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CHAPTER 1

VELOCITIES AND STRESSES IN TURBULENT FLOWS

Introduction

Viscosity

The manner in which a liquid flows depends on several factors. Among these is the viscosity of the liquid, which is a measure of its resistance to steady (relatively slow) flow. The viscosity determines, in other words, the stress (i.e., force per unit area) of one fluid layer moving smoothly past an adjacent layer: in practical terms, one requires a higher external pressure to force glycerol as compared with water through a tube at some given rate. This stress τ, resisting the flow of one fluid layer past another, becomes greater at higher shear rates. One defines the viscosity μ with these facts in mind:

$$\mu = \tau/(dv_x/dy) \qquad *(1.1)$$

where v_x is the velocity of the liquid in the x direction (the flow direction in our convention), and y is distance measured perpendicular to the direction of flow. (The asterisk on the equation number indicates that the equation is used frequently, and is worth memorizing.) Some typical values of μ are listed in Table 1-I.

TABLE 1-I

PROPERTIES OF VARIOUS NEWTONIAN FLUIDS AT 20°C

Fluid	Viscosity μ (N sec m^{-2}, i.e., kg m^{-1} sec^{-1})	Density ϱ (kg m^{-3})	Kinematic viscosity ν ($= \mu/\varrho$) (m^2 sec^{-1})
Water	0.0010	1,000	1.0×10^{-6}
Ethanol	0.0017	790	2.2×10^{-6}
Toluene	0.00059	867	0.68×10^{-6}
Mercury	0.00155	13,550	1.2×10^{-7}
Glycerol	0.83	1,260	6.5×10^{-4}
Air	1.8×10^{-5}	1.2	15.2×10^{-6}
Carbon dioxide	1.4×10^{-5}	1.84	7.6×10^{-6}
Ethane	0.9×10^{-5}	1.26	7.1×10^{-6}
Hydrogen	0.87×10^{-5}	0.084	1.03×10^{-4}

Laminar Flow in a Pipe

For a fluid flowing slowly through a pipe, the velocity profile, as is well known, is found to be parabolic. Such flow is easily represented quantitatively as follows. Denoting by v_x the velocity in the x direction (parallel to the wall of the pipe) at some distance r from the center of the pipe, and denoting by Δp the pressure difference across the ends of the pipe of length L and radius a, then the total drag force acting on any cylindrical element of fluid is $2\pi rL\tau$, or, by Eq. (1.1), $2\pi rL\mu(dv_x/dy)$.

This force is balanced, in steady flow, by the pressure difference Δp which must be applied across the ends of the pipe to keep the liquid moving, i.e., the drag force is balanced by the driving force across the ends of the pipe, $\pi r^2 \Delta p$. Hence one obtains the well-known relation

$$v_x = (\Delta p)(a^2 - r^2)/4\mu L \tag{1.2}$$

Note that $v_x = 0$ at the solid wall ($r = a$), i.e., there is no fluid slippage, and v_x is maximum at the center of the pipe (where $r = 0$). Equation (1.2) is the equation of a parabola, and is illustrated by Fig. 1-1(b, c).

The velocity gradient at any distance y from the wall is obtained by differentiating Eq. (1.2), having substituted $r = a - y$:

$$-(dv_x/dy) = 2(y - a)(\Delta p)/4\mu L$$

At the wall, $y = 0$, and thus the velocity gradient is given by

$$(dv_x/dy)_{y=0} = a(\Delta p)/2\mu L \tag{1.3}$$

At very small distances y away from the wall,

$$v_x \approx \int_0^y [a(\Delta p)/2\mu L]\, dy$$

or

$$v_x \approx a(\Delta p)y/2\mu L \tag{1.3a}$$

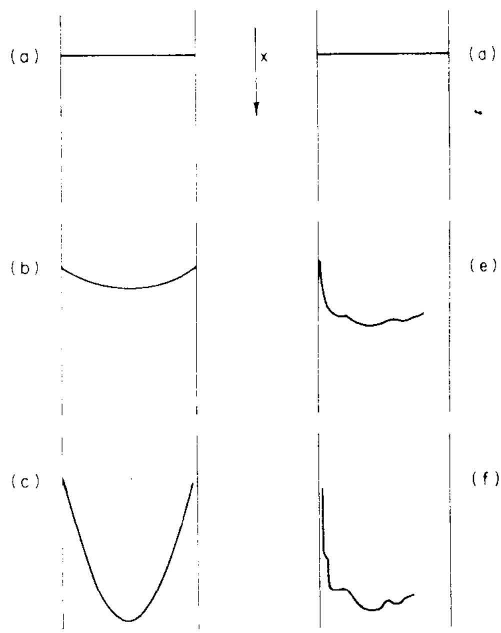

FIG. 1-1. Profile for flow in a smooth pipe, using a tracer technique which produces a streak of dye photochemically at $t = 0$, with photographs taken after various time intervals. The dye streak is initially perpendicular to the direction of flow. Results for laminar flow are at (a) $t = 0$, (b) 6.9, (c) 47 msec. For turbulent flow ($Re = 13{,}800$), results are at (d) $t = 0$, (e) 3, (f) 8.3 msec [From Frantisak *et al.* (1969). By permission of the American Chemical Society.]

The ratio μ/a, it should be noted here, is of primary importance in laminar flow. Equation (1.3a) shows that close to the wall, for fluid in laminar flow, v_x is directly proportional to y, the distance out from the wall. In such circumstances, Eq. (1.1) can accordingly be written in the approximate form

$$\tau_0 = \mu(v_x/y) \tag{1.4}$$

which is valid for laminar flow close to the wall, τ_0 being the stress of the flowing liquid on the wall.

Alternatively, near $y = 0$, the stress τ_0 across the liquid film and onto the wall is constant, and is given by

$$\tau_0 = a(\Delta p)/2L \tag{1.5}$$

The volume flow rate (e.g., in $m^3\ sec^{-1}$) through the pipe is defined by the integral

$$V = \int_0^a 2\pi r v_x \, dr$$

which, by Eq. (1.2), yields

$$V = (\Delta p)\pi a^4/8\mu L$$

The mean velocity of flow v_m is simply V divided by the cross-sectional area (πa^2) of the pipe, i.e.,

$$v_m = (\Delta p)a^2/8\mu L \tag{1.6}$$

which is exactly half the maximum velocity, i.e., the velocity at the center of the tube [cf. Eq. (1.2) when $r = 0$]:

$$v_x(\text{center}) = (\Delta p)a^2/4\mu L \tag{1.7}$$

Turbulent Flow

As the flow rate of the liquid is increased, the *laminar flow* pattern (with its steady advance in separate layers) is not maintained: the flow becomes unsteady, with chaotic movements of parts of the liquid in different directions superimposed on the main flow of the liquid as in Figs. 1-1e, f and Fig. 1-2. Such movement of any particular element of fluid is now very complicated, and it can only be described in terms of averages. This is called *turbulent flow*.

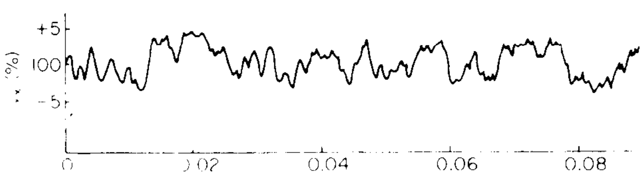

FIG. 1-2. Typical fluctuations in turbulent flow at the center of a narrow pipe in which air is flowing at a mean velocity of 12 m sec^{-1}. [After Wattendorf and Kuethe (1934).]

In turbulent flow, transfers of momentum between neighboring pulses of the fluid are of primary importance, as is discussed in detail later. These inertial (momentum) effects in turbulent flow (as contrasted with purely viscous effects in laminar flow) cause the velocity and density ϱ of the flowing fluid to assume great importance. Turbulent flow replaces laminar flow when these inertial effects, as characterized by ϱv^2, are great compared with the viscous effects, these being characterized by $\mu v/a$ (see preceding subsection). For the flow of fluid in a pipe of diameter d ($d = 2a$), Reynolds (1883) used the ratio $\varrho v_m{}^2/(\mu v_m/d)$ to characterize the change of flow from laminar to turbulent. This dimensionless ratio is known as the Reynolds number Re:

$$Re = v_m \varrho d/\mu = v_m d/\nu \qquad *(1.8)$$

Here, ν conveniently denotes the ratio μ/ϱ, which occurs frequently in hydrodynamic theory: it is called the "kinematic viscosity," and has the dimensions of [length]^2[time]$^{-1}$. Some typical values are listed in Table 1-I.

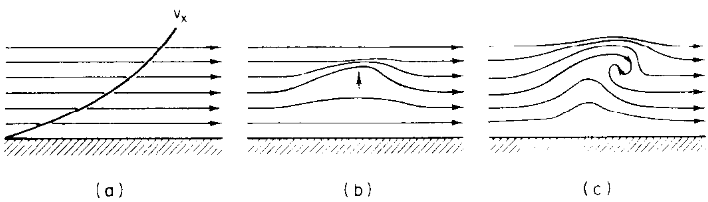

FIG. 1-3. Effect of random disturbance on fluid in laminar flow. (a) The flow is pure laminar, with a parabolic velocity profile as given by Eq. (1.2). (b) A random disturbance is shown, the density and velocity gradient effects (momentum effect) being as shown by the arrow. (c) More momentum is being transferred into the disturbance than is being damped out, and an eddy is forming.

More precisely, one can see (Fig. 1-3) that laminar motion will be stable only as long as there is no net transfer of energy from the primary flow into any superposed random disturbance. The lateral pressure gradient associated with any disturbance to the streamline flow pattern will augment the disturbance in direct proportion to the density ϱ and the velocity gradient (measured by v_m/a to an order of magnitude). On the other hand, the disturbance will be damped by the viscosity μ and the closeness of the solid walls (measured by some inverse function of a). Dimensional analysis gives for the ratio of the augmentation to damping factors $\varrho(v_m/a)/\mu a^{-2}$, which is of the form of Eq. (1.8). The arithmetical factor involved in replacing a by d is of no account, since the values of Re are found empirically by comparison with experimental data.

Experiments with many fluids in smooth, circular pipes of different diameters have confirmed that *Re* does indeed characterize the velocity of flow at which laminar flow breaks down to turbulent flow. At Reynolds numbers up to about 2000, the flow of fluid in a smooth pipe is always laminar. Between 2000 and 4000 (the so-called "transition region"), there is usually a gradual change to turbulent flow, though in a pipe with a tapered inlet, laminar flow can be made to persist to much higher *Re* values. But, generally, turbulence is fully established when $Re > 4000$. A more definite limit can be specified for decreasing flow rates: when *Re* falls below 2000, the flow is *always* laminar. For smooth geometries other than that of a uniform circular pipe, the "characteristic length" d in Eq. (1.8) can be suitably assigned. Because ν for water is 15 times less than for air, it is sometimes convenient to simulate the turbulent gas flow in large-scale equipment by the flow of water in a small-scale laboratory model.

Figures 1-4 and 1-5a show the turbulence eddies in a pipe as viewed by an observer moving at the mean flow rate (from left to right). Near the wall, the strong velocity gradient within the fluid tears the fluid into small eddies. Some of these migrate toward the center of the pipe, where larger eddies are also to be found.

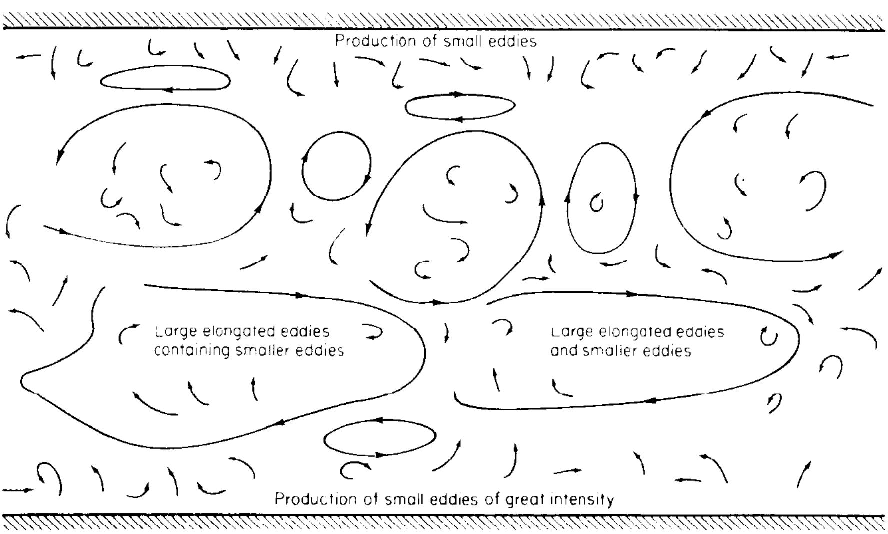

FIG. 1-4. Schematic representation of local instantaneous flow patterns in turbulent flow in a pipe. These eddies are actually superimposed on a much faster overall mean flow, from left to right: in this picture, the observer is considered to be moving at the same velocity as the mean flow. The small eddies, formed in the high-shear region near the walls, diffuse into the "core" fluid, decaying in intensity as they do so.

The critical Reynolds numbers of a few thousand quoted here are for the special case of relatively smooth, straight pipes. At the other extreme, eddies form downstream of protrusions and solid bodies at much lower local Reynolds numbers. For spheres, for example, in a stream of fluid, eddies form when Re_s (i.e., Re based on the diameter of the sphere) is of the order unity (Chapter 8), as is seen in Fig. 1.5c. But these eddies are not chaotic in their motions at such low Reynolds numbers: true turbulence (with a spectrum of eddy sizes and chaotic motion) sets in only if $Re_s > 1000$. For *sharp* protrusions, however, turbulence can be initiated at lower Reynolds numbers.

For fluids flowing in a pipe, turbulence is not necessarily established very close to the inlet: in general, the eddy structure of Fig. 1-4 is established only at a distance x_{ent} (the so-called entry length) downstream from the entrance to the pipe. Only after this point is the characteristic turbulence developed at the center of the tube, and only after this point are the mean velocities (in given regions of the pipe) independent of the distance along the pipe. In general, it can be safely assumed that this situation is achieved at a distance of 100 diameters from the inlet, i.e., that $x_{ent} < 100d$. Though for smooth pipes x_{ent}/d approaches this value of 100, rough pipes, bent pipes, or pipes with sharp edges have x_{ent}/d values usually much smaller than 100, e.g., as low as 30 or even less.

For flow in a corrugated pipe (Fig. 1-5d), vortex shedding at the angularities can induce strong eddying at Re values of only a few hundred. This is used in heat exchanger design for very viscous liquids, for which only moderate flow rates can readily be achieved: the eddy flow in narrow tubes then leads to good heat transfer (Chapter 3), even though Re may be perhaps 300 or 500. The sharpness of the angularities is very important: eddies are shed much more readily from sharp corners than from smooth ones.

In turbulent flow, the instantaneous velocity in the x direction is given by

$$v_x = \bar{v}_x \pm v_x' \tag{1.9}$$

where $\bar{v}_x$ is the time-averaged velocity at any point in the flowing fluid and v_x' is the instantaneous fluctuation velocity. The fluctuation velocities are sometimes positive, sometimes negative (see Fig. 1-1e, f), and the time average of them is zero, i.e., $\bar{v}_x' = 0$. It is convenient to express the amplitude of the fluctuation velocities in the x direction as $\overline{(v_x')^2}$, which is the mean of the squares of the fluctuation velocities, this being necessarily positive. One can take the square root of the mean of the squares to get a root mean square fluctuation velocity, denoted $\tilde{v}_x'$. Thus $\tilde{v}_x' = \left[\overline{(v_x')^2}\right]^{1/2}$,

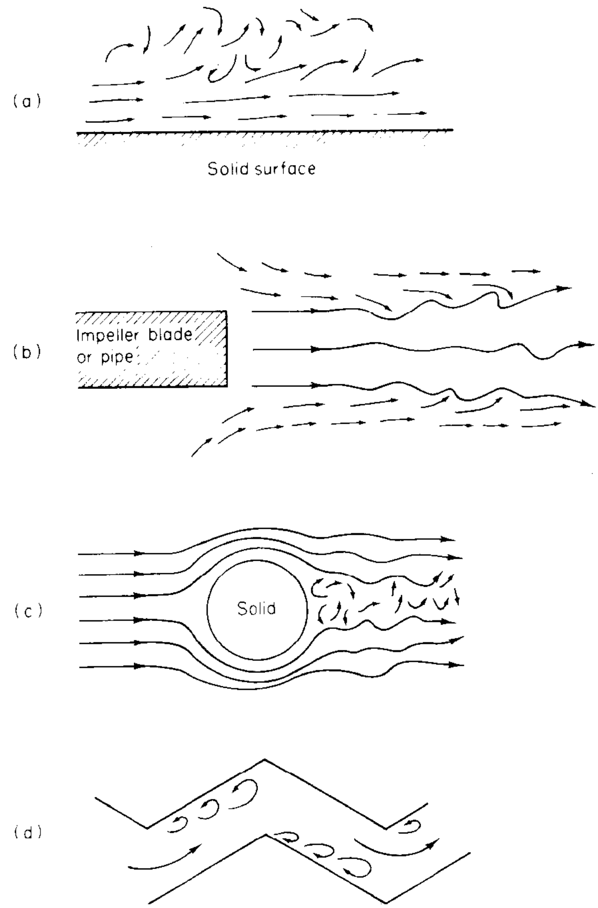

FIG. 1-5. The three ways of inducing turbulence. (a) Fluid is flowing rapidly past a solid surface. (b) Rapidly flowing fluid is ejected (from an impeller blade, or from a nozzle as a submerged jet) into a slow-moving or stationary fluid. (c) A solid object (sphere or rod or blade) and the fluid are in relative motion, producing form drag and eddy shedding behind the solid object, or (d) behind the protrusion or angularity.

where $\tilde{v}_x'$ is always positive. The fluctuation velocity $\tilde{v}_x'$ is a measure of the *intensity of the turbulence* in the x direction.

Similarly,

$$v_y = \bar{v}_y \pm v_y'$$

where in turbulent flow $\bar{v}_y'$ 0, but $\left[\overline{(v_y')^2}\right]^{1/2}$ $(= \tilde{v}_y')$ is not zero. The product $\overline{v_x'v_y'}$ (taken at any point in the fluid) will also generally not be zero for pipe flow, since the fluctuations are correlated, i.e., they refer to the same eddy. The nomenclature implies that the product $v_x'v_y'$ has been time-averaged.

It is the fluctuation velocities which, as we shall see, account for much of the dissipation of energy, mass transfer, and heat transfer in turbulent liquids.

Types of Turbulent Flow

Turbulence is set up in one of three possible ways, as shown in Fig. 1-5. The rapid flow of a fluid past a solid wall can lead to unstable, self-amplifying velocity fluctuations, these forming in the fluid close to the wall (where the velocity gradient is high) and then spreading outward into the rest of the fluid stream. Figures 1-4 and 1-5a show the eddies in a pipe containing turbulent fluid. In addition, and rather similarly, the velocity gradients between a fast-moving stream of fluid and slower-moving fluid can also set up turbulence eddies. Finally, the relative movement of a body such as an angularity, a stirrer blade, or a falling sphere or cylinder causes eddies to be set up in the wake: this increases the resistance to movement of the blade ("form drag").

Turbulence induced by the rapid *flow of fluid through a pipe* is commonly encountered in engineering practice, both in relation to the drag coefficient of the flow and in relation to the heat transfer. The velocity gradients and the strong influence of the walls produce a velocity profile rather flat over most of the tube, but steep near the walls, where there is a strong correlation of v_x' and v_y'.

Turbulence is also induced by a stream of *fluid flowing over a flat plate*: the velocity gradient near the solid plate is quite high, and again v_x' and v_y' correlate strongly here. The turbulence is confined to a boundary layer in the vicinity of the plate, though this boundary layer increases in thickness with the distance from the leading edge of the plate.

Flow in open channels (e.g., rivers) is usually turbulent in practice, with large eddies and large fluctuations in velocity. Geometric factors are very important here.

In *homogeneous turbulence*, each fluctuation component is independent of the position in space. In practice, it is difficult to produce homogeneous turbulence except over short distances.

In *isotropic turbulence*, all the fluctuation components are equal, i.e.,

$$\tilde{v}_x' = \tilde{v}_y' = \tilde{v}_z'$$

and, if the fluctuations are thus random in space, then there is no correlation between the fluctuations in different directions, and so

$$\overline{v_x' v_y'} = 0$$

and similarly for the terms involving v_z'. Isotropic turbulent flow is relatively easily analyzed mathematically, though in many practical applications of turbulence, isotropy does not exist. However, there is a *tendency* for turbulence to be isotropic, as at the edge of a boundary layer or near the axis of a pipe (Fig. 1-4), where the mean velocity gradient is locally nearly zero.

The turbulence arising from *flow through a grid or screen* or "honeycomb" structure of bars has simple properties at a sufficient distance downstream. It is found to be *isotropic*, i.e., $\tilde{v}_x' = \tilde{v}_y' = \tilde{v}_z'$: the directional components associated with the bars have now disappeared. This turbulence decays with time, however, so that it becomes less with increasing distance downstream. It is thus not homogeneous.

In *stirred tanks*, the turbulence can be very intense near the tips of the rotor blades. Though it may be locally more or less isotropic near the tips, the turbulence is necessarily inhomogeneous over the tank as a whole. Most of the turbulence arises from the velocity gradients, where high-velocity liquid is flung off the impeller blades on to much slower-moving liquid, but some arises from the high shear over the blades themselves and form separation behind each blade or baffle. The latter mode of turbulence generation is important if the baffles are situated close to the impeller.

In *free turbulent jets*, sometimes termed submerged jets, the fluid is expelled from a nozzle into a mass of miscible fluid more or less at rest. The free jet spreads out through a cone of half-angle about 10°, entraining a considerable amount of surrounding fluid in the outer part of the jet. The eddy sizes are relatively uniform across any given section of such a jet.

In *restrained turbulent jets*, the turbulent fluid is ejected from a nozzle into an immiscible fluid. The turbulence eddies protrude from the sides of the free jet, restrained by surface tension from breaking away completely unless the eddy velocities are particularly great. This type of turbulence decays very rapidly due to the fluctuations being damped by the elastic forces associated with the surface tension.

Pipe-Flow Turbulence

The One-Seventh-Power Approximation for the Mean Velocity Profile

It is known from experiment that the time-averaged velocity profile for turbulent flow in a pipe is fairly flat, except in the vicinity of the wall. This is seen from the results in Fig. 1-1c, f. Later (pp. 22–27), we shall analyze separately the precise flow patterns both in the core of the turbulent liquid and close to the wall. But for many purposes a simple power law is a good approximation, over the core of the pipe and to within a small distance of the wall. For values of Re up to 10^5, this approximation is

$$\bar{v}_x/\bar{v}_x(\text{center}) = (y/a)^{1/7} \qquad *(1.10)$$

where $\bar{v}_x$ is the mean velocity (averaging out the turbulent fluctuations) in the x direction at any particular value of y. The distance y is measured from the wall of the tube, which is of radius a. The maximum velocity occurs, of course, at the center of the tube. Equation (1.10) can be used up to $y = a$, but not for $y > a$, for which region one uses a similar profile from the opposite wall. At values of Re higher than 10^5, the exponent falls below $\frac{1}{7}$, to values of $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{1}{10}$ at extremely high Re values. Clearly, this type of representation is an approximation.

From Eq. (1.10), one finds the mean velocity of flow v_m as for laminar flow: the total volume flow rate V is given by

$$V = \int_0^a 2\pi r \bar{v}_x \, dr$$

where $r = a - y$. Using Eq. (1.10) and integrating, one obtains

$$V = 2\pi\bar{v}_x(\text{center})(49/120)a^2$$

and hence

$$v_m = V/\pi a^2 = 0.817\bar{v}_x(\text{center}) \qquad *(1.11)$$

The distance y_m at which the flow rate is equal to the mean rate (i.e., $\bar{v}_x = v_m$) is found from Eq. (1.10) in the form

$$0.817\bar{v}_x(\text{center})/\bar{v}_x(\text{center}) = (y_m/a)^{1/7}$$

whence

$$y_m = 0.24a \qquad (1.12)$$

for turbulent flow.

The flow profile near the wall is of interest: the flow gradient [from differentiating the admittedly approximate Eq. (1.10)] is

$$d\bar{v}_x/dy = \bar{v}_x(\text{center})/7a^{1/7}y^{6/7}$$

That the empirical power-law equation is limited in application is now clear: the velocity gradient must, of course, be zero at $y = a$ and finite at $y = 0$, and the above differential does not achieve these limits. But it does suggest that if y is small (near the wall), then the velocity gradient will become very steep. Later, we shall see that in fact there is always a thin, viscous (pseudo-laminar) layer adjacent to a solid wall, and that there is indeed a steep velocity gradient in this region.

Fluctuations and Tangential Stresses in Turbulent Flow

It is well known that the tangential stress (i.e., resistance to flow) is very much higher for a liquid in turbulent flow than for one in laminar flow. This extra drag stress in turbulent flow arises from the continuous interchange of "lumps" of fluid between adjacent regions of the fluid, this mass exchange involving gains and losses of momentum. Each lump of fluid, suddenly leaving a region moving at a certain velocity, carries its momentum with it to the next region of fluid, which may be moving at a different mean velocity. Lumps of fluid transposed laterally from a faster-moving part of the fluid will tend to accelerate the slower part at which they are arriving, and conversely, as in Fig. 1-6.

Quantitatively, the mass of fluid translated from a slower-moving region to a faster layer further from the wall, per unit area per unit time, is given by $\varrho v_y'$, where v_y' is the momentary cross-current fluctuation velocity, here taken as positive (away from the wall). If at the same instant the velocity difference between the regions becomes v_x', the rate of momentum change per unit area due to the transfer is given by $\varrho v_y' v_x'$. Actually, the signs of v_y' and v_x' will always be opposite, as explained later, i.e., $-\varrho v_y' v_x'$ will be positive. This momentum transfer can be represented as a stress τ (i.e., an equivalent force per unit area which equals the rate of change of momentum per unit area). Thus at the instant of time,

$$\tau = -\varrho v_x' v_y'$$

The time average of $-\varrho v_x' v_y'$, represented $-\varrho \overline{v_x' v_y'}$, is called the Reynolds stress: the latter is thus defined by

$$\tau = -\varrho \overline{v_x' v_y'}$$

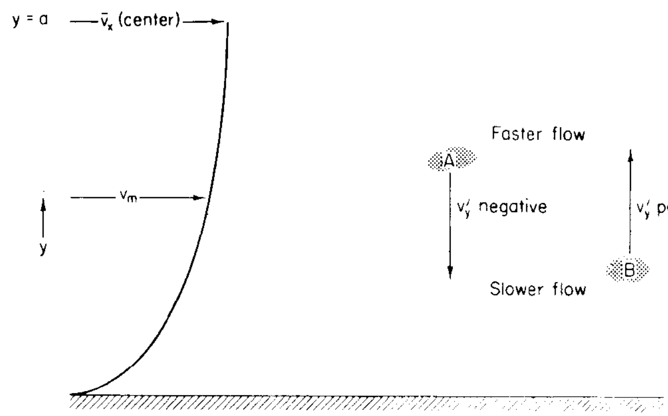

FIG. 1-6. Velocity profile and eddy fluctuations for fluid in turbulent flow in a pipe. A "lump" of fluid, such as that marked *A*, transposed from a faster-moving part of the stream to a slower part, will accelerate the latter. Conversely, a slower-moving lump *B* suddenly transposed to a faster-moving region retards the latter. Assuming that momentum is conserved, the movement of lump *B* causes a change of momentum at level y_2 given by Eq. (1.15). The corresponding stress is expressed by Eq. (1.16).

In practice, the drag stress exerted by a turbulent fluid flowing through a pipe is high, i.e., $-v_x'v_y'$ is high. This implies that v_x' and $-v_y'$ must be strongly correlated with each other: a random independent variation of each would lead to zero stress, as the example shown in Table 1-II of equally probable possibilities demonstrates.

TABLE 1-II

v_x'	v_y'	$v_x'v_y'$
+1	+1	+1
+1	−1	−1
+1	0	0
0	+1	0
0	−1	0
0	0	0
−1	+1	−1
−1	−1	+1
−1	0	0
		$\overline{v_x'v_y'} = 0$