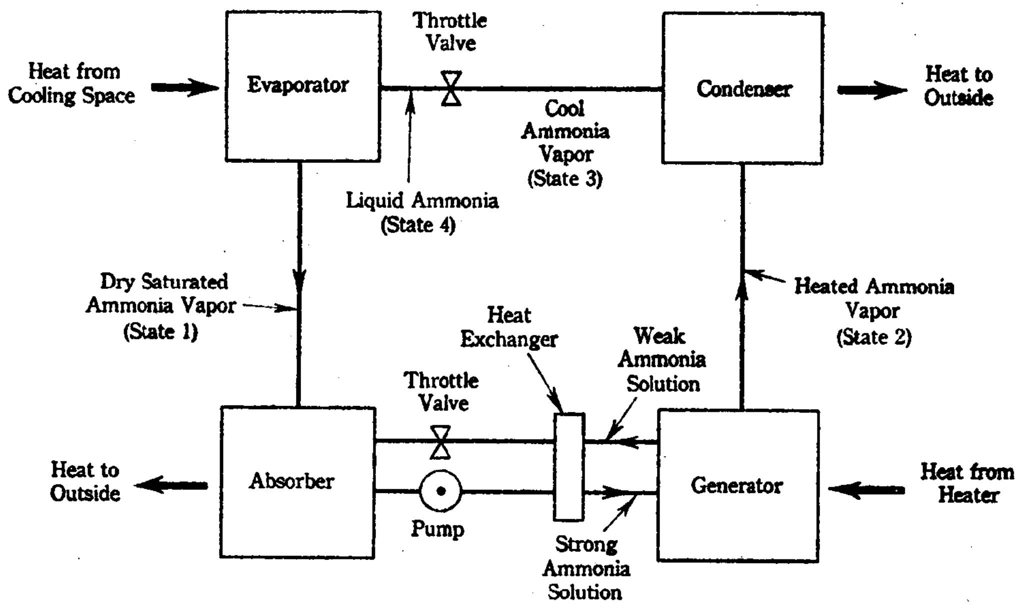


Fig. 1. Block diagram of absorption refrigerator.

ABSORPTION CONSTANT. A synonym for the optical absorption index. (See **absorption coefficient for light; index of refraction, complex.**)

ABSORPTION DISCONTINUITY. The discontinuities of the **absorption coefficient** κ_ν (in terms of frequency) or α_λ (using the wavelength notation), of a medium, correspond to spectral absorption lines and are often associated with anomalies in other frequency (wavelength) dependent properties of the medium, e.g., the refractive index.

ABSORPTION, EXPONENTIAL. The intensity of a beam of light of frequency ν passing through a homegeneous absorbing substance, decreases exponentially according to

$$I = I_0 e^{-\kappa_\nu s}$$

where I_0 and I are the initial and final intensity respectively, κ_ν the **absorption coefficient** for frequency ν, and s the thickness of the absorbing layer (see also **optical depth**).

Note that in wavelength notation, the coefficient κ_ν becomes $\alpha(\lambda)$, α_λ or simply α.

ABSORPTION FACTOR. Ratio of absorbed to unabsorbed radiation. (See also **optical depth.**)

ABSORPTION OF RADIANT ENERGY. The transformation of radiant energy to a different form of energy by the intervention of matter.

ABSORPTION REFRIGERATOR. A refrigerator in which the work of compression is considerably reduced by dissolving the refrigerant in a suitable liquid before compression. Thus the compression of a liquid is substituted for that of a gas. To compensate for it, it is necessary to introduce a source of heat at a temperature higher than that of the surroundings (i.e., of the cooling water available). (See Figures 1, 2 and 3.)

The mixture of gaseous and liquid ammonia is evaporated in the *evaporator* until it becomes dry saturated (state 1). After evaporation, the dry saturated vapor is passed into an *absorber* where it is absorbed in water, rejecting heat Q_s to the surroundings. The temperature of the water is only a little above atmospheric and the solubility of ammonia in water

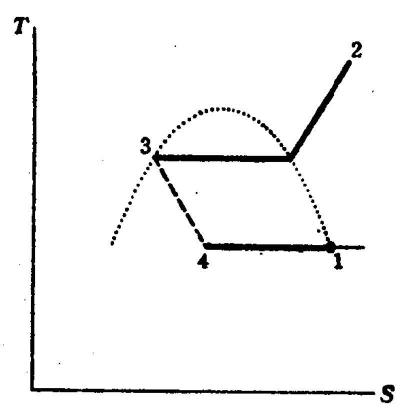


Fig. 2. Temperature-entropy diagram of absorption refrigerator.

is very high. The liquid solution is compressed in a pump, the process requiring a very small amount of work only ($W \simeq 0$). The liquid solution is then heated in the *generator*

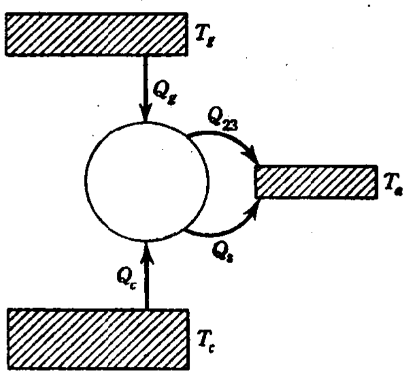


Fig. 3. Heat flow diagram of absorption refrigerator.

at a temperature T_g. This causes the ammonia to be driven off from the solution at the expense of heat Q_g absorbed. At this temperature the solubility of ammonia in water is considerably reduced. In order to maintain steady-state operation a weak solution is throttled back to the absorber, and the weak and strong solutions are made to exchange heat in a counter-flow heat exchanger (heat regeneration) for the sake of economy. Gaseous, usually superheated, ammonia at state 2 is used to complete a conventional refrigeration cycle: (a) cooling at constant pressure from state 2 to state 3 in the *condenser*, with the possibility of undercooling below T_3; (b) throttling in a throttle valve to state 4, etc.

Energy is transferred at three levels: T_g in the generator, T_c in the evaporator (cold chamber), and T_a in the absorber and condenser. We have $T_g > T_a > T_c$. By the first law of **thermodynamics**

$$Q_g + Q_c = Q_{23} + Q_s.$$

In the ideal case there is no entropy increase due to the operation of the refrigerator and hence

$$\frac{Q}{T_g} - \frac{Q_c}{T_c} + \frac{Q_{23} + Q_s}{T_a} \geq 0.$$

The performance of the refrigerator can be judged by the ratio Q_c/Q_g of the refrigerating effect, Q_c, to the quantity of heat, Q_g, added at the highest temperature. It is seen that

$$\frac{Q_c}{Q_g} \leq \frac{T_c(T_g - T_a)}{T_g(T_c - T_a)}.$$

The so-called *Electrolux refrigerator* constitutes a special form of the absorption refrigerator in which the pump has been replaced by a syphon, and the throttling valve has been eliminated. Instead of throttling through a valve, ammonia is mixed with inert hydrogen so that its partial pressure is decreased, giving the same effect as throttling. Owing to its great simplicity, it is widely used in domestic refrigerators. (See Figure 4.)

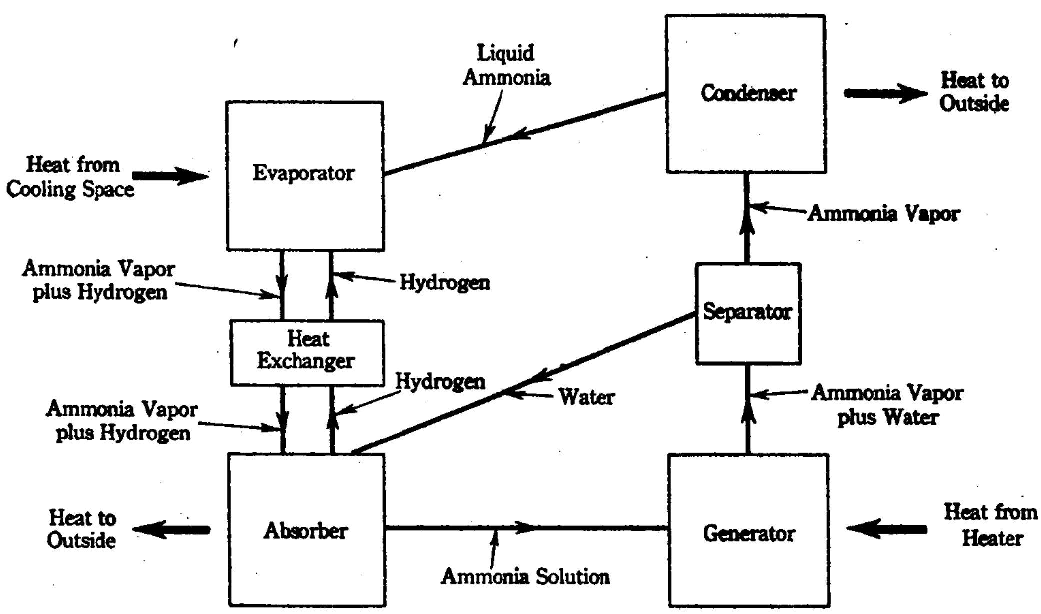


Fig. 4. Block diagram of Electrolux refrigerator.

The concentrated solution of ammonia in water is boiled in the *generator* and the syphon action through the connector tube makes it rise to the *separator*. The gaseous ammonia flows to the *condenser*, whereas the liquid water, while cooling, returns to the *absorber* through a liquid trap. The gaseous ammonia is cooled in the *condenser* shaped like a finned heat exchanger, and eventually condenses. The liquid ammonia flows through a trap to the *evaporator* filled with hydrogen so that its partial pressure decreases to that corresponding to the low temperature prevailing. In the process, the evaporator absorbs heat Q from the cooling space at that reduced temperature. The mixture of hydrogen and ammonia flows down through a heat exchanger to the absorber where the ammonia vapor enters into solution freeing the hydrogen in the process. The hydrogen rises to the evaporator, while the solution of ammonia in water flows to the generator to complete the cycle.

ABSORPTIVE POWER. That part of the radiation of a given wavelength which is absorbed by a body. When radiation falls on a surface, a portion A_λ of it is absorbed, a portion R_λ is reflected and a portion Z_λ is transmitted, so that

$$A_\lambda + R_\lambda + Z_\lambda = 1,$$

where A_λ is the absorptive power or absorptivity, R_λ is the reflecting power or reflectivity, Z_λ is the transmitting power or transmissivity at wavelength λ. Sometimes the adjective "monochromatic" is added to the above terms to emphasize that they relate to a definite wavelength.

ABSORPTIVITY. See **absorptive power.**

ABVOLT. See **electromagnetic units.**

ACCELERATING CONVERGENCE OF SERIES AND SEQUENCES. See **summation of series.**

ACCELERATION. The rate of change of the **velocity** with respect to the time is called acceleration. It is expressed mathematically by $\frac{d\mathbf{v}}{dt}$, the vector derivative of the velocity, $\mathbf{v}$ with respect to the time, t. If the motion is in a straight line whose position is clearly understood, it is convenient to treat the velocity v and the acceleration $\frac{dv}{dt}$ as scalars with appropriate algebraic signs; otherwise they must be treated by vector methods.

Acceleration may be rectilinear or curvilinear, depending upon whether the path of motion is a straight line or a curved line. A body which moves along a curved path has acceleration components at every point. One component is in the direction of the tangent to the curve and is equal to the rate of change of the speed at the point. For uniform circular motion this component is zero. The second component is normal to the tangent and is equal to the square of the tangential **speed** divided by the **radius of curvature** at the point. This normal component which is directed toward the center of curvature also equals the square of the **angular velocity** multiplied by the radius of curvature. The acceleration due to **gravity** is equal to an increase in the velocity of about 32.2 ft per sec per sec at the earth's surface and is of prime importance since it is the ratio of the weight to the **mass** of a body. (For examples of acceleration in both curved and linear motion, see **kinematics.**)

ACCELERATION, ANGULAR. See **angular acceleration.**

ACCELERATION, CENTRIPETAL. See **centripetal acceleration.**

ACCELERATION, COMPATIBLE. Another name for **Coriolis acceleration.**

ACCELERATION, COMPLEMENTARY. Another name for **Coriolis acceleration.**

ACCELERATION, COMPOSITION LAW OF (RELATIVITY). Let a'_x, a'_y, a'_z be the components of the acceleration in Σ' and w'_x, w'_y, w'_z the velocity components in Σ' and a_x, a_y, a_z the components of the acceleration in Σ (v the velocity of Σ' with respect to Σ in the common x-direction) then

$$a_x = \left[\frac{c\sqrt{c^2 - v^2}}{c^2 + vw'_x}\right]^3 \cdot a'_x$$

$$a_y = \left[\frac{c\sqrt{c^2 - v^2}}{c^2 + vw'_x}\right]^2 \cdot \left[a'_y - \frac{vw'_y a'_x}{c^2 + vw'_x}\right]$$

$$a_z = \left[\frac{c\sqrt{c^2 - v^2}}{c^2 + vw'_x}\right]^2 \cdot \left[a'_z - \frac{vw'_z a'_x}{c^2 + vw'_x}\right].$$

ACCELERATION, CORIOLIS. See **Coriolis acceleration.**

ACCELERATION, NORMAL. See **centripetal acceleration.**

ACCELERATION OF GRAVITY. (1) The ratio of the weight of a material particle to its mass at any specific point in an approximately uniform gravitational **field.** This is the **acceleration** with which a body would fall in the absence of all other disturbing forces, such as those due to friction.

(2) Specifically, the acceleration with which a body falls *in vacuo* at a given point on or near a given point on the earth's surface. This acceleration, frequently denoted by g, varies by less than one percent over the entire surface of the earth. Its "average value" has been defined by the International Commission of Weights and Measures as 9.80665 M/S^2 or 32.174 ft/S^2. Its value at the poles is 9.8321 M/S^2 and at the equator 9.7799 M/S^2.

ACCELERATION POTENTIAL. The scalar function (if it exists) whose **gradient** is equal to the total acceleration of a fluid. In meteorology, this must include the **Coriolis acceleration:**

$$\frac{d\mathbf{V}}{dt} + 2\mathbf{\Omega} \times \mathbf{V} = -\nabla\chi$$

where $\mathbf{V}$ is the velocity, $\mathbf{\Omega}$ the angular velocity of the earth, and χ the acceleration potential.

So defined, this potential exists if and only if the atmosphere is **barotropic,** in which case $\chi = \pi + gz$, where π is the **barotropic pressure function,** g, the **acceleration of gravity,** and z, the vertical coordinate.

However, if some thermodynamic variable σ be taken as vertical coordinate, the horizontal acceleration will have a potential

$$\frac{\partial \mathbf{V}_H}{\partial t} + \mathbf{V}_H \cdot \nabla_\sigma \mathbf{V}_H + 2\mathbf{\Omega} \times \mathbf{V}_H = -\nabla_\sigma \chi,$$

where $\mathbf{V}_H$ is the horizontal velocity and ∇_σ the horizontal del operator in the σ-surface. In the case of pressure as vertical coordinate, $\sigma = p$, and $\chi = gz$. In the case of potential temperature θ as vertical coordinate, $\sigma = \theta$ and $\chi = c_pT + gz$, where c_p is the specific heat of air at constant pressure and T the absolute temperature. Note that the acceleration potential acts as the **stream function** in the case of **geostrophic equilibrium.** (See **velocity potential; potential.**)

ACCELERATION, TANGENTIAL. The component of acceleration along the path of motion is called the tangential acceleration. Its magnitude is dv/dt, the time rate of change of the **speed along the path.**

ACCELERATOR, PARTICLE. A device designed to give to charged particles, such as electrons or positive ions, the high energies necessary to penetrate the **Coulomb barrier** of the target nucleus.

ACCEPTOR BOND. See **donor bond.**

ACCESS TIME. (1) The time interval, characteristic of a memory or storage device, between the instant at which **information** is requested of the **memory** and the instant at which this information begins to be available in useful form. (2) The time interval between the instant at which information is available for storage and the instant at which it is effectively stored.

ACCOMMODATION COEFFICIENT. Let E_i be the average kinetic energy of the gaseous molecules which collide with a wall whose temperature corresponds to an equilibrium kinetic energy E_1 per molecule. If E_i and E_1 differ, the average kinetic energy of the reflected molecules will in general have a value E_r intermediate between E_i and E_1. Knudsen has defined an *accommodation coefficient* α, which expresses how much of the maximum possible kinetic energy exchange does actually occur:

$$\alpha = \frac{E_i - E_r}{E_i - E_1}.$$

Similar accommodation coefficients can be introduced for the internal degrees of freedom, or even for chemical reactions.

ACCOMPANYING FLUID. The fluid in irrotational motion which travels with a moving vortex system. It surrounds the **substance of the vortex** and is itself surrounded by fluid in irrotational motion which does not travel with it but which passes by it as if it were a solid body. Hill's **spherical vortex** has no accompanying fluid.

ACCUMULATION POINT. One of a set such that any neighborhood of this point, no matter how small, contains a member of the set. All the points of a set which is everywhere dense are accumulation points.

ACCURACY. The degree of exactness actually possessed by an approximation, measurement, etc. It may be contrasted with precision, which is the degree of exactness with which the quantity is expressed. For example, as a value of π, the number 3.1428 is more precise than accurate.

ACHROMATIC. In the accepted colorimetric sense: (1) for primary light sources, the color of the **equi-energy spectrum** ($x = y = z = 1/3$) is taken as achromatic; (2) for surface colors the light source serving as illuminant is taken as achromatic. On this basis, an ideal white surface is always defined as achromatic whatever may be the color of the light.

In optical design: (3) an optical system is achromatic if it is approximately corrected for chromatic aberration in the sense that the focal length is the same for two distinct wavelengths.

ACHROMATIC LIGHT, SPECIFIED. (1) Light of the same chromaticity as that having an **equi-energy spectrum.** (2) The colorimetric standard illuminants *A*, *B* and *C*, the spectral energy distributions of which were specified by the C.I.E. in 1931, with various scientific applications in view:

Standard A. Incandescent electric lamp of color temperature 2854°K.
Standard B. Standard A combined with a specified liquid filter, to give a light of color temperature approximately 4800°K.
Standard C. Standard A combined with a specified liquid filter to give a light of color temperature approximately 6500°K.

(3) Any other specified white light.

ACIDS AND BASES. DEFINITIONS OF BRØNSTED AND LEWIS. The two most common definitions of acids and bases are those of Brønsted and Lewis.

According to Brønsted, an acid is a proton donor; a base, a proton acceptor. For example NH_3 and H_2O are acids in the reactions:

$$NH_3 \rightarrow NH_2^- + H^+; \quad H_2O \rightarrow OH^- + H^+.$$

They act as bases in the reactions:

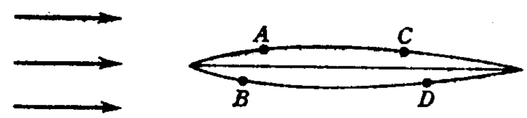
$$NH_3 + H^+ \rightarrow NH_4^+; \quad H_2O + H^+ \rightarrow H_3O^+.$$

In a more general definition, Lewis calls a base any substance with a free doublet of electrons which it is capable of sharing with an electron pair acceptor, which is called an acid. For example, in the reaction:

$$(C_2H_5)_2O: + BF_3 \rightarrow (C_2H_5)_2O{:}BF_3$$

the ethyl ether molecule is a base, the boron trifluoride an acid. The resulting complex is a *Lewis salt.*

ACKERET THEORY. A theory of two-dimensional airfoils in supersonic flow. Since the theory is based on the linearized equation, it is valid only when the perturbation velocities are small, and for Mach numbers that are not either near 1 or large enough to make the hypersonic similarity parameter $M\tau$ of order 1. The limitation to small perturbation velocities means that the theory can only be applied to thin airfoils, with sharp leading and trailing edges, at small angles of incidence.



Ackeret theory.

The theory gives the pressure coefficient at any point on an airfoil, where the surface is inclined at an angle θ to the stream, as

$$C_p = \frac{2\theta}{\sqrt{M^2 - 1}}, \tag{1}$$

where M is the Mach number of the stream. In Equation (1), the angle θ is considered to be positive if its sign corresponds to deflection of the stream at a concave corner. Thus, in the figure, θ and C_p are positive for the points A and B, and negative for C and D.

By integration of Equation (1) over the surface of an airfoil, a number of simple results relating to lift, drag and pitching moment may be obtained. Thus for a thin airfoil of any shape, at an incidence α,

$$C_L = \frac{4\alpha}{\sqrt{M^2 - 1}}, \tag{2}$$

and for a symmetrical airfoil the center of pressure is at the half-chord point.

Drag coefficients calculated from Equation (1) do not include any boundary-layer effects and are, in fact, coefficients of wave drag. For a thin flat plate the wave drag coefficient is

$$C_D = \alpha C_L = \frac{4\alpha^2}{\sqrt{M^2 - 1}}. \tag{3}$$

Equation (3) expresses the simple condition that, in inviscid flow, the resultant force on a thin flat plate acts in a direction normal to the plate.

The total wave drag coefficient of an airfoil with thickness is the sum of two terms, the first given by Equation (3) and the second associated with the thickness of the airfoil. For a series of airfoils related by an **affine transformation,** the drag coefficient due to thickness at a given Mach number is proportional to the square of the thickness/chord ratio.

ACOUSTIC ABSORPTION COEFFICIENT. See **sound absorption coefficient of surfaces.**

ACOUSTIC ABSORPTIVITY. See **sound absorption coefficient of surfaces.**

ACOUSTIC AMPLITUDE EQUATIONS, FINITE. See **finite amplitude equations (acoustic).**

ACOUSTIC ATTENUATION COEFFICIENT. When a plane harmonic sound wave is propagated through an attenuating medium, it is generally found that the amplitude of the wave decays exponentially with distance. The absolute value of the natural logarithm of the ratio of the peak sound pressures (or the peak particle velocities) at two points, unit distance apart, is called the acoustic attenuation coefficient or, more strictly, the acoustic amplitude attenuation coefficient. In the absence of scattering, this quantity can be related to other measures of internal friction (see **friction, internal**). The acoustic attenuation coefficient is generally measured in nepers per cm.

ACOUSTIC ATTENUATION CONSTANT. See **acoustic attenuation coefficient.**

ACOUSTIC CAPACITANCE. See **compliance, acoustic.**

ACOUSTIC CENTER, EFFECTIVE. The point from which spherically divergent sound waves, emitted by an acoustic generator, appear to diverge.

ACOUSTIC COMPLIANCE. See **compliance, acoustic.**

ACOUSTIC DAMPING. See **damping, acoustic.**

ACOUSTIC ENERGY EQUATION. See **Franklin equation.**

ACOUSTIC IMPEDANCE. See **impedance, acoustic.**

ACOUSTIC INERTANCE. See **inertance, acoustic.**

ACOUSTIC MASS. See **mass, acoustic.**

ACOUSTIC MEAN FREE PATH. See **mean free path, acoustic.**

ACOUSTIC OHM. See **acoustic units.**

ACOUSTIC PHASE CONSTANT. The acoustic phase constant is the imaginary part of the **acoustic propagation constant.** The commonly used unit is the radian per section or per unit distance. In the case of a symmetrical structure, the imaginary parts of both the transfer constant and the acoustic propagation constant are identical, and have been called the "wavelength constant."

ACOUSTIC POWER GENERATED. See **radiation, acoustic.**

ACOUSTIC PRINCIPLE OF SIMILARITY. For any acoustical system involving **diffraction** phenomena, it is possible to construct a new system on a different scale which will perform in similar fashion, provided that the wavelength of the sound is altered in the same ratio as the linear dimensions of the original system.

ACOUSTIC PROPAGATION CONSTANT. Of a uniform system or of a section of a system of recurrent structures, the natural logarithm of the complex ratio of the steady-state particle velocities (see **velocity, particle**), **volume velocities,** or pressures at two points separated by unit distance in the uniform system (assumed to be of infinite length), or at two successive corresponding points in the system of recurrent structures (assumed to be of infinite length). The ratio is determined by dividing the value at the point nearer the transmitting end by the corresponding value at the more remote point.

ACOUSTIC RADIATION. See **radiation, acoustic.**

ACOUSTIC REACTANCE. See **reactance, acoustic.**

ACOUSTIC RECIPROCITY THEOREM. See **reciprocity theorem, acoustical.**