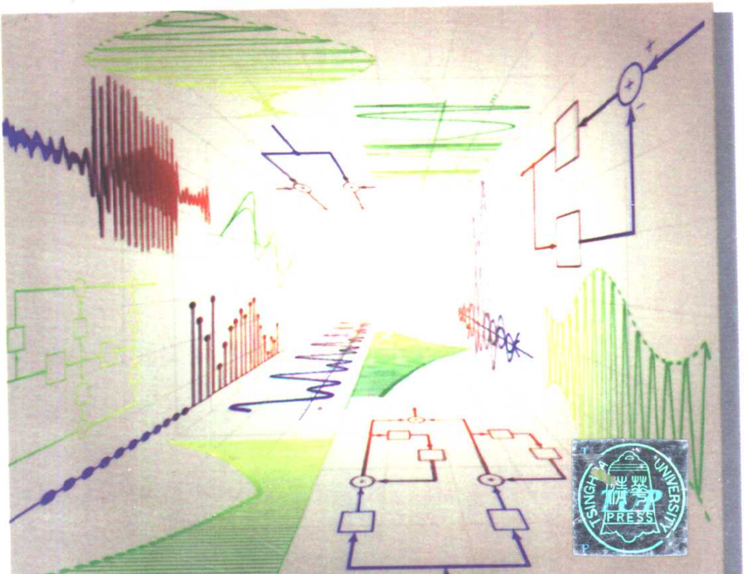
電子工程系列丛书（影印版）

SIGNALS & SYSTEMS

SECOND EDITION

信号与系统

第 2 版



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清华大学出版社与 Prentice Hall 出版公司合作推出的“大学计算机教育丛书(影印版)”和“ATM 与 B-ISDN 技术丛书(影印版)”受到了广大读者的欢迎。很多读者通过电话、信函、电子邮件对我们的工作以积极评价,并提出了不少极好的建议,令我们感动和鼓舞。我们除了继续努力完善上述两套丛书以外,还将努力拓宽影印图书的专业范围,以更好地满足读者的需要。

电子工程是信息科学的基础,高等学校新的教学要求指出,计算机专业和电子学专业的学生应相互学习渗透到彼此的专业领域,拓宽知识面,以适应信息技术飞速发展的时代。培养通晓相关专业领域知识的人才,成为面向新世纪的理工科教育的迫切要求。为此,我们挑选了与信息科学、电子学有关的国外优秀著作,组成电子工程系列丛书(影印版),奉献给国内读者。我们希望这套新的丛书能为国内的大专院校师生和科研单位的工作人员提供新的知识和营养,也衷心期待着读者对我们一如既往的支持。

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PREFACE

This book is the second edition of a text designed for undergraduate courses in signals and systems. While such courses are frequently found in electrical engineering curricula, the concepts and techniques that form the core of the subject are of fundamental importance in all engineering disciplines. In fact, the scope of potential and actual applications of the methods of signal and system analysis continues to expand as engineers are confronted with new challenges involving the synthesis or analysis of complex processes. For these reasons we feel that a course in signals and systems not only is an essential element in an engineering program but also can be one of the most rewarding, exciting, and useful courses that engineering students take during their undergraduate education.

Our treatment of the subject of signals and systems in this second edition maintains the same general philosophy as in the first edition but with significant rewriting, restructuring, and additions. These changes are designed to help both the instructor in presenting the subject material and the student in mastering it. In the preface to the first edition we stated that our overall approach to signals and systems had been guided by the continuing developments in technologies for signal and system design and implementation, which made it increasingly important for a student to have equal familiarity with techniques suitable for analyzing and synthesizing both continuous-time and discrete-time systems. As we write the preface to this second edition, that observation and guiding principle are even more true than before. Thus, while students studying signals and systems should certainly have a solid foundation in disciplines based on the laws of physics, they must also have a firm grounding in the use of computers for the analysis of phenomena and the implementation of systems and algorithms. As a consequence, engineering curricula now reflect a blend of subjects, some involving continuous-time models and others focusing on the use of computers and discrete representations. For these reasons, signals and systems courses that bring discrete-time and continuous-time concepts together in a unified way play an increasingly important role in the education of engineering students and in their preparation for current and future developments in their chosen fields.

It is with these goals in mind that we have structured this book to develop in parallel the methods of analysis for continuous-time and discrete-time signals and systems. This approach also offers a distinct and extremely important pedagogical advantage. Specifically, we are able to draw on the similarities between continuous- and discrete-time methods in order to share insights and intuition developed in each domain. Similarly, we can exploit the differences between them to sharpen an understanding of the distinct properties of each.

In organizing the material both originally and now in the second edition, we have also considered it essential to introduce the student to some of the important uses of the basic methods that are developed in the book. Not only does this provide the student with an appreciation for the range of applications of the techniques being learned and for directions for further study, but it also helps to deepen understanding of the subject. To achieve this goal we include introductory treatments on the subjects of filtering, communications, sampling, discrete-time processing of continuous-time signals, and feedback. In fact, in one of the major changes in this second edition, we have introduced the concept of frequency-domain filtering very early in our treatment of Fourier analysis in order to provide both motivation for and insight into this very important topic. In addition, we have again included an up-to-date bibliography at the end of the book in order to assist the student who is interested in pursuing additional and more advanced studies of the methods and applications of signal and system analysis.

The organization of the book reflects our conviction that full mastery of a subject of this nature cannot be accomplished without a significant amount of practice in using and applying the tools that are developed. Consequently, in the second edition we have significantly increased the number of worked examples within each chapter. We have also enhanced one of the key assets of the first edition, namely the end-of-chapter homework problems. As in the first edition, we have included a substantial number of problems, totaling more than 600 in number. A majority of the problems included here are new and thus provide additional flexibility for the instructor in preparing homework assignments. In addition, in order to enhance the utility of the problems for both the student and the instructor we have made a number of other changes to the organization and presentation of the problems. In particular, we have organized the problems in each chapter under several specific headings, each of which spans the material in the entire chapter but with a different objective. The first two sections of problems in each chapter emphasize the mechanics of using the basic concepts and methods presented in the chapter. For the first of these two sections, which has the heading Basic Problems with Answers, we have also provided answers (but not solutions) at the end of the book. These answers provide a simple and immediate way for the student to check his or her understanding of the material. The problems in this first section are generally appropriate for inclusion in homework sets. Also, in order to give the instructor additional flexibility in assigning homework problems, we have provided a second section of Basic Problems for which answers have not been included.

A third section of problems in each chapter, organized under the heading of Advanced Problems, is oriented toward exploring and elaborating upon the foundations and practical implications of the material in the text. These problems often involve mathematical derivations and more sophisticated use of the concepts and methods presented in the chapter. Some chapters also include a section of Extension Problems which involve extensions of material presented in the chapter and/or involve the use of knowledge from applications that are outside the scope of the main text (such as advanced circuits or mechanical systems). The overall variety and quantity of problems in each chapter will hopefully provide students with the means to develop their understanding of the material and instructors with considerable flexibility in putting together homework sets that are tailored to the specific needs of their students. A solutions manual is also available to instructors through the publisher.

Another significant additional enhancement to this second edition is the availability of the companion book *Explorations in Signals and Systems Using MATLAB* by Buck, Daniel, and Singer. This book contains MATLAB™-based computer exercises for each topic in the text, and should be of great assistance to both instructor and student.

Students using this book are assumed to have a basic background in calculus as well as some experience in manipulating complex numbers and some exposure to differential equations. With this background, the book is self-contained. In particular, no prior experience with system analysis, convolution, Fourier analysis, or Laplace and z-transforms is assumed. Prior to learning the subject of signals and systems most students will have had a course such as basic circuit theory for electrical engineers or fundamentals of dynamics for mechanical engineers. Such subjects touch on some of the basic ideas that are developed more fully in this text. This background can clearly be of great value to students in providing additional perspective as they proceed through the book.

The Foreword, which follows this preface, is written to offer the reader motivation and perspective for the subject of signals and systems in general and our treatment of it in particular. We begin Chapter 1 by introducing some of the elementary ideas related to the mathematical representation of signals and systems. In particular we discuss transformations (such as time shifts and scaling) of the independent variable of a signal. We also introduce some of the most important and basic continuous-time and discrete-time signals, namely real and complex exponentials and the continuous-time and discrete-time unit step and unit impulse. Chapter 1 also introduces block diagram representations of interconnections of systems and discusses several basic system properties such as causality, linearity and time-invariance. In Chapter 2 we build on these last two properties, together with the sifting property of unit impulses to develop the convolution-sum representation for discrete-time linear, time-invariant (LTI) systems and the convolution integral representation for continuous-time LTI systems. In this treatment we use the intuition gained from our development of the discrete-time case as an aid in deriving and understanding its continuous-time counterpart. We then turn to a discussion of causal, LTI systems characterized by linear constant-coefficient differential and difference equations. In this introductory discussion we review the basic ideas involved in solving linear differential equations (to which most students will have had some previous exposure) and we also provide a discussion of analogous methods for linear difference equations. However, the primary focus of our development in Chapter 2 is not on methods of solution, since more convenient approaches are developed later using transform methods. Instead, in this first look, our intent is to provide the student with some appreciation for these extremely important classes of systems, which will be encountered often in subsequent chapters. Finally, Chapter 2 concludes with a brief discussion of singularity functions—steps, impulses, doublets, and so forth—in the context of their role in the description and analysis of continuous-time LTI systems. In particular, we stress the interpretation of these signals in terms of how they are defined under convolution—that is, in terms of the responses of LTI systems to these idealized signals.

Chapters 3 through 6 present a thorough and self-contained development of the methods of Fourier analysis in both continuous and discrete time and together represent the most significant reorganization and revision in the second edition. In particular, as we indicated previously, we have introduced the concept of frequency-domain filtering at a much earlier point in the development in order to provide motivation for and a concrete application of the Fourier methods being developed. As in the first edition, we begin the discussions in Chapter 3 by emphasizing and illustrating the two fundamental reasons for the important role Fourier analysis plays in the study of signals and systems in both continuous and discrete time: (1) extremely broad classes of signals can be represented as weighted sums or integrals of complex exponentials; and (2) the response of an LTI system to a complex exponential input is the same exponential multiplied by a complex-number characteristic of the system. However, in contrast to the first edition, the focus of attention in Chapter 3 is on Fourier series representations for periodic signals in both continuous time and discrete time. In this way we not only introduce and examine many of the properties of Fourier representations without the additional mathematical generalization required to obtain the Fourier transform for aperiodic signals, but we also can introduce the application to filtering at a very early stage in the development. In particular, taking advantage of the fact that complex exponentials are eigenfunctions of LTI systems, we introduce the frequency response of an LTI system and use it to discuss the concept of frequency-selective filtering, to introduce ideal filters, and to give several examples of nonideal filters described by differential and difference equations. In this way, with a minimum of mathematical preliminaries, we provide the student with a deeper appreciation for what a Fourier representation means and why it is such a useful construct.

Chapters 4 and 5 then build on the foundation provided by Chapter 3 as we develop first the continuous-time Fourier transform in Chapter 4 and, in a parallel fashion, the discrete-time Fourier transform in Chapter 5. In both chapters we derive the Fourier transform representation of an aperiodic signal as the limit of the Fourier series for a signal whose period becomes arbitrarily large. This perspective emphasizes the close relationship between Fourier series and transforms, which we develop further in subsequent sections and which allows us to transfer the intuition developed for Fourier series in Chapter 3 to the more general context of Fourier transforms. In both chapters we have included a discussion of the many important properties of Fourier transforms, with special emphasis placed on the convolution and multiplication properties. In particular, the convolution property allows us to take a second look at the topic of frequency-selective filtering, while the multiplication property serves as the starting point for our treatment of sampling and modulation in later chapters. Finally, in the last sections in Chapters 4 and 5 we use transform methods to determine the frequency responses of LTI systems described by differential and difference equations and to provide several examples illustrating how Fourier transforms can be used to compute the responses for such systems. To supplement these discussions (and later treatments of Laplace and z-transforms) we have again included an Appendix at the end of the book that contains a description of the method of partial fraction expansion.

Our treatment of Fourier analysis in these two chapters is characteristic of the parallel treatment we have developed. Specifically, in our discussion in Chapter 5, we are able to build on much of the insight developed in Chapter 4 for the continuous-time case, and toward the end of Chapter 5 we emphasize the complete duality in continuous-time and discrete-time Fourier representations. In addition, we bring the special nature of each domain into sharper focus by contrasting the differences between continuous- and discrete-time Fourier analysis.

As those familiar with the first edition will note, the lengths and scopes of Chapters 4 and 5 in the second edition are considerably smaller than their first edition counterparts. This is due not only to the fact that Fourier series are now dealt with in a separate chapter but also to our moving several topics into Chapter 6. The result, we believe, has several significant benefits. First, the presentation in three shorter chapters of the basic concepts and results of Fourier analysis, together with the introduction of the concept of frequency-selective filtering, should help the student in organizing his or her understanding of this material and in developing some intuition about the frequency domain and appreciation for its potential applications. Then, with Chapters 3-5 as a foundation, we can engage in a more detailed look at a number of important topics and applications. In Chapter 6 we take a deeper look at both the time- and frequency-domain characteristics of LTI systems. For example, we introduce magnitude-phase and Bode plot representations for frequency responses and discuss the effect of frequency response phase on the time domain characteristics of the output of an LTI system. In addition, we examine the time- and frequency-domain behavior of ideal and nonideal filters and the tradeoffs between these that must be addressed in practice. We also take a careful look at first- and second-order systems and their roles as basic building blocks for more complex system synthesis and analysis in both continuous and discrete time. Finally, we discuss several other more complex examples of filters in both continuous and discrete time. These examples together with the numerous other aspects of filtering explored in the problems at the end of the chapter provide the student with some appreciation for the richness and flavor of this important subject. While each of the topics in Chapter 6 was present in the first edition, we believe that by reorganizing and collecting them in a separate chapter following the basic development of Fourier analysis, we have both simplified the introduction of this important topic in Chapters 3-5 and presented in Chapter 6 a considerably more cohesive picture of time- and frequency-domain issues.

In response to suggestions and preferences expressed by many users of the first edition we have modified notation in the discussion of Fourier transforms to be more consistent with notation most typically used for continuous-time and discrete-time Fourier transforms. Specifically, beginning with Chapter 3 we now denote the continuous-time Fourier transform as $X(j\omega)$ and the discrete-time Fourier transform as $X(e^{j\omega})$. As with all options with notation, there is not a unique best choice for the notation for Fourier transforms. However, it is our feeling, and that of many of our colleagues, that the notation used in this edition represents the preferable choice.

Our treatment of sampling in Chapter 7 is concerned primarily with the sampling theorem and its implications. However, to place this subject in perspective we begin by discussing the general concepts of representing a continuous-time signal in terms of its samples and the reconstruction of signals using interpolation. After using frequency-domain methods to derive the sampling theorem, we consider both the frequency and time domains to provide intuition concerning the phenomenon of aliasing resulting from undersampling. One of the very important uses of sampling is in the discrete-time processing of continuous-time signals, a topic that we explore at some length in this chapter. Following this, we turn to the sampling of discrete-time signals. The basic result underlying discrete-time sampling is developed in a manner that parallels that used in continuous time, and the applications of this result to problems of decimation and interpolation are described. Again a variety of other applications, in both continuous and discrete time, are addressed in the problems.

Once again the reader acquainted with our first edition will note a change, in this case involving the reversal in the order of the presentation of sampling and communications. We have chosen to place sampling before communications in the second edition both because we can call on simple intuition to motivate and describe the processes of sampling and reconstruction from samples and also because this order of presentation then allows us in Chapter 8 to talk more easily about forms of communication systems that are closely related to sampling or rely fundamentally on using a sampled version of the signal to be transmitted.

Our treatment of communications in Chapter 8 includes an in-depth discussion of continuous-time sinusoidal amplitude modulation (AM), which begins with the straightforward application of the multiplication property to describe the effect of sinusoidal AM in the frequency domain and to suggest how the original modulating signal can be recovered. Following this, we develop a number of additional issues and applications related to sinusoidal modulation, including frequency-division multiplexing and single-sideband modulation. Many other examples and applications are described in the problems. Several additional topics are covered in Chapter 8. The first of these is amplitude modulation of a pulse train and time-division multiplexing, which has a close connection to the topic of sampling in Chapter 7. Indeed we make this tie even more explicit and provide a look into the important field of digital communications by introducing and briefly describing the topics of pulse-amplitude modulation (PAM) and intersymbol interference. Finally, our discussion of frequency modulation (FM) provides the reader with a look at a nonlinear modulation problem. Although the analysis of FM systems is not as straightforward as for the AM case, our introductory treatment indicates how frequency-domain methods can be used to gain a significant amount of insight into the characteristics of FM signals and systems. Through these discussions and the many other aspects of modulation and communications explored in the problems in this chapter we believe that the student can gain an appreciation both for the richness of the field of communications and for the central role that the tools of signals and systems analysis play in it.

Chapters 9 and 10 treat the Laplace and z-transforms, respectively. For the most part, we focus on the bilateral versions of these transforms, although in the last section of each chapter we discuss unilateral transforms and their use in solving differential and difference equations with nonzero initial conditions. Both chapters include discussions on: the close relationship between these transforms and Fourier transforms; the class of rational transforms and their representation in terms of poles and zeros; the region of convergence of a Laplace or z-transform and its relationship to properties of the signal with which it is associated; inverse transforms using partial fraction expansion; the geometric evaluation of system functions and frequency responses from pole-zero plots; and basic transform properties. In addition, in each chapter we examine the properties and uses of system functions for LTI systems. Included in these discussions are the determination of system functions for systems characterized by differential and difference equations; the use of system function algebra for interconnections of LTI systems; and the construction of cascade, parallel- and direct-form block-diagram representations for systems with rational system functions.

The tools of Laplace and z-transforms form the basis for our examination of linear feedback systems in Chapter 11. We begin this chapter by describing a number of the important uses and properties of feedback systems, including stabilizing unstable systems, designing tracking systems, and reducing system sensitivity. In subsequent sections we use the tools that we have developed in previous chapters to examine three topics that are of importance for both continuous-time and discrete-time feedback systems. These are root locus analysis, Nyquist plots and the Nyquist criterion, and log-magnitude/phase plots and the concepts of phase and gain margins for stable feedback systems.

The subject of signals and systems is an extraordinarily rich one, and a variety of approaches can be taken in designing an introductory course. It was our intention with the first edition and again with this second edition to provide instructors with a great deal of flexibility in structuring their presentations of the subject. To obtain this flexibility and to maximize the usefulness of this book for instructors, we have chosen to present thorough, in-depth treatments of a cohesive set of topics that forms the core of most introductory courses on signals and systems. In achieving this depth we have of necessity omitted introductions to topics such as descriptions of random signals and state space models that are sometimes included in first courses on signals and systems. Traditionally, at many schools, such topics are not included in introductory courses but rather are developed in more depth in follow-on undergraduate courses or in courses explicitly devoted to their investigation. Although we have not included an introduction to state space in the book, instructors of introductory courses can easily incorporate it into the treatments of differential and difference equations that can be found throughout the book. In particular, the discussions in Chapters 9 and 10 on block diagram representations for systems with rational system functions and on unilateral transforms and their use in solving differential and difference equations with initial conditions form natural points of departure for the discussions of state-space representations.

A typical one-semester course at the sophomore-junior level using this book would cover Chapters 1-5 in reasonable depth (although various topics in each chapter are easily omitted at the discretion of the instructor) with selected topics chosen from the remaining chapters. For example, one possibility is to present several of the basic topics in Chapters 6-8 together with a treatment of Laplace and z-transforms and perhaps a brief introduction to the use of system function concepts to analyze feedback systems. A variety of alternate formats are possible, including one that incorporates an introduction to state space or one in which more focus is placed on continuous-time systems by de-emphasizing Chapters 5 and 10 and the discrete-time topics in Chapters 3, 7, 8, and 11.

In addition to these course formats this book can be used as the basic text for a thorough, two-semester sequence on linear systems. Alternatively, the portions of the book not used in a first course on signals and systems can, together with other sources, form the basis for a subsequent course. For example, much of the material in this book forms a direct bridge to subjects such as state space analysis, control systems, digital signal processing, communications and statistical signal processing. Consequently, a follow-on course can be constructed that uses some of the topics in this book together with supplementary material in order to provide an introduction to one or more of these advanced subjects. In fact, a new course following this model has been developed at MIT and has proven not only to be a popular course among our students but also a crucial component of our signals and systems curriculum.

As it was with the first edition, in the process of writing this book we have been fortunate to have received assistance, suggestions, and support from numerous colleagues, students and friends. The ideas and perspectives that form the heart of this book have continued to evolve as a result of our own experiences in teaching signals and systems and the influences of the many colleagues and students with whom we have worked. We would like to thank Professor Ian T. Young for his contributions to the first edition of this book and to thank and welcome Professor Hamid Nawab for the significant role he played in the development and complete restructuring of the examples and problems for this second edition. We also express our appreciation to John Buck, Michael Daniel and Andrew Singer for writing the MATLAB companion to the text. In addition, we would like to thank Jason Oppenheim for the use of one of his original photographs and Vivian Berman for her ideas and help in arriving at a cover design. Also, as indicated on the acknowledgment page that follows, we are deeply grateful to the many students and colleagues who devoted a significant number of hours to a variety of aspects of the preparation of this second edition.

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Cambridge, Massachusetts

FOREWORD

The concepts of signals and systems arise in a wide variety of fields, and the ideas and techniques associated with these concepts play an important role in such diverse areas of science and technology as communications, aeronautics and astronautics, circuit design, acoustics, seismology, biomedical engineering, energy generation and distribution systems, chemical process control, and speech processing. Although the physical nature of the signals and systems that arise in these various disciplines may be drastically different, they all have two very basic features in common. The signals, which are functions of one or more independent variables, contain information about the behavior or nature of some phenomenon, whereas the systems respond to particular signals by producing other signals or some desired behavior. Voltages and currents as a function of time in an electrical circuit are examples of signals, and a circuit is itself an example of a system, which in this case responds to applied voltages and currents. As another example, when an automobile driver depresses the accelerator pedal, the automobile responds by increasing the speed of the vehicle. In this case, the system is the automobile, the pressure on the accelerator pedal the input to the system, and the automobile speed the response. A computer program for the automated diagnosis of electrocardiograms can be viewed as a system which has as its input a digitized electrocardiogram and which produces estimates of parameters such as heart rate as outputs. A camera is a system that receives light from different sources and reflected from objects and produces a photograph. A robot arm is a system whose movements are the response to control inputs.

In the many contexts in which signals and systems arise, there are a variety of problems and questions that are of importance. In some cases, we are presented with a specific system and are interested in characterizing it in detail to understand how it will respond to various inputs. Examples include the analysis of a circuit in order to quantify its response to different voltage and current sources and the determination of an aircraft's response characteristics both to pilot commands and to wind gusts.

In other problems of signal and system analysis, rather than analyzing existing systems, our interest may be focused on designing systems to process signals in particular ways. One very common context in which such problems arise is in the design of systems to enhance or restore signals that have been degraded in some way. For example, when a pilot is communicating with an air traffic control tower, the communication can be degraded by the high level of background noise in the cockpit. In this and many similar cases, it is possible to design systems that will retain the desired signal, in this case the pilot's voice, and reject (at least approximately) the unwanted signal, i.e., the noise. A similar set of objectives can also be found in the general area of image restoration and image enhancement. For example, images from deep space probes or earth-observing satellites typically represent degraded versions of the scenes being imaged because of limitations of the imaging equipment, atmospheric effects, and errors in signal transmission in returning the images to earth. Consequently, images returned from space are routinely processed by systems to compensate for some of these degradations. In addition, such images are usually processed to enhance certain features, such as lines (corresponding, for example, to river beds or faults) or regional boundaries in which there are sharp contrasts in color or darkness.

In addition to enhancement and restoration, in many applications there is a need to design systems to extract specific pieces of information from signals. The estimation of heart rate from an electrocardiogram is one example. Another arises in economic forecasting. We may, for example, wish to analyze the history of an economic time series, such as a set of stock market averages, in order to estimate trends and other characteristics such as seasonal variations that may be of use in making predictions about future behavior. In other applications, the focus may be on the design of signals with particular properties. Specifically, in communications applications considerable attention is paid to designing signals to meet the constraints and requirements for successful transmission. For example, long distance communication through the atmosphere requires the use of signals with frequencies in a particular part of the electromagnetic spectrum. The design of communication signals must also take into account the need for reliable reception in the presence of both distortion due to transmission through the atmosphere and interference from other signals being transmitted simultaneously by other users.

Another very important class of applications in which the concepts and techniques of signal and system analysis arise are those in which we wish to modify or control the characteristics of a given system, perhaps through the choice of specific input signals or by combining the system with other systems. Illustrative of this kind of application is the design of control systems to regulate chemical processing plants. Plants of this type are equipped with a variety of sensors that measure physical signals such as temperature, humidity, and chemical composition. The control system in such a plant responds to these sensor signals by adjusting quantities such as flow rates and temperature in order to regulate the ongoing chemical process. The design of aircraft autopilots and computer control systems represents another example. In this case, signals measuring aircraft speed, altitude, and heading are used by the aircraft's control system in order to adjust variables such as throttle setting and the position of the rudder and ailerons. These adjustments are made to ensure that the aircraft follows a specified course, to smooth out the aircraft's ride, and to enhance its responsiveness to pilot commands. In both this case and in the previous example of chemical process control, an important concept, referred to as feedback, plays a major role, as measured signals are fed back and used to adjust the response characteristics of a system.

The examples in the preceding paragraphs represent only a few of an extraordinarily wide variety of applications for the concepts of signals and systems. The importance of these concepts stems not only from the diversity of phenomena and processes in which they arise, but also from the collection of ideas, analytical techniques, and methodologies that have been and are being developed and used to solve problems involving signals and systems. The history of this development extends back over many centuries, and although most of this work was motivated by specific applications, many of these ideas have proven to be of central importance to problems in a far larger variety of contexts than those for which they were originally intended. For example, the tools of Fourier analysis, which form the basis for the frequency-domain analysis of signals and systems, and which we will develop in some detail in this book, can be traced from problems of astronomy studied by the ancient Babylonians to the development of mathematical physics in the eighteenth and nineteenth centuries.

In some of the examples that we have mentioned, the signals vary continuously in time, whereas in others, their evolution is described only at discrete points in time. For example, in the analysis of electrical circuits and mechanical systems we are concerned with signals that vary continuously. On the other hand, the daily closing stock market average is by its very nature a signal that evolves at discrete points in time (i.e., at the close of each day). Rather than a curve as a function of a continuous variable, then, the closing stock market average is a sequence of numbers associated with the discrete time instants at which it is specified. This distinction in the basic description of the evolution of signals and of the systems that respond to or process these signals leads naturally to two parallel frameworks for signal and system analysis, one for phenomena and processes that are described in continuous time and one for those that are described in discrete time.

The concepts and techniques associated both with continuous-time signals and systems and with discrete-time signals and systems have a rich history and are conceptually closely related. Historically, however, because their applications have in the past been sufficiently different, they have for the most part been studied and developed somewhat separately. Continuous-time signals and systems have very strong roots in problems associated with physics and, in the more recent past, with electrical circuits and communications. The techniques of discrete-time signals and systems have strong roots in numerical analysis, statistics, and time-series analysis associated with such applications as the analysis of economic and demographic data. Over the past several decades, however, the disciplines of continuous-time and discrete-time signals and systems have become increasingly entwined and the applications have become highly interrelated. The major impetus for this has come from the dramatic advances in technology for the implementation of systems and for the generation of signals. Specifically, the continuing development of high-speed digital computers, integrated circuits, and sophisticated high-density device fabrication techniques has made it increasingly advantageous to consider processing continuous-time signals by representing them by time samples (i.e., by converting them to discrete-time signals). As one example, the computer control system for a modern high-performance aircraft digitizes sensor outputs such as vehicle speed in order to produce a sequence of sampled measurements which are then processed by the control system.

Because of the growing interrelationship between continuous-time signals and systems and discrete-time signals and systems and because of the close relationship among the concepts and techniques associated with each, we have chosen in this text to develop the concepts of continuous-time and discrete-time signals and systems in parallel. Since many of the concepts are similar (but not identical), by treating them in parallel, insight and intuition can be shared and both the similarities and differences between them become better focused. In addition, as will be evident as we proceed through the material, there are some concepts that are inherently easier to understand in one framework than the other and, once understood, the insight is easily transferable. Furthermore, this parallel treatment greatly facilitates our understanding of the very important practical context in which continuous and discrete time are brought together, namely the sampling of continuous-time signals and the processing of continuous-time signals using discrete-time systems.

As we have so far described them, the notions of signals and systems are extremely general concepts. At this level of generality, however, only the most sweeping statements can be made about the nature of signals and systems, and their properties can be discussed only in the most elementary terms. On the other hand, an important and fundamental notion in dealing with signals and systems is that by carefully choosing subclasses of each with particular properties that can then be exploited, we can analyze and characterize these signals and systems in great depth. The principal focus in this book is on the particular class of linear time-invariant systems. The properties of linearity and time invariance that define this class lead to a remarkable set of concepts and techniques which are not only of major practical importance but also analytically tractable and intellectually satisfying.

As we have emphasized in this foreword, signal and system analysis has a long history out of which have emerged some basic techniques and fundamental principles which have extremely broad areas of application. Indeed, signal and system analysis is constantly evolving and developing in response to new problems, techniques, and opportunities. We fully expect this development to accelerate in pace as improved technology makes possible the implementation of increasingly complex systems and signal processing techniques. In the future we will see signals and systems tools and concepts applied to an expanding scope of applications. For these reasons, we feel that the topic of signal and system analysis represents a body of knowledge that is of essential concern to the scientist and engineer. We have chosen the set of topics presented in this book, the organization of the presentation, and the problems in each chapter in a way that we feel will most help the reader to obtain a solid foundation in the fundamentals of signal and system analysis; to gain an understanding of some of the very important and basic applications of these fundamentals to problems in filtering, sampling, communications, and feedback system analysis; and to develop some appreciation for an extremely powerful and broadly applicable approach to formulating and solving complex problems.