教 育 部 高 等 教 育 司 推 荐
国外优秀信息科学与技术系列教学用书

数 值 分 析

（第七版　影印版）

NUMERICAL ANALYSIS

(Seventh Edition)

Richard L. Burden
J. Douglas Faires

高 等 教 育 出 版 社
Higher Education Press
BROOKS/COLE
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前　　言

20世纪末，以计算机和通信技术为代表的信息科学和技术，对世界的经济、军事、科技、教育、文化、卫生等方面的发展产生了深刻的影响，由此而兴起的信息产业已经成为世界经济发展的支柱。进入21世纪，各国为了加快本国的信息产业，加大了资金投入和政策扶持。

为了加快我国信息产业的进程，在我国《国民经济和社会发展第十个五年计划纲要》中，明确提出“以信息化带动工业化，发挥后发优势，实现社会生产力的跨越式发展。”信息产业的国际竞争将日趋激烈。在我国加入WTO后，我国信息产业将面临国外竞争对手的严峻挑战。竞争成败最终将取决于信息科学和技术人才的多少与优劣。

在20世纪末，我国信息产业虽然得到迅猛发展，但与国际先进国家相比，差距还很大。为了赶上并超过国际先进水平，我国必须加快信息技术人才的培养，特别要培养一大批具有国际竞争能力的高水平的信息技术人才，促进我国信息产业和国家信息化水平的全面提高。为此，教育部高等教育司根据教育部吕福源副部长的意见，在长期重视推动高等学校信息科学和技术教学的基础上，将实施超前发展战略，采取一些重要举措，加快推动高等学校的信息科学和技术等相关专业的教学工作。在大力宣传、推荐我国专家编著的面向21世纪和“九五”重点的信息科学和技术课程教材的基础上，在有条件的高等学校的某些信息科学和技术课程中推动使用国外优秀教材的影印版进行英语或双语教学，以缩短我国在计算机教学上与国际先进水平的差距，同时也有助于强化我国大学生的英语水平。

为了达到上述目的，在分析一些出版社已影印相关教材，一些学校已试用影印教材进行教学的基础上，教育部高等教育司组织并委托高等教育出版社开展国外优秀信息科学和技术优秀教材及其教学辅助材料的引进研究与影印出版的试点工作。为推动用影印版教材进行教学创造条件。

本次引进的系列教材的影印出版工作，是在对我国高校信息科学和技术专业的课程与美国高校的进行对比分析的基础上展开的；所影印出版的教材均由我国主要高校

的信息科学和技术专家组成的专家组，从国外近两年出版的大量最新教材中精心筛选评审通过的内容新、有影响的优秀教材；影印教材的定价原则上应与我国大学教材价格相当。

教育部高等教育司将此影印系列教材推荐给高等学校，希望有关教师选用，使用后有什么意见和建议请及时反馈。也希望有条件的出版社，根据影印教材的要求，积极参加此项工作，以便引进更多、更新、更好的外国教材和教学辅助材料。

同时，感谢国外有关出版公司对此项引进工作的配合，欢迎更多的国外公司关心并参与此项工作。

教育部高等教育司

二〇〇一年四月

Preface

About the Text

We have developed this material for a sequence of courses on the theory and application of numerical approximation techniques. The text is designed primarily for junior-level mathematics, science, and engineering majors who have completed at least the first year of the standard college calculus sequence. Familiarity with the fundamentals of matrix algebra and differential equations is also useful, but adequate introductory material on these topics is presented in the text so that those courses need not be prerequisites.

Previous editions of *Numerical Analysis* have been used in a wide variety of situations. In some cases, the mathematical analysis underlying the development of approximation techniques was emphasized rather than the methods themselves; in others, the emphasis was reversed. The book has also been used as the core reference for courses at the beginning graduate level in engineering and computer science programs, and in first-year courses in introductory analysis offered at international universities. We have tried to adapt the book to fit these diverse users without compromising our original purpose:

> *To give an introduction to modern approximation techniques; to explain how, why, and when they can be expected to work; and to provide a firm basis for future study of numerical analysis and scientific computing.*

The book contains sufficient material for a full year of study, but we expect many readers to use the text only for a single-term course. In such a course, students learn to identify the types of problems that require numerical techniques for their solution and see examples of the error propagation that can occur when numerical methods are applied. They accurately approximate the solutions of problems that cannot be solved exactly and learn techniques for estimating error bounds for the approximations. The remainder of the text serves as a reference for methods that are not considered in the course. Either the full-year or single-course treatment is consistent with the aims of the text.

Virtually every concept in the text is illustrated by example, and this edition contains more than 2,000 class-tested exercises ranging from elementary applications of methods and algorithms to generalizations and extensions of the theory. In addition, the exercise sets include many applied problems from diverse areas of engineering, as well as from the physical, computer, biological, and social sciences. The applications chosen demonstrate concisely how numerical methods can be, and often must be, applied in real-life situations.

A number of software packages have been developed to produce symbolic mathematical computations. Predominant among them in the academic environment are Derive®, Maple®, and Mathematica®. Student versions of these software packages are available at reasonable prices for most common computer systems. Although there are significant differences among the packages, both in performance and price, all can perform standard algebra and calculus operations. Having a symbolic computation package available can be very useful in the study of approximation techniques. The results in most of our examples and exercises have been generated using problems for which exact values *can* be determined, since this permits the performance of the approximation method to be monitored. Exact solutions can often be obtained quite easily using symbolic computation. In addition, for many numerical techniques the error analysis requires bounding a higher ordinary or partial derivative of a function, which can be a tedious task and one that is not particularly instructive once the techniques of calculus have been mastered. Derivatives can be quickly obtained symbolically, and a little insight often permits a symbolic computation to aid in the bounding process as well.

We have chosen Maple as our standard package because of its wide distribution, but Derive or Mathematica can be substituted with only minor modifications. Examples and exercises have been added whenever we felt that a computer algebra system would be of significant benefit, and we have discussed the approximation methods that Maple employs when it is unable to solve a problem exactly.

New for This Edition

The seventh edition includes two new major sections. The Preconditioned Conjugate Gradient method has been added to Chapter 7 to provide a more complete treatment of the numerical solution to systems of linear equations. It is presented as an iterative approximation technique for solving positive definite linear systems. In this form, it is particularly useful for approximating the solution to large sparse systems.

In Chapter 10 we have added a section on Homotopy and Continuation methods. These methods provide a distinctly different technique for approximating the solutions to nonlinear systems of equations, one that has attracted a great deal of attention recently.

We have also added extensive listings of Maple code throughout the book, since reviewers found this feature useful in the sixth edition. We have updated all the Maple code to Release 6, the version that will be current by the time the book is printed. Those familiar with our past editions will find that virtually every page has been improved in some way. All the references have been updated and revised, and new exercises have been added. We hope you will find these changes beneficial to the teaching and study of numerical analysis; most have been motivated by changes in the presentation of the material to our own students.

Another important modification in this edition is a web site at

http://www.as.ysu.edu/~faires/Numerical-Analysis/

On this web site we will place updated programs as the software changes and post responses to comments made by users of the book. We can also add new material that might be included in subsequent editions in the form of PDF files that users can download. Our hope is that this will extend the life of the seventh edition while keeping the material in the book up to date.

Algorithms

As in previous editions, we give a detailed, structured algorithm without program listing for each method in the text. The algorithms are in a form that can be coded, even by those with limited programming experience.

This edition includes a disk containing programs for solutions to representative exercises using the algorithms. The programs for each algorithm are written in Fortran, Pascal, and C. In addition, we have coded the programs using Maple and Mathematica, as well as in MATLAB®, a computer software package that is widely used for linear algebra applications. This should ensure that a set of programs is available for most common computing systems.

A *Student Study Guide* is available with this edition that illustrates the calls required for these programs, which is useful for those with limited programming experience. The study guide also contains worked-out solutions to many of the problems.

Brooks/Cole can provide instructors with an *Instructor's Manual* that provides answers and solutions to all the exercises in the book. Computation results in the *Instructor's Manual* were regenerated for this edition, using the programs on the disk to ensure compatibility among the various programming systems.

The algorithms in the text lead to programs that give correct results for the examples and exercises in the text, but no attempt was made to write general-purpose professional software. Specifically, the algorithms are not always written in a form that leads to the *most efficient* program in terms of either time or storage requirements. When a conflict occurred between writing an extremely efficient algorithm and writing a slightly different one that better illustrates the important features of the method, the latter path was invariably taken.

About the Program Disk

The CD on the inside back cover of the book contains programs for all the algorithms in the book, in numerous formats, as well as samples of the Student Study Guide for the book in both the PostScript® (PS) and the Adobe® Portable Document (PDF) formats.

For each algoithm there is a C, Fortran, Maple, Mathematica, MATLAB, and Pascal program, and for some of these systems there are multiple programs that depend on the particular version of the software that is being run. Every program is illustrated with a sample problem that is closely correlated to the text. This permits the program to be run initially in the language of your choice to see the form of the input and output. The programs can then be modified for other problems by making minor changes. The form of the input and output are, as nearly as possible, the same in each of the programming systems. This permits an instructor using the programs to discuss them generically, without regard to the particular programming system an individual student uses.

The programs are designed to run on a minimally configured computer. All that is required is a computer running MS-DOS®, Windows®, or the Macintosh® operating system. You will, however, need appropriate software, such as a compiler for Pascal, Fortran, and C, or one of the computer algebra systems (Maple, Mathematica, and MATLAB). There are six subdirectories on the disk, one for each of the computer languages and the accompanying data files.

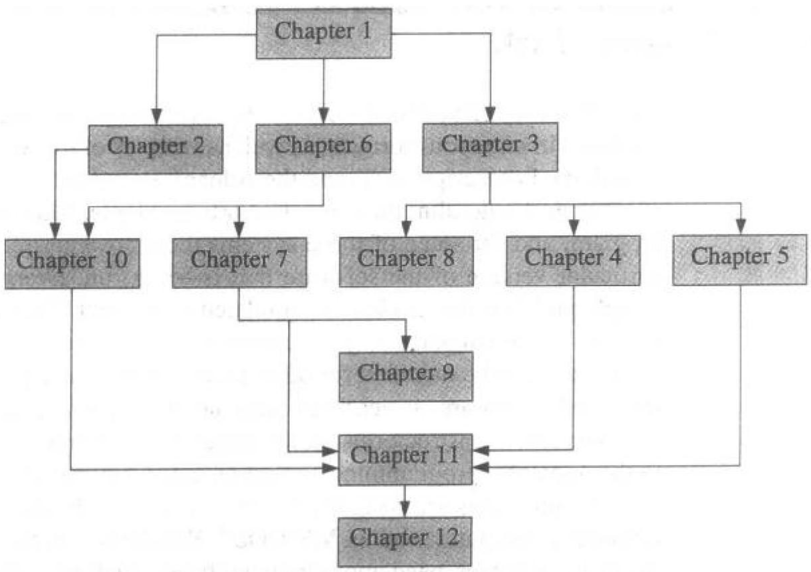
All of the programs are given as ASCII files or worksheets. They can be altered using any editor or word processor that creates a standard ASCII file. (These are also commonly called "Text Only" files.)

Extensive README files are included with the program files so that the peculiarities of the various programming systems can be individually addressed. The README files are presented both in ASCII format and as PDF files. As new software is developed, the algorithms will be updated and placed on the web site for the book.

Suggested Course Outlines

Numerical Analysis is designed to allow instructors flexibility in the choice of topics, as well as in the level of theoretical rigor and in the emphasis on applications. In line with these aims, we provide detailed references for the results that are not demonstrated in the text and for the applications that are used to indicate the practical importance of the methods. The text references cited are those most likely to be available in college libraries and have been updated to reflect the most recent edition at the time this book was placed into production. We also include quotations from original research papers when we feel this material is accessible to our intended audience.

The following flowchart indicates chapter prerequisites. The only deviation from this chart is described in the footnote at the bottom of the first page of Section 3.4. Most of the possible sequences that can be generated from this chart have been taught by the authors at Youngstown State University.



Acknowledgments

We feel most fortunate to have had so many of our students and colleagues communicate with us regarding their impressions of earlier editions of this book. All of these comments are taken very seriously; we have tried to include all the suggestions that are in line with the philosophy of the book, and are extremely grateful to all those that have taken the time to contact us and inform us of improvements we can make in subsequent versions.

We would particularly like to thank the following, whose efforts we greatly appreciate.

Glen Granzow, Idaho State University
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We want especially to thank our friend and former student Jim Baglama of Ball State University. Jim agreed to be an extensive reviewer for this edition and was particularly helpful in updating our survey sections and references to software. It is most gratifying to see one's students move through the profession.

Also moving through his profession, but in a completly different manner, is our Editor and Publisher Gary Ostedt. Gary has been an outstanding manager of our projects and a good personal friend. We will very much miss his direction and assistance, and would like to take this opportunity to wish him all the best in his upcoming retirement from Brooks/Cole.

As has been our practice in past editions of the book, we have used student help at Youngstown State University in preparing the seventh edition. Our able assistant for this edition was Laurie Marinelli, whom we thank for all her work. We would also like to express gratitude to our colleagues on the faculty and administration of Youngstown State University for providing us the opportunity and facilities to complete this project.

Finally, we would like to thank all those who have used and adopted the various editions of *Numerical Analysis* over the years. It has been wonderful to hear from so many students, and new faculty, who used our book for their first exposure to the study of numerical methods. We hope this edition continues the trend and adds to the enjoyment of students studying numerical analysis. If you have any suggestions for improvements that can be incorporated into future editions of the book, we would be grateful for your comments. We can be contacted by electronic mail at the addresses listed below.

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CHAPTER

1

Mathematical Preliminaries

■ ■ ■

IN beginning chemistry courses, we see the *ideal gas law*,

$$PV = NRT,$$

which relates the pressure P, volume V, temperature T, and number of moles N of an "ideal" gas. In this equation, R is a constant that depends on the measurement system.

Suppose two experiments are conducted to test this law, using the same gas in each case. In the first experiment,

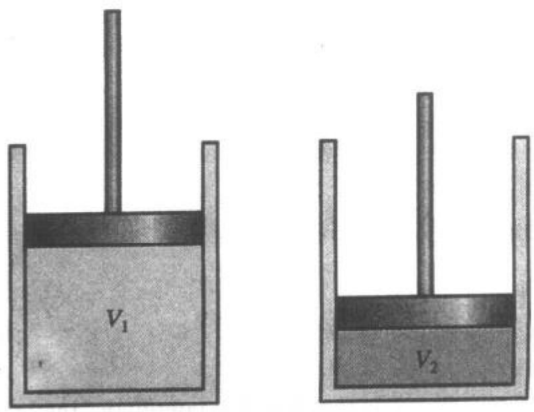
$$P = 1.00 \text{ atm}, \qquad V = 0.100 \text{ m}^3,$$

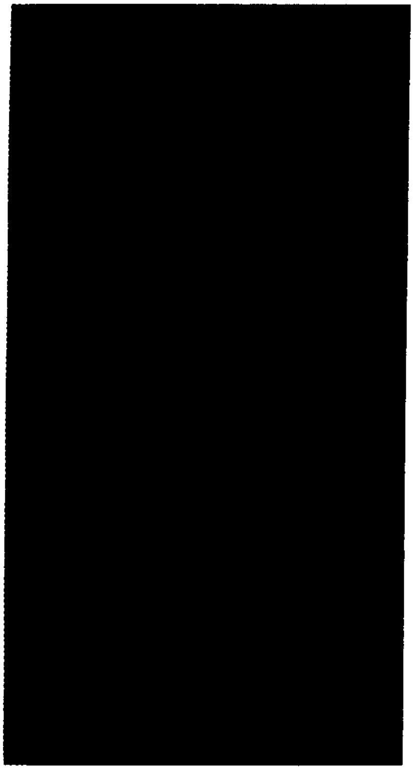
$$N = 0.00420 \text{ mol}, \qquad R = 0.08206.$$

The ideal gas law predicts the temperature of the gas to be

$$T = \frac{PV}{NR} = \frac{(1.00)(0.100)}{(0.00420)(0.08206)} = 290.15 \text{ K} = 17^\circ\text{C}.$$

When we measure the temperature of the gas, we find that the true temperature is 15°C.





We then repeat the experiment using the same values of *R* and *N*, but increase the pressure by a factor of two and reduce the volume by the same factor. Since the product *PV* remains the same, the predicted temperature is still 17°C, but we find that the actual temperature of the gas is now 19°C.

Clearly, the ideal gas law is suspect, but before concluding that the law is invalid in this situation, we should examine the data to see whether the error can be attributed to the experimental results. If so, we might be able to determine how much more accurate our experimental results would need to be to ensure that an error of this magnitude could not occur.

Analysis of the error involved in calculations is an important topic in numerical analysis and is introduced in Section 1.2. This particular application is considered in Exercise 28 of that section.

This chapter contains a short review of those topics from elementary single-variable calculus that will be needed in later chapters, together with an introduction to convergence, error analysis, and the machine representation of numbers.

1.1 Review of Calculus

The concepts of *limit* and *continuity* of a function are fundamental to the study of calculus.

Definition 1.1 A function f defined on a set X of real numbers has the **limit** L at x_0, written

$$\lim_{x \to x_0} f(x) = L,$$

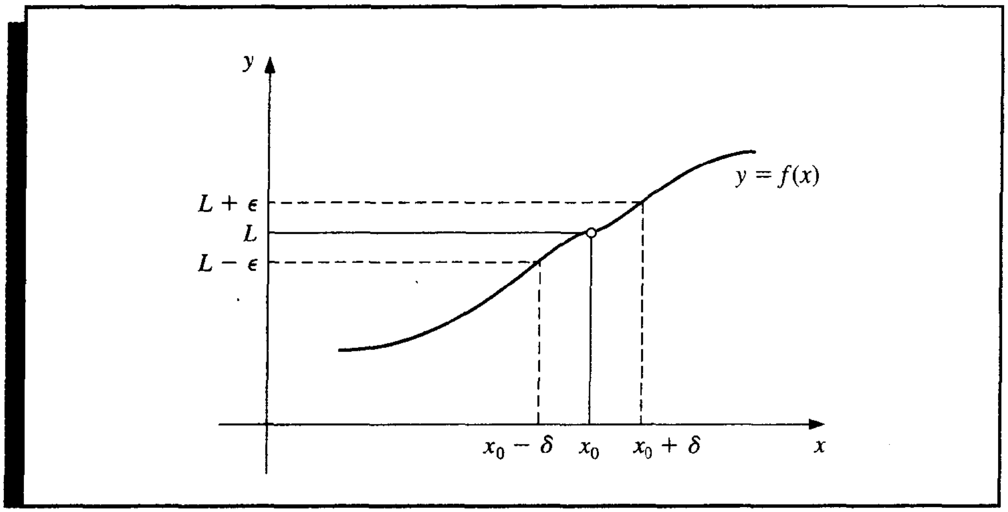
if, given any real number $\epsilon > 0$, there exists a real number $\delta > 0$ such that $|f(x) - L| < \epsilon$, whenever $x \in X$ and $0 < |x - x_0| < \delta$. (See Figure 1.1.) ■

Definition 1.2 Let f be a function defined on a set X of real numbers and $x_0 \in X$. Then f is **continuous** at x_0 if

$$\lim_{x \to x_0} f(x) = f(x_0).$$

The function f is continuous on the set X if it is continuous at each number in X. ■

Figure 1.1



$C(X)$ denotes the set of all functions that are continuous on X. When X is an interval of the real line, the parentheses in this notation are omitted. For example, the set of all functions continuous on the closed interval $[a, b]$ is denoted $C[a, b]$.

The *limit of a sequence* of real or complex numbers is defined in a similar manner.

Definition 1.3 Let $\{x_n\}_{n=1}^{\infty}$ be an infinite sequence of real or complex numbers. The sequence $\{x_n\}_{n=1}^{\infty}$ has the **limit** x (**converges to** x) if, for any $\epsilon > 0$, there exists a positive integer $N(\epsilon)$ such that $|x_n - x| < \epsilon$, whenever $n > N(\epsilon)$. The notation

$$\lim_{n\to\infty} x_n = x, \quad \text{or} \quad x_n \to x \quad \text{as} \quad n \to \infty,$$

means that the sequence $\{x_n\}_{n=1}^{\infty}$ converges to x. ■

The following theorem relates the concepts of convergence and continuity.

Theorem 1.4 If f is a function defined on a set X of real numbers and $x_0 \in X$, then the following statements are equivalent:

a. f is continuous at x_0;

b. If $\{x_n\}_{n=1}^{\infty}$ is any sequence in X converging to x_0, then $\lim_{n\to\infty} f(x_n) = f(x_0)$.

■

The functions we consider when discussing numerical methods are assumed to be continuous since this is a minimal requirement for predictable behavior. Functions that are not continuous can skip over points of interest, which can cause difficulties when attempting to approximate a solution to a problem. More sophisticated assumptions about a function generally lead to better approximation results. For example, a function with a smooth