
玉蜻蜓

PRACTICAL OPTIMIZATION

PHILIP E. GILL
WALTER MURRAY
MARGARET H. WRIGHT

Systems Optimization Laboratory
Department of Operations Research
Stanford University
California, USA

ACADEMIC PRESS INC (LONDON) LTD
24/28 Oval Road,
London NW1

United States Edition published by
ACADEMIC PRESS INC
111 Fifth Avenue
New York, New York 10003

Copyright © 1981 by
ACADEMIC PRESS INC (LONDON) LTD
Second Printing 1982

All rights reserved
No part of this book may be reproduced in any form by photostat, microfilm, or any other means, without written permission from the publishers

British Library Cataloguing in Publication Data
Gill, P E
Practical optimization
1 Mathematics optimization
I Title II Murray, W III Wright, M H
515 QA402 5

ISBN 0-12-283952 8

LCCCN 81-66366

Printed in Great Britain by
St Edmundsbury Press, Bury St Edmunds, Suffolk

PREFACE

As researchers at the National Physical Laboratory and Stanford University, and as contributors to the Numerical Algorithms Group (NAG) software library, we have been involved for many years in the development of numerical methods and software for the solution of optimization problems. Within the past twenty years, there has been a dramatic increase in the efficiency and reliability of optimization methods for almost all problem categories. However, this improved capability has been achieved by the use of more complicated ideas, particularly from the areas of numerical linear algebra and finite-precision calculation.

The best methods available today are extremely complex, and their manner of operation is far from obvious, especially to users from other disciplines. This book is intended as a treatment — necessarily broad — of the subject of *practical optimization*. The word "practical" is included in the title in order to convey our concern not only with the motivation for optimization methods, but also with details of implementation that affect the performance of a method in practice. In particular, we believe that some consideration of the effects of finite-precision computation is essential in order for any description of a method to be useful. We also believe that it is important to discuss the linear algebraic processes that are used to perform certain portions of all optimization methods.

This book is meant to be largely self-contained; we have therefore devoted one chapter to a description of the essential results from numerical linear algebra and the analysis of rounding errors in computation, and a second chapter to a treatment of optimality conditions.

Selected methods for unconstrained, linearly constrained, and nonlinearly constrained optimization are described in three chapters. This discussion is intended to present an overview of the methods, including the underlying motivation as well as particular theoretical and computational features. Illustrations have been used wherever possible in order to stress the geometric interpretation of the methods. The methods discussed are primarily those with which we have had extensive experience and success; other methods are described that provide special insights or background. References to methods not discussed and to further details are given in the extensive Notes and Bibliography at the end of each section. The methods have been presented in sufficient detail to allow this book to be used as a text for a university-level course in numerical optimization.

Two chapters are devoted to selected less formal, but nonetheless crucial, topics that might be viewed as "advice to users". For example, some suggestions concerning modelling are included because we have observed that an understanding of optimization methods can have a beneficial effect on the modelling of the activities to be optimized. In addition, we have presented an extensive discussion of topics that are crucial in using and understanding a numerical optimization method — such as selecting a method, interpreting the computed results, and diagnosing (and, if possible, curing) difficulties that may cause an algorithm to fail or perform poorly.

62958

In writing this book, the authors have had the benefit of advice and help from many people. In particular, we offer special thanks to our friend and colleague Michael Saunders, not only for many helpful comments on various parts of the book, but also for all of his characteristic good humour and patience when the task of writing this book seemed to go on forever. He has played a major role in much of the recent work on algorithms for large-scale problems described in Chapters 5 and 6.

We gratefully acknowledge David Martin for his tireless help and support for the optimization group at the National Physical Laboratory, and George Dantzig for his efforts to assemble the algorithms group at the Systems Optimization Laboratory, Stanford University.

We thank Brian Hinde, Susan Hodson, Enid Long and David Rhead for their work in developing and testing many of the algorithms described in this book.

We are grateful to Greg Dobson, David Fuchs, Stefania Gai, Richard Stone and Wes Winkler, who have been helpful in many ways during the preparation of this manuscript. The clarity of certain parts of the text has been improved because of helpful comments from Paulo Benevides-Soares, Andy Conn, Laureano Escudero, Don Iglehart, James Lyness, Jorge Moré, Michael Overton and Danny Sorensen.

We thank Clive Hall for producing the computer-generated figures on the Laserscan plotter at the National Physical Laboratory. The remaining diagrams were expertly drawn by Nancy Cimina.

This book was typeset by the authors using the TEX mathematical typesetting system of Don Knuth*. We are grateful to Don Knuth for his efforts in devising the TEX system, and for making available to us various TEX macros that improved the quality of the final text. We also thank him for kindly allowing us to use the Alphatype CRS typesetter, and Chris Tucci for his substantial help in producing the final copy.

Finally, we offer our deepest personal thanks to those closest to us, who have provided encouragement, support and humour during the time-consuming process of writing this book.

Stanford University
May, 1981

P. E. G.
W. M.
M. H. W.

*D. E. Knuth, *TEX and METAFONT, New Directions in Typesetting*, American Mathematical Society and Digital Press, Bedford, Massachusetts (1979).

此为试读，需要完整PDF请访问：www.ertongbook.com

CONTENTS

Those sections marked with "*" contain material of a rather specialized nature that may be omitted on first reading.

此为试读，需要完整PDF请访问： www.ertongbook.com

CHAPTER ONE

INTRODUCTION

Mankind always sets itself only such problems as it can solve. . . .

—KARL MARX (1859)

1.1. DEFINITION OF OPTIMIZATION PROBLEMS

An optimization problem begins with a set of independent variables or parameters, and often includes conditions or restrictions that define acceptable values of the variables. Such restrictions are termed the *constraints* of the problem. The other essential component of an optimization problem is a single measure of "goodness", termed the *objective function*, which depends in some way on the variables. The solution of an optimization problem is a set of allowed values of the variables for which the objective function assumes an "optimal" value. In mathematical terms, optimization usually involves *maximizing* or *minimizing*; for example, we may wish to maximize profit or minimize weight.

Problems in all areas of mathematics, applied science, engineering, economics, medicine, and statistics can be posed in terms of optimization. In particular, mathematical models are often developed in order to analyze and understand complex phenomena. Optimization is used in this context to determine the form and characteristics of the model that corresponds most closely to reality. Furthermore, most decision-making procedures involve explicit solution of an optimization problem to make the "best" choice. In addition to their role *per se*, optimization problems often arise as critical sub-problems within other numerical processes. This situation is so common that the existence of the optimization problem may pass unremarked — for example, when an optimization problem must be solved to find points where a function reaches a certain critical value.

Throughout this book, it will be assumed that the ultimate objective is to *compute* the solution of an optimization problem. In order to devise solution techniques, it is helpful to assume that optimization problems can be posed in a standard form. It is clearly desirable to select a standard form that arises naturally from the nature of most optimization problems, in order to reduce the need for re-formulation. The general form of optimization problem to be considered may be expressed in mathematical terms as:

NCP		
	$\underset{x \in \Re^n}{\text{minimize}}$	$F(x)$
	subject to	$c_i(x) = 0, \quad i = 1, 2, \ldots, m';$
		$c_i(x) \geq 0, \quad i = m' + 1, \ldots, m.$

The objective function F and constraint functions $\{c_i\}$ (which, taken together, are termed the *problem functions*) are real-valued scalar functions.

To illustrate some of the flavour and diversity of optimization problems, we consider two specific examples. Firstly, the layout of the text of this book was designed by the TEX computer typesetting system using optimization techniques. The aim of the system is to produce a visually pleasing arrangement of text with justified margins. Two means are available to achieve this objective: the spaces between letters, words, lines and paragraphs can be adjusted, and words can be split between lines by hyphenation. An "ideal" spacing is specified for every situation in which a space may occur. These spaces may then be stretched or compressed within given limits of acceptability, subject to penalties for increasing the amount of deviation from the ideal. Varying penalties are also imposed to minimize undesirable features, such as two consecutive lines that end with hyphenated words or a page that begins with a displayed equation. The process of choosing a good text layout includes many of the elements of a general optimization problem: a single function that measures quality, parameters that can be adjusted in order to achieve the best objective, and restrictions on the form and extent of the allowed variation.

We next consider a simplified description of a real problem that illustrates the convenience of the standard form. The problem is to design the nose cone of a vehicle, such that, when it travels at hypersonic speeds, the air drag is minimized. Hence, the function to be optimized is the drag, and the parameters to be adjusted are the specifications of the structure of the nose cone. In order to be able to compute the objective function, it is necessary first to devise a model of the nose cone in terms of the chosen parameters. For this problem, the nose cone is represented as a series of conical sections, with a spherical section as the front piece and a fixed final radius R. Figure 1a illustrates the chosen model, and shows the parameters to be adjusted. Although the idealized model deviates from a real-world nose cone, the approximation should not noticeably impair the quality of the solution, provided that the number of conical sections is sufficiently large.

The next step is to formulate the drag as a scalar function of the eight parameters $\alpha_1, \ldots, \alpha_4$, $r_1, \ldots, r_4$. The function $D(\alpha_1, \ldots, \alpha_4, r_1, \ldots, r_4)$ will be assumed to compute an estimate of the drag on the nose cone for a set of particular values of the free variables, and thus D will be the objective function of the resulting optimization problem.

In order to complete the formulation of the problem, some restrictions must be imposed on the values of the design parameters in order for the mathematical model to be meaningful and for the optimal solution to be implementable. In particular, the radii $r_1, \ldots, r_4$ must not be negative, so that the constraints

$$r_i \geq 0, \quad i = 1, \ldots, 4,$$

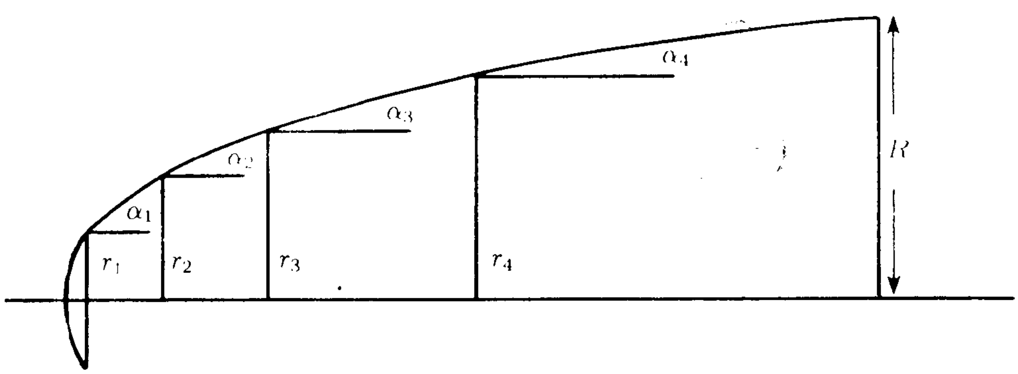

Figure 1a. Cross-section of a conical representation of a nose cone.

此为试读，需要完整PDF请访问：www.ertongbook.com