教育部高等教育司推荐
国外优秀信息科学与技术系列教学用书

离散数学结构

（第四版 影印版）

DISCRETE MATHEMATICAL STRUCTURES

(Fourth Edition)

Bernard Kolman
Robert C. Busby
Sharon Cutler Ross

高等教育出版社
Higher Education Press
Prentice Hall
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Bernard Kolman 等

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前　言

20世纪末，以计算机和通信技术为代表的信息科学和技术，对世界的经济、军事、科技、教育、文化、卫生等方面的发展产生了深刻的影响，由此而兴起的信息产业已经成为世界经济发展的支柱。进入21世纪，各国为了加快本国的信息产业，加大了资金投入和政策扶持。

为了加快我国信息产业的进程，在我国《国民经济和社会发展第十个五年计划纲要》中，明确提出“以信息化带动工业化，发挥后发优势，实现社会生产力的跨越式发展。”信息产业的国际竞争将日趋激烈。在我国加入WTO后，我国信息产业将面临国外竞争对手的严峻挑战。竞争成败最终将取决于信息科学和技术人才的多少与优劣。

在20世纪末，我国信息产业虽然得到迅猛发展，但与国际先进国家相比，差距还很大。为了赶上并超过国际先进水平，我国必须加快信息技术人才的培养，特别要培养一大批具有国际竞争能力的高水平的信息技术人才，促进我国信息产业和国家信息化水平的全面提高。为此，教育部高等教育司根据教育部吕福源副部长的意见，在长期重视推动高等学校信息科学和技术教学的基础上，将实施超前发展战略，采取一些重要举措，加快推动高等学校的信息科学和技术等相关专业的教学工作。在大力宣传、推荐我国专家编著的面向21世纪和“九五”重点的信息科学和技术课程教材的基础上，在有条件的高等学校的某些信息科学和技术课程中推动使用国外优秀教材的影印版进行英语或双语教学，以缩短我国在计算机教学上与国际先进水平的差距，同时也有助于强化我国大学生的英语水平。

为了达到上述目的，在分析一些出版社已影印相关教材，一些学校已试用影印教材进行教学的基础上，教育部高等教育司组织并委托高等教育出版社开展国外优秀信息科学和技术优秀教材及其教学辅助材料的引进研究与影印出版的试点工作。为推动用影印版教材进行教学创造条件。

本次引进的系列教材的影印出版工作，是在对我国高校信息科学和技术专业的课程与美国高校的进行对比分析的基础上展开的；所影印出版的教材均由我国主要高校

的信息科学和技术专家组成的专家组，从国外近两年出版的大量最新教材中精心筛选评审通过的内容新、有影响的优秀教材；影印教材的定价原则上应与我国大学教材价格相当。

教育部高等教育司将此影印系列教材推荐给高等学校，希望有关教师选用，使用后有什么意见和建议请及时反馈。也希望有条件的出版社，根据影印教材的要求，积极参加此项工作，以便引进更多、更新、更好的外国教材和教学辅助材料。

同时，感谢国外有关出版公司对此项引进工作的配合，欢迎更多的国外公司关心并参与此项工作。

教育部高等教育司

二〇〇一年四月

To the memory of Lillie
B.K.

To my wife, Patricia, and our sons, Robert and Scott
R.C.B.

To Bill and bill
S.C.R.

PREFACE

Discrete mathematics is a difficult course to teach and to study at the freshman and sophomore level for several reasons. It is a hybrid course. Its content is mathematics, but many of its applications and more than half its students are from computer science. Thus careful motivation of topics and previews of applications are important and necessary strategies. Moreover, the number of substantive and diverse topics covered in the course is high, so that student must absorb them rather quickly. At the same time, the student may also be expected to develop proof-writing skills.

APPROACH

First, we have limited both the areas covered and the depth of coverage to what we deemed prudent in a *first course* taught at the freshman and sophomore level. We have identified a set of topics that we feel are of genuine use in computer science and elsewhere and that can be presented in a logically coherent fashion. We have presented an introduction to these topics along with an indication of how they can be pursued in greater depth.

For example, we cover the simpler finite-state machines, not Turing machines. We have limited the coverage of abstract algebra to a discussion of semigroups and groups and have given application of these to the important topics of finite-state machines and error-detecting and error-correcting codes. Error-correcting codes, in turn, have been primarily restricted to simple linear codes.

Second, the material has been organized and interrelated to minimize the mass of definitions and the abstraction of some of the theory. Relations and digraphs are treated as two aspects of the same fundamental mathematical idea, with a directed graph being a pictorial representation of a relation. This fundamental idea is then used as the basis of virtually all the concepts introduced in the book, including functions, partial orders, graphs, and algebraic structures. Whenever possible, each new idea introduced in the text uses previously encountered material and, in turn, is developed in such a way that it simplifies the more complex ideas that follow. Thus partial orders, lattices, and Boolean algebras develop from general relations. This material in turn leads naturally to other algebraic structures.

WHAT IS NEW IN THE FOURTH EDITION

We continue to be pleased by the reception given to earlier editions of this book. We still believe that the book works well in the classroom because of the unifying role played by two key concepts: relations and digraphs. For this edition we have modified the order of topics slightly and made extensive revisions of the exercise sets. The discourse on proof has been expanded in several ways. One of these is the insertion of comments on nearly every proof in the book. Whatever changes we have made, our goal continues to be that of maximizing the clarity of presentation. As the audience for an introductory discrete mathematics course changes and as the course is increasingly used as a bridge course, we have added the following features.

- A new section, Transport Networks, introduces this topic using ideas from Chapter 4.
- A new section, Matching Problems, applies the techniques of transport networks to a broad class of problems.
- The section on mathematical induction now includes the strong form of induction as well.
- The discussion of proofs and proof techniques is now woven throughout the book with comments on most proofs, more exercises related to the mechanics of proving statements, and Tips for Proofs sections. Tips for Proofs highlight the types of proofs commonly seen for that chapter's material and methods for selecting fruitful proof strategies.
- A Self-Test is provided for each chapter with answers for all problems given at the back of the book.
- Exercise Sets have a broader range of problems: more routine problems and more challenging problems. More exercises focus on the mechanics of proof and proof techniques. As with writing in general, students learn to write proofs not only by reading, analyzing, and recognizing the structure of proofs, but especially by writing, re-writing, and writing more proofs themselves.

EXERCISES

The exercises form an integral part of the book. Many are computational in nature, whereas others are of a theoretical type. Many of the latter and the experiments, to be further described below, require verbal solutions. Exercises to help develop proof-writing skills ask the student to analyze proofs, amplify arguments, or complete partial proofs. Answers to all odd-numbered exercises appear in the back of the book. Solutions to all exercises appear in the **Instructor's Manual**, which is available (to instructors only) gratis from the publisher. The Instructor's Manual also includes notes on the pedagogical ideas underlying each chapter, goals and grading guidelines for the experiments further described below, and a test bank.

EXPERIMENTS

Appendix B contains a number of assignments that we call experiments. These provide an opportunity for discovery and exploration, or a more-in-depth look at various topics discussed in the text. These are suitable for group work. Content prerequisites for each experiment are given in the Instructor's Manual.

END OF CHAPTER MATERIAL

Each chapter contains Tips for Proofs, a summary of Key Ideas, a set of Coding Exercises, and a Self-Test covering the chapter's material.

CONTENT

Chapter 1 contains a miscellany of basic material required in the course. This includes sets, subsets, and their operations; sequences; division in the integers; matrices; and mathematical structures. A goal of this chapter is to help students develop skills in identifying patterns on many levels. Chapter 2 covers logic and related material, including methods of proof and mathematical induction. Although the discussion of proof is based on this chapter, the commentary continues throughout the book. Chapter 3, on counting, deals with permutations, combinations, the pigeonhole principle, elements of probability, and recurrence relations. Chapter 4 presents basic types and properties of relations, along with their representation as directed graphs. Connections with matrices and other data structures are also explored in this chapter. The power of multiple representations for the concept of relation is fully exploited. Chapter 5 deals with the notion of a function and gives important examples of functions, including functions of special interest in computer science. An introduction to the growth of functions is developed.

Chapter 6 covers partially ordered sets, including lattices and Boolean algebras. Chapter 7 introduces directed and undirected trees along with applications of these ideas. Elementary graph theory is the focus of Chapter 8. New to this edition are sections on Transport Networks and Matching Problems; these build on the foundation of Chapter 4.

In Chapter 9 we give the basic theory of semigroups and groups. These ideas are applied in Chapters 10 and 11. Chapter 10 is devoted to finite-state machines. It complements and makes effective use of ideas developed in previous chapters. Chapter 11 treats the subject of binary coding.

Appendix A discusses algorithms and pseudocode. The simplified pseudocode presented here is used in some text examples and exercises; these may be omitted without loss of continuity. Appendix B gives a collection of experiments dealing with extensions or previews of topics in various parts of the course.

USE OF THIS TEXT

This text can be used by students in mathematics as an introduction to the fundamental ideas of discrete mathematics, and as a foundation for the development of more advanced mathematical concepts. If used in this way, the topics dealing with specific computer science applications can be ignored or selected independently as important examples. The text can also be used in a computer science or computer engineering curriculum to present the foundations of many basic computer-related concepts, and provide a coherent development and common theme for these ideas.

The instructor can easily develop a suitable course by referring to the chapter prerequisites, which identify material needed by that chapter.

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B.K.
R.C.B.
S.C.R.

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1

FUNDAMENTALS

Prerequisites: There are no formal prerequisites for this chapter; the reader is encouraged to read carefully and work through all examples.

In this chapter we introduce some of the basic tools of discrete mathematics. We begin with sets, subsets, and their operations, notions with which you may already be familiar. Next we deal with sequences, using both explicit and recursive patterns. Then we review some of the basic divisibility properties of the integers. Finally we introduce matrices and matrix operations. This gives us the background needed to begin our exploration of mathematical structures.

1.1 SETS AND SUBSETS

Sets

A **set** is any well-defined collection of objects called the **elements** or **members of the set**. For example, the collection of all wooden chairs, the collection of all one-legged black birds, or the collection of real numbers between zero and one is each a set. Well-defined just means that it is possible to decide if a given object belongs to the collection or not. Almost all mathematical objects are first of all sets, regardless of any additional properties they may possess. Thus set theory is, in a sense, the foundation on which virtually all of mathematics is constructed. In spite of this, set theory (at least the informal brand we need) is quite easy to learn and use.

One way of describing a set that has a finite number of elements is by listing the elements of the set between braces. Thus the set of all positive integers that are less than 4 can be written as

$$\{1, 2, 3\}. \tag{1}$$

The order in which the elements of a set are listed is not important. Thus $\{1, 3, 2\}$, $\{3, 2, 1\}$, $\{3, 1, 2\}$, $\{2, 1, 3\}$, and $\{2, 3, 1\}$ are all representations of the set given in (1). Moreover, repeated elements in the listing of the elements of a set can be ignored. Thus, $\{1, 3, 2, 3, 1\}$ is another representation of the set given in (1).

We use uppercase letters such as A, B, C to denote sets, and lowercase letters such as a, b, c, x, y, z, t to denote the members (or elements) of sets.

We indicate the fact that x is an element of the set A by writing $x \in A$, and we indicate the fact that x is not an element of A by writing $x \notin A$.

EXAMPLE 1 Let $A = \{1, 3, 5, 7\}$. Then $1 \in A$, $3 \in A$, but $2 \notin A$. ■

Sometimes it is inconvenient or impossible to describe a set by listing all its elements. Another useful way to define a set is by specifying a property that the elements of the set have in common. We use the notation P(x) to denote a sentence or statement P concerning the variable object x. The set defined by P(x), written $\{x \mid P(x)\}$, is just the collection of all objects for which P is sensible and true. For example, $\{x \mid x$ is a positive integer less than $4\}$ is the set $\{1, 2, 3\}$ described in (1) by listing its elements.

EXAMPLE 2 The set consisting of all the letters in the word "byte" can be denoted by {b, y, t, e} or by $\{x \mid x$ is a letter in the word "byte"$\}$. ■

EXAMPLE 3 We introduce here several sets and their notations that will be used throughout this book.

(a) $Z^+ = \{x \mid x$ is a positive integer$\}$.
Thus Z^+ consists of the numbers used for counting: $1, 2, 3 \ldots$.

(b) $N = \{x \mid x$ is a positive integer or zero$\}$.
Thus N consists of the positive integers and zero: $0, 1, 2, \ldots$.

(c) $Z = \{x \mid x$ is an integer$\}$.
Thus Z consists of all the integers: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

(d) $\mathbb{Q} = \{x \mid x$ is a rational number$\}$.
Thus $\mathbb{Q}$ consists of numbers that can be written as $\frac{a}{b}$, where a and b are integers and b is not 0.

(e) $\mathbb{R} = \{x \mid x$ is a real number$\}$.

(f) The set that has no elements in it is denoted either by { } or the symbol $\varnothing$ and is called the **empty set**. ■

EXAMPLE 4 Since the square of a real number is always nonnegative,

$$\{x \mid x \text{ is a real number and } x^2 = -1\} = \varnothing. \quad ■$$

Sets are completely known when their members are all known. Thus we say two sets A and B are **equal** if they have the same elements, and we write $A = B$.

EXAMPLE 5 If $A = \{1, 2, 3\}$ and $B = \{x \mid x$ is a positive integer and $x^2 < 12\}$, then $A = B$. ■

EXAMPLE 6 If $A = \{$BASIC, PASCAL, ADA$\}$ and $B = \{$ADA, BASIC, PASCAL$\}$, then $A = B$. ■

Subsets

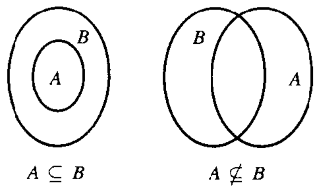

Figure 1.1

If every element of A is also an element of B, that is, if whenever $x \in A$ then $x \in B$, we say that A is a **subset** of B or that A is **contained in** B, and we write $A \subseteq B$. If A is not a subset of B, we write $A \not\subseteq B$. (See Figure 1.1.)

Diagrams, such as those in Figure 1.1, which are used to show relationships between sets, are called **Venn diagrams** after the British logician John Venn. Venn diagrams will be used extensively in Section 1.2.

EXAMPLE 7

We have $Z^+ \subseteq Z$. Moreover, if $\mathbb{Q}$ denotes the set of rational numbers, then $Z \subseteq \mathbb{Q}$. ■

EXAMPLE 8

Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 5\}$, and $C = \{1, 2, 3, 4, 5\}$. Then $B \subseteq A$, $B \subseteq C$, and $C \subseteq A$. However, $A \not\subseteq B$, $A \not\subseteq C$, and $C \not\subseteq B$. ■

EXAMPLE 9

If A is any set, then $A \subseteq A$. That is, every set is a subset of itself. ■

EXAMPLE 10

Let A be a set and let $B = \{A, \{A\}\}$. Then, since A and $\{A\}$ are elements of B, we have $A \in B$ and $\{A\} \in B$. It follows that $\{A\} \subseteq B$ and $\{\{A\}\} \subseteq B$. However, it is not true that $A \subseteq B$. ■

For any set A, since there are no elements of $\varnothing$ that are not in A, we have $\varnothing \subseteq A$. (We will look at this again in Section 2.1.)

It is easy to see that $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

The collection of everything, it turns out, cannot be considered a set without presenting serious logical difficulties. To avoid this and other problems, which need not concern us here, we will assume that for each discussion there is a "universal set" U (which will vary with the discussion) containing all objects for which the discussion is meaningful. Any other set mentioned in the discussion will automatically be assumed to be a subset of U. Thus, if we are discussing real numbers and we mention sets A and B, then A and B must (we assume) be sets of real numbers, not matrices, electronic circuits, or rhesus monkeys. In most problems, a universal set will be apparent from the setting of the problem. In Venn diagrams, the universal set U will be denoted by a rectangle, while sets within U will be denoted by circles as shown in Figure 1.2.

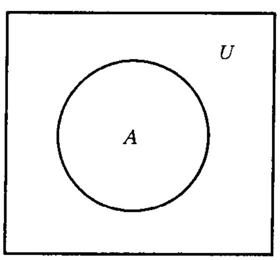

Figure 1.2

A set A is called **finite** if it has n distinct elements, where $n \in N$. In this case, n is called the **cardinality** of A and is denoted by $|A|$. Thus, the sets of Examples 1, 2, 4, 5, and 6 are finite. A set that is not finite is called **infinite**. The sets introduced in Example 3 (except $\varnothing$) are infinite sets.

If A is a set, then the set of all subsets of A is called the **power set** of A and is denoted by $P(A)$.

EXAMPLE 11

Let $A = \{1, 2, 3\}$. Then $P(A)$ consists of the following subsets of A: $\{\ \}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, and $\{1, 2, 3\}$ (or A). In a later section, we will count the number of subsets that a set can have. ■

1.1 Exercises

1. Let $A = \{1, 2, 4, a, b, c\}$. Identify each of the following as true or false.

(a) $2 \in A$ (b) $3 \in A$ (c) $c \notin A$
(d) $\varnothing \in A$ (e) $\{\ \} \notin A$ (f) $A \in A$

2. Let $A = \{x \mid x$ is a real number and $x < 6.\}$. Identify each of the following as true or false.

(a) $3 \in A$ (b) $6 \in A$ (c) $5 \notin A$
(d) $8 \notin A$ (e) $-8 \in A$ (f) $3.4 \notin A$

3. In each part, give the set of letters in each word by listing the elements of the set.

(a) AARDVARK (b) BOOK
(c) MISSISSIPPI

4. Give the set by listing its elements.

(a) The set of all positive integers that are less than ten.
(b) $\{x \mid x \in Z \text{ and } x^2 < 12\}$

5. Let $A = \{1, \{2, 3\}, 4\}$. Identify each of the following as true or false.

(a) $3 \in A$ (b) $\{1, 4\} \subseteq A$ (c) $\{2, 3\} \subseteq A$
(d) $\{2, 3\} \in A$ (e) $\{4\} \in A$ (f) $\{1, 2, 3\} \subseteq A$

In Exercises 6 *through* 9, *write the set in the form* $\{x \mid P(x)\}$, *where* $P(x)$ *is a property that describes the elements of the set.*

6. $\{2, 4, 6, 8, 10\}$ **7.** $\{a, e, i, o, u\}$

8. $\{1, 8, 27, 64, 125\}$ **9.** $\{-2, -1, 0, 1, 2\}$

10. Let $A = \{1, 2, 3, 4, 5\}$. Which of the following sets are equal to A?

(a) $\{4, 1, 2, 3, 5\}$ (b) $\{2, 3, 4\}$ (c) $\{1, 2, 3, 4, 5, 6\}$
(d) $\{x \mid x \text{ is an integer and } x^2 \le 25\}$
(e) $\{x \mid x \text{ is a positive integer and } x \le 5\}$
(f) $\{x \mid x \text{ is a positive rational number and } x \le 5\}$

11. Which of the following sets are the empty set?

(a) $\{x \mid x \text{ is a real number and } x^2 - 1 = 0\}$
(b) $\{x \mid x \text{ is a real number and } x^2 + 1 = 0\}$
(c) $\{x \mid x \text{ is a real number and } x^2 = -9\}$
(d) $\{x \mid x \text{ is a real number and } x = 2x + 1\}$
(e) $\{x \mid x \text{ is a real number and } x = x + 1\}$

12. List all the subsets of $\{a, b\}$.

13. List all the subsets of {BASIC, PASCAL, ADA}.

14. List all the subsets of $\{\ \}$.

15. Let $A = \{1, 2, 5, 8, 11\}$. Identify each of the following as true or false.

(a) $\{5, 1\} \subseteq A$ (b) $\{8, 1\} \in A$
(c) $\{1, 8, 2, 11, 5\} \not\subseteq A$ (d) $\varnothing \subseteq A$
(e) $\{1, 6\} \not\subseteq A$ (f) $\{2\} \subseteq A$
(g) $\{3\} \notin A$ (h) $A \subseteq \{11, 2, 5, 1, 8, 4\}$

16. Let $A = \{x \mid x \text{ is an integer and } x^2 < 16\}$. Identify each of the following as true or false.

(a) $\{0, 1, 2, 3\} \subseteq A$ (b) $\{-3, -2, -1\} \subseteq A$
(c) $\{\ \} \subseteq A$
(d) $\{x \mid x \text{ is an integer and } |x| < 4\} \subseteq A$
(e) $A \subseteq \{-3, -2, -1, 0, 1, 2, 3\}$

17. Let $A = \{1\}$, $B = \{1, a, 2, b, c\}$, $C = \{b, c\}$, $D = \{a, b\}$, and $E = \{1, a, 2, b, c, d\}$. For each part, replace the symbol $\square$ with either $\subseteq$ or $\not\subseteq$ to give a true statement.

(a) $A \square B$ (b) $\varnothing \square A$ (c) $B \square C$
(d) $C \square E$ (e) $D \square C$ (f) $B \square E$

In Exercises 18 *through* 20, *find the set of smallest cardinality that contains the given sets as subsets.*

18. $\{a, b, c\}$, $\{a, d, e, f\}$, $\{b, c, e, g\}$

19. $\{1, 2\}$, $\{1, 3\}$, $\varnothing$

20. $\{2, 4, 6, \ldots, 20\}$, $\{3, 6, 9, \ldots, 21\}$

21. Is it possible to have two different (appropriate) universal sets for a collection of sets? Would having different universal sets create any problems? Explain.

22. Use the Venn diagram in Figure 1.3 to identify each of the following as true or false.

(a) $A \subseteq B$ (b) $B \subseteq A$ (c) $C \subseteq B$
(d) $x \in B$ (e) $x \in A$ (f) $y \in B$

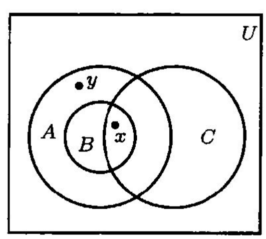

Figure 1.3