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PART II HEAT TRANSFER BY CONVECTION

part I

Heat Conduction

In the next several chapters we shall examine the physical phenomena, the fundamental laws, the thermophysical properties, and the characteristic mathematical formulations that are significant to the process of heat conduction in matter. The flow of heat in opaque solids takes place exclusively by the conduction process. In transparent or translucent solids conduction and radiation transfer can occur, and in liquids and gases the transfer processes of conduction, convection, and radiation can occur simultaneously. In the developments that follow the major interest will be with solids, and the results of the problems considered in the chapters on heat conduction can be applied to all solid bodies. In those specific cases where heat exchange by convection is prevented and exchange by radiation minimized, the principles of heat conduction can be applied to liquids and gases as well.

The treatment of heat conduction has been separated into five specific topics: (1) *theory of heat conduction and heat-conduction equations*, (2) *thermal conductivity and its measurement*, (3) *steady heat conduction*, (4) *unsteady heat conduction, and* (5) *heat conduction with moving boundaries*. These subdivisions are necessarily arbitrary but serve as an aid in the presentation of the material.

Heat conduction from the macroscopic, phenomenological point of view can be understood without a companion understanding of the mechanism of heat conduction as proposed by the theories of modern solid-state physics and kinetic theory. However, today, an acceptable understanding of the science of heat and heat transfer requires a familiarity with both the microscopic and macroscopic points of view. Many of the advances in technology from which we benefit today have been the result of intelligent application of knowledge of the behavior of materials on a microscopic as well as on a macroscopic scale. To note two examples: the effect of alloying on the reduction of thermal conductivity of metals has been used to advantage in the selection of materials for low-temperature work, and knowledge of the material behavior on a microscopic scale has been of major importance in the development of heat-shield materials for atmospheric reentry.

Within the section on heat conduction it is appropriate to treat in some detail, from both microscopic and macroscopic points of view, the important transport property *thermal conductivity*. In the discussion of heat conduction it is the thermal conductivity of solids that appears most important; however, heat conduction is an ever-present process in nominally heat-convection processes, and for this reason the discussion of thermal conductivity includes the liquid and gas as well as the solid phases of matter.

In the other chapters, where heat conduction is treated from the macroscopic, phenomenological sense, the thermophysical properties, such as the thermal conductivity, are presumed to be known. In such a treatment it is well to realize that although the physical notions may be simple, the mathematical techniques required to obtain usable results, i.e., temperature distributions, heat rates, temperature-time histories, etc., are generally complex and in many cases quite difficult. In the treatment here, the mathematical complexities have not been avoided; however, an attempt has been made to keep the presentation from becoming a mathematical treatise. For more complicated problems and problems with diverse boundary conditions the reader is referred to the excellent book by Carslaw and Jaeger.[1]

Certain elementary problems in heat conduction have been treated only briefly in this edition. The reader wishing more detail than shown here should consult the introductory volume by Eckert and Gross[2] or the more advanced work by Eckert and Drake.[3]

With some exceptions, the thermophysical properties which occur in the heat-conduction problems have been considered to be independent of temperature. Such practice not only simplifies the mathematical treatment but also is a reasonable approximation in many physical problems where the temperature variation is not large. In problems involving large temperature differences, chemical reactions, or phase changes, the neglect of temperature dependency of the thermophysical properties may be a serious omission. Therefore, each problem must be carefully considered physically before the assumption of constant properties is applied.

Certain heat-conduction problems involve the convection (or the radiation) mode of heat transfer, usually in the statement of some boundary condition. In the consideration of heat-conduction problems in which heat convection (or radiation) plays a part, it will be assumed that the heat-transfer coefficients are known. The nature of these heat-transfer coefficients as well as the methods for their determination are considered in the chapters on convection heat transfer and thermal radiation.

[1] H. S. Carslaw and J. C. Jaeger, "Conduction of Heat in Solids," 2d ed., Oxford University Press, New York, 1959.

[2] E. R. G. Eckert and J. F. Gross, "Introduction to Heat and Mass Transfer," McGraw-Hill, New York, 1963.

[3] E. R. G. Eckert and R. M. Drake, Jr., "Heat and Mass Transfer," 2d ed., McGraw-Hill, New York, 1959.

1

Theory of Heat Conduction and Heat-conduction Equations

1-1 THE CONCEPT OF HEAT CONDUCTION

The currently accepted theory of heat is closely associated with the internal energy of matter, which in thermodynamics is referred to as the energy related to the physical and chemical state of the body—the orientation and motion of the molecules and atoms within the body. Although incomplete, the dynamic theory of heat permits some important conclusions to be drawn which are quite generally confirmed by experiment:

1. Since heat as energy is associated with translational, rotational, and vibrational motions of the molecules, atoms, and their components, heat transfer by conduction must be strictly related to these motions.
2. Increased temperature increases the intensity and frequency of molecular and atomic motions; therefore, the conduction of heat should increase with increasing temperature. (There are exceptions to this statement.)

3. Changes from the denser solid phases to the liquid and gaseous phases result in lower thermal conductivities and thus smaller heat-conduction effects.

These general patterns of the behavior of materials are discussed in considerable detail in Chap. 2 in terms of the modern theories of the solid state and the kinetic theory of matter. The description of the heat-transfer mechanism by means of a dynamic theory and in terms of a meaningful molecular model has presented the physicist with some of the most complicated problems in theoretical physics. In recent years improvement of the fundamental theories and implementation of the theoretical treatment by means of accurate and detailed calculations made possible by modern computers have greatly advanced the knowledge and understanding of the transport properties. And while there remain large gaps in knowledge in certain areas, liquids in particular, the general understanding of these properties is fairly complete. In fact, in the case of gases at high temperatures the calculated results are superior to the experimental measurements.

While there remains some conjecture in regard to the precise physical model for the mechanism of the transfer of heat energy, for any event and for any theory the energy in transition is referred to as *heat* and the process of energy transfer is known as *conduction*.

It is not necessary to understand fully the mechanism of heat conduction in matter to proceed with the mathematical developments which lead to practical results. For even though the physicomathematical models representing the microscopic behavior are not yet perfect, one relies upon the fact that the practical developments of the heat-conduction processes resulted from a hypothesis based upon experimental observations. Subsequent use of this hypothesis as a basis for mathematical analysis to obtain results which have been experimentally verified is sufficient to establish the particular law which is characteristic of the transfer itself. The basic law so established is entirely consistent with the laws of thermodynamics.

1-2 THE FUNDAMENTAL LAW OF HEAT CONDUCTION

To be consistent thermodynamically is to require, by virtue of the second law of thermodynamics, that heat will be transferred from one body to another body (or from one part of a body to another part of the same body) only when the bodies are at different temperatures and that the heat will flow from the location of the highest temperature to the location of the lowest temperature; i.e., a temperature gradient exists, and the energy in the form of heat flows in the direction of decreasing temperature. The first law of thermodynamics states that the flowing thermal energy is conserved in the absence of heat sources or sinks. To this end a body may have a temperature distribution which is dependent upon the space coordinates and time of observation:

$$t = f(x,y,z,\tau)$$

One may suppose that within this body is a surface such that, when observed at a certain time, each point on it has an identical temperature. Such a surface is called an *isothermal surface.* One can further visualize other isothermal surfaces within this body which differ from one another by being hotter or colder by temperature increments $\pm\,\delta t$, respectively. These isothermal surfaces never intersect, because no point in the body can exist at two different temperatures at the same time. The body is thus visualized as being composed of a number of arbitrarily thin, isothermal shells that, of course, vary with time.

In the discussion which follows, unless stated differently, we shall consider only *isotropic solids*, i.e., solids whose properties and constitution in the neighborhood of any point are invariant with the direction from the point. In such a case, and because of the symmetry involved, the heat flow at a point is along a path perpendicular to the isothermal surface through the point. For a nonisotropic solid the heat flow is not necessarily in a direction perpendicular to the isothermal surface through the point, a situation which will be discussed in a subsequent paragraph.

The hypothesis which forms the basis for the mathematical formulation of the law of heat conduction had its foundation in the results of a simple experiment. A plate of some solid is bounded by planes sufficiently large to be supposed infinite in extent. The bounding planes are maintained experimentally at different but uniform temperatures, the temperature difference being not so great as to measurably change the thermophysical properties of the plate material. After some sufficient time the heat flow and temperature distribution in the plate become invariant with time, or *steady*. The heat flow can then be measured to be

$$Q = \frac{kA(t_1 - t_2)}{d} \tag{1-1}$$

where Q, the heat flow, is shown to be proportional to the area A of the plate surface and to the temperature difference $t_1 - t_2$ of the plate surface and inversely proportional to the plate thickness d. The constant of proportionality is the thermal conductivity k. Strictly speaking, the thermal conductivity is not a constant but depends upon temperature and to some extent pressure. Thermal conductivity is a *transport property* of the material. The nature of the thermal conductivity and its variation with physical parameters such as material composition, pressure, and temperature will be discussed in Chap. 2.

At a point in the interior of the plate, Eq. (1-1) can be written more generally as

$$\frac{Q}{A} = \frac{\partial \mathbf{H}}{\partial r} = -k\,\frac{\partial t}{\partial n} \tag{1-2}$$

where **H** is the heat-flow vector field. Equation (1-2) can be interpreted with the aid of Fig. 1-1. The heat flux Q/A flows along the normal n to the area A in the direction of the decreasing temperature, i.e., the negative thermal gradient. The negative sign in Eq. (1-2) indicates that the heat flow is in the direction of the negative gradient and serves to make the heat flux positive in that sense. Again, the proportionality factor is the thermal conductivity k, a property of the material through which the heat flows.

The form of Eq. (1-2), which in fact serves to define the thermal conductivity, implies that the process of heat conduction is a random process, a diffusion of energy. The heat energy does not enter one side of the plate and travel directly to the other side but moves randomly through the plate as the result of frequent collisions. If the heat were propagated through the plate without the random character, the expression for the heat flux in Eq. (1-2) would show a dependence only on the temperature difference and not on the temperature gradient, thus being independent of the thickness of the plate d. It is the random nature of the conduction process that brings the temperature gradient into the expression for the heat flux.

The random nature of the conduction process can be demonstrated in the following way: In the kinetic theory of gases under certain approximations, as will be shown elsewhere in this text, the thermal conductivity of certain solids can be shown as

$$k = \tfrac{1}{3}\rho\bar{w}\lambda c_v \tag{1-3}$$

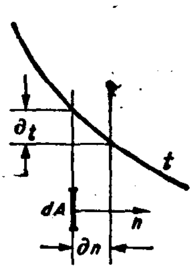

Fig. 1-1 Heat-conduction system.

where c_v = heat capacity at constant volume
$\bar{w}$ = average carrier velocity
ρ = mass density
λ = mean free path of carrier between collisions

If Eqs. (1-1) and (1-3) are combined,

$$q = \frac{Q}{A} = \frac{1}{3}\rho c_v(t_1 - t_2)\frac{\bar{w}\lambda}{d} \tag{1-4}$$

The expression for the thermal flux as shown by Eq. (1-4) can be interpreted as follows: $\rho c_v(t_1 - t_2)$ is the excess energy on one side of the plate over that of the other side. This energy is propagated across the plate at an effective transport velocity $\bar{w}\lambda/d$, which is just the carrier velocity reduced by the ratio of the mean free path to the significant dimension of the plate d. As will be shown in Chap. 2, the carriers are the individual molecules in the case of a gas; in the case of solids (and to a great extent liquids) the carriers are free electrons and phonons.

Equation (1-2) can be rewritten for the case of an infinitesimal area as

$$dQ = -k\,dA\,\frac{\partial t}{\partial n} \tag{1-5}$$

Equations (1-2) and (1-5) are generally attributed to the French mathematician *Jean Baptiste Fourier* and in his honor are designated the *Fourier heat-conduction equations.*

The heat flow per unit area per unit time across any surface is called the *heat flux* q and has units of watts per square meter. The heat flux is a *vector;* i.e., its magnitude and direction must be specified. The heat flux can be calculated for any point in reference to any arbitrary direction through the point if the area normal to the desired direction is considered.

In Fig. 1-2 are shown the isotherms t and $t + dt$ in a body. The normal to these isotherms is designated by the axis n, which is also normal to the differential area dA. The heat flux can be calculated in the direction of the normal n and in the arbitrary direction s as shown below:

$$q_n = \frac{dQ}{dA} = -k\frac{\partial t}{\partial n}$$

$$q_s = \frac{dQ}{dA\,\cos\alpha} = -k\frac{\partial t}{\partial s}$$

Since $n = s\cos\alpha$,

$$q_s = -k\frac{\partial t}{\partial n}\cos\alpha \tag{1-6}$$

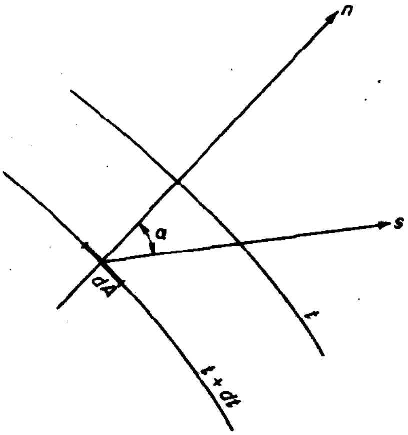

Fig. 1-2 Direction of heat flow.

or, in other words, q_s is a component of the heat-flux vector q_n. From Eq. (1-6) it can be seen that the greatest heat flux is that which is calculated along the normal to the isothermal surfaces. In particular, if the component fluxes are related to the planes of the (x,y,z) coordinate system, the heat fluxes are

$$q_x = -k\frac{\partial t}{\partial x} \qquad q_y = -k\frac{\partial t}{\partial y} \qquad q_z = -k\frac{\partial t}{\partial z} \tag{1-7}$$

The heat fluxes shown in Eq. (1-7) are components of the heat-flux vector

$$\mathbf{q} = i\mathbf{q}_x + j\mathbf{q}_y + k\mathbf{q}_z \tag{1-8}$$

Effect of variable thermal conductivity It should be noted here that the thermal conductivity k is not a constant but, in fact, is a function of the temperature for all phases and in liquids and gases depends also upon the pressure, especially when near the critical state. The thermal conductivity in wood and crystals also varies markedly in direction.

The dependence of thermal conductivity on temperature for small, select temperature ranges can be acceptably expressed in a linear form:

$$k = k_0(1 + at)$$

where k_0 is the value of the thermal conductivity at some reference condition and a is the temperature coefficient and is positive or negative depending upon the material in question. Figure 1-3 shows the effect on the temperature gradient in a body as a result of the positive or negative characteristic of a.

It can readily be seen that a linear temperature gradient exists only when the thermal conductivity is a constant.