AN OUTLINE OF MECHANICS

吴 镇 编

上海交通大学出版社

内 容 简 介

本书是编者按照1980年审订的高等工业学校《理论力学教学大纲》(草案)所编写的《理论力学》的英文摘要。其内容包括静力学、运动学、动力学与分析力学基础四部分；其章节与中文教材基本上相对应。这只是一本"纲要"，没有包括例题与习题。

本书可以作为高等工业学校学生学习理论力学课程时的辅助教材或课外读物，也可供理论力学教师教学时的参考。

理论力学纲要

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上海交通大学出版社出版

(淮海中路1984弄19号)

新华书店上海发行所发行

上海群众印刷厂排版

开本787×1092毫米1/32　印张 5　字数 120000

1986年5月第1版　1986年5月第1次印刷

印数：1—5000

统一书号：15324.7　科技书目：125-178

定价：1.10元

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AN OUTLINE OF MECHANICS

Mechanics may be defined as that science which describes and predicts the conditions of rest or motion of bodies under the action of forces. It is commonly divided into *statics* and *dynamics*, the former dealing with bodies at rest, the latter with bodies in motion. For convenience, dynamics is subdivided into two branches called *kinematics* and *kinetics*. In kinematics, we are concerned only with the purely geometric features of motion and in kinetics, we study the relations between the motions of bodies and the forces acting on them.

PART I. STATICS

Chapter 1. Fundamental Concepts and Principles of Statics

§ 1-1 Rigid Body

Statics deals with the conditions of equilibrium of bodies which are acted upon by forces.

Physical bodies, such as engineering structures and machine parts, are never absolutely rigid and deform slightly under the action of the loads which they carry. Such deformation, however, is usually very small and can be completely ignored in the investigation of conditions of equilibrium. Thus, in statics, we make the assumption that we are dealing with rigid bodies. A *rigid body* is defined as one which does not deform at all under the action of applied loads. Problems in which the effect of small deformations in physical bodies must be taken into account are generally treated in books on Strength of Materials.

§ 1-2 Force

A *force* is the action of one body on another which changes or tends to change the state of motion of the body acted on. The idea of force implies the mutual actions of two bodies. A force, therefore never exists alone.

For the complete definition of a force, we must know: (1) its magnitude, (2) its point of application, and (3) its direction. These three quantities which completely define the force are called its *specifications*, or *characteristics*.

The point of application of a force, acting upon a body, is that point in the body at which the force can be assumed to be concentrated. Physically it will be impossible to concentrate a force at a single point, i. e., every force must have some finite area or volume over which its action is distributed.

The direction of a force is defined by the *line of action* and the *sense* of the force. The line of action is the infinite straight line along which the force acts and the sense of the force may be indicated by an arrow-head.

Any quantity, such as force which possesses direction as well as magnitude, is called a *vector quantity* and can be represented graphically by a *vector*. A vector is a portion of a straight line having an arrow representing its direction and a length representing its magnitude.

§ 1-3 Principles of Statics

The study of statics rests on five fundamental principles based on experimental evidence. These principles may be stated as follows:

First principle: If two forces, represented by vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, are applied to a rigid body at point A, their action is equivalent to the action of one force, represented by the vector $\overrightarrow{AD}$ obtained as the diagonal of the parallelogram constructed on the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

The force $\overrightarrow{AD}$ is called the *resultant* of the two forces $\overrightarrow{AB}$ and $\overrightarrow{AC}$. The force $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called *components* of the force $\overrightarrow{AD}$. Thus a force is equivalent to its components and vice versa.

This principle is called the *principle of the parallelogram of forces*, and was first formulated by Stevinus in 1586.

Second principle: If a rigid body is held in equilibrium by two forces only, the two forces must have the same magnitude, the same line of action, and opposite sense.

Third principle: The action of a given system of forces will in no way be changed if we add to, or subtract from, these forces any other system of forces in equilibrium.

Using the above principle, it can be proved that the point of application of a force **F**, acting upon a rigid body at point

A, may be transmitted to any other point *B* on its line of action without changing the action of the force on the body. Thus, in the case of forces acting on a rigid body, however, the point of application of the force does not matter, as long as the line of action remains unchanged. In other words, forces acting on a rigid body are vectors which may be allowed to slide along their line of action; such vectors are called *sliding vectors.*

This statement is called *the theorem of transmissibility of a force.*

The use of this theorem is limited to those problems of statics in which we are interested only in the conditions of equilibrium of a rigid body and not in the internal forces to which it is subjected.

Fourth principle: There are mutual actions between any two bodies such that the forces of action and reaction have the same magnitude, the same line of action, and opposite senses.

Fifth principle: If a freely deformable body subjected to the action of a force system is in equilibrium, the state of equilibrium will not be disturbed if the body solidifies.

This principle is called *the principle of solidification.*

§ 1-4 Free-Body Diagram

A *free-body diagram* is a diagram in which are shown an isolated (free) body and all the forces exerted by other bodies on the given body.

The word *free* in the name "free-body diagram" emphasizes the idea that all the bodies exerting forces on the given body are removed or withdrawn and are replaced by the forces they exert.

In drawing a free-body diagram of a given body, certain assumptions are frequently made as to the nature of the forces exerted by the other bodies on the given body. The assumptions usually made are the following:

(a) If a surface of contact at which a force is applied by one body to another body has only a small degree of roughness, it may be assumed to be smooth (frictionless), and hence the action (or reaction) of the one body on the other is directed normal to the surface of contact.

(b) A body that possesses only a small degree of bending stiffness, such as a cord, a rope, a chain, etc., may be considered to be perfectly flexible, and hence the pull of such a body on any other body is directed along the axis of the flexible body.

§ 1-5 Types of Force Systems

Any number of forces treated as a group constitute a *force system*. The force systems may be classified as follows:

- Force systems
  - coplanar (forces in a plane)
    - concurrent
    - parallel
    - general case
  - non-coplanar (forces in space)
    - concurrent
    - parallel
    - general case

A force system is said to be *coplanar* when all the forces lie in the same plane.

A force system is said to be *concurrent* if the action lines of all the forces intersect in a common point.

A *parallel* force system is one in which the action lines of the forces are parallel, the senses of the forces not necessarily being the same.

If a force system applied to a body produces no external effect on the body, the forces are said to be in *equilibrium*.

Two force systems are said to be *equivalent* if they will produce the same external effect when applied in turn to a given body. The *resultant* of a force system is the simplest equivalent system to which the system will reduce. The resultant of a force system is frequently a single force. For some force systems, however, the simplest equivalent system is composed of two equal, non-collinear, parallel forces of opposite sense, called a *couple*. And still other force systems reduce to a force and a couple as the simplest equivalent system.

The process of reducing a force system to a simpler equivalent system is called *composition*. The process of expanding a force or a force system into a less simple equivalent system is called *resolution*. A *component* of a force is one of the two or more forces into which the given force may be resolved.

Chapter 2. Concurrent Forces

§ 2-1 Composition of Concurrent Forces

A system of concurrent forces can be reduced to a single resultant force. This resultant can be found by successive application of the principle of the parallelogram of forces. This resultant can also be obtained by successive geometric addition of the free vectors representing the given forces.

The polygon, constructed by arranging the given forces in tip-to-tail fashion, is called the *force polygon*. The resultant is given by the closing side of the force polygon. This is known as the *polygon rule* for the addition of concurrent forces, or vectors. It should be noted that the order in which the forces, or vectors, are added is immaterial. That is, vector addition obeys the commutative law.

Thus, we may say that the resultant of any system of concurrent forces is obtained as the geometric sum of the given forces. This statement may be expressed by the vector equation:

$$\mathbf{R} = \Sigma \mathbf{F}.$$

If the force polygon closes, the resultant of the given system is equal to zero, and the given system of forces is in equilibrium.

Geometric or graphical methods are generally not practical in the case of forces in space.

§ 2-2 Method of Projections

The resultant of a system of concurrent forces can also be obtained by the method of projections.

We can prove that the projection on any axis of the resultant of a system of concurrent forces is equal to the algebraic sum of the projections of the given forces on the same axis. Thus, the projections of the resultant on the rectangular coordinate axes x, y and z are:

$$R_x = \Sigma F_x, \ R_y = \Sigma F_y, \ R_z = \Sigma F_z,$$

where F_x, F_y, F_z are the projections of the given forces on the coordinate axes x, y, z, respectively. The magnitude and direction of the resultant can be computed from the following equations:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2};$$

$$\cos\alpha = \frac{\Sigma F_x}{R}, \ \cos\beta = \frac{\Sigma F_y}{R}, \ \cos\gamma = \frac{\Sigma F_z}{R}.$$

§ 2-3 Equilibrium of Concurrent Forces

If a system of concurrent forces is in equilibrium, their resultant must be zero. This condition can be formulated by the vector equation:

$$\Sigma \mathbf{F} = 0.$$

That is, the vector sum of the given forces is zero.

This vector equation can be expressed by a closed force polygon. Thus, the condition of equilibrium of concurrent forces may be stated as follows: All the given forces form a closed polygon. It is called the geometric or graphical condition of equilibrium of concurrent forces.

The vector equation $\Sigma \mathbf{F} = 0$ is also equivalent to three algebraic equations:

$$\Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma F_z = 0.$$

That is, the algebraic sums of the projections of the given forces on the rectangular coordinate axes are all equal to zero. These independent algebraic equations are called the *equations of equilibrium* for a system of concurrent forces and give the algebraic or analytical conditions of equilibrium of concurrent forces.

By applying these equations to any system of concurrent forces which are in equilibrium, three unknown quantities can always be calculated.

For a coplanar, concurrent system of forces, there are only two independent equations of equilibrium as follows:

$$\Sigma F_x = 0, \ \Sigma F_y = 0.$$

They may be used to solve problems involving no more than two unknowns.

If the number of unknown quantities in a force system is greater than the number of equations of equilibrium for that system, the system is said to be *statically indeterminate*.

§ 2-4 Equilibrium of Three Forces

If three non-parallel forces acting in one plane are in equilibrium, their lines of action must intersect in one point.

In engineering problems of statics, especially in discussing the equilibrium of constrained bodies, we very often encounter the case of three forces in one plane which are in equilibrium. Thus, knowing the lines of action of two of the forces and the point of application of the third, the action line of the third can always be determined, provided that the point of intersection of the known lines of action does not coincide with the point of application of the third force.