Complexity

Hierarchical structures and scaling in physics

复杂性

物理学中的递阶结构和标度

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This is a comprehensive discussion of complexity as it arises in physical, chemical, and biological systems, as well as in mathematical models of nature. Common features of these apparently unrelated fields are emphasised and incorporated into a uniform mathematical description, with the support of a large number of detailed examples and illustrations.

The quantitative study of complexity is a rapidly developing subject with special impact in the fields of physics, mathematics, information science, and biology. Because of the variety of the approaches, no comprehensive discussion has previously been attempted. The aim of this book is to illustrate the ways in which complexity manifests itself and to introduce a sequence of increasingly sharp mathematical methods for the classification of complex behaviour. The authors offer a systematic, critical, ordering of traditional and novel complexity measures, relating them to well-established physical theories, such as statistical mechanics and ergodic theory, and to mathematical models, such as measure-preserving transformations and discrete automata. A large number of fully worked-out examples with new, unpublished results is presented. This study provides a classification of patterns of different origin and specifies the conditions under which various forms of complexity can arise and evolve. An even more important result than the definition of explicit complexity indicators is, however, the establishment of general criteria for the identification of anologies among seemingly unrelated fields and for the inference of effective mathematical models.

This book will be of interest to graduate students and researchers in physics (nonlinear dynamics, fluid dynamics, solid-state, cellular automata, stochastic processes, statistical mechanics and thermodynamics), mathematics (dynamical systems, ergodic and probability theory), information and computer science (coding, information theory and algorithmic complexity), electrical engineering and theoretical biology.

Preface

The intuitive notion of complexity is well expressed by the usual dictionary definition: "a complex object is an arrangement of parts, so intricate as to be hard to understand or deal with" (Webster, 1986). A scientist, when confronted with a complex problem, feels a sensation of distress that is often not attributable to a definite cause: it is commonly associated with the inability to discriminate the fundamental constituents of the system or to describe their interrelations in a concise way. The behaviour is so involved that any specifically designed finite model eventually departs from the observation, either when time proceeds or when the spatial resolution is sharpened. This elusiveness is the main hindrance to the formulation of a "theory of complexity", in spite of the generality of the phenomenon.

The problem of characterizing complexity in a quantitative way is a vast and rapidly developing subject. Although various interpretations of the term have been advanced in different disciplines, no comprehensive discussion has yet been attempted. The fields in which most efforts have been originally concentrated are automata and information theories and computer science. More recently, research in this topic has received considerable impulse in the physics community, especially in connection with the study of phase transitions and chaotic dynamics. Further interest has been raised by the discovery of "glassy" behaviour and by the construction of the first mathematical models in evolutionary biology and neuroscience.

The aim of this book is to illustrate the ways in which complexity manifests itself in nature and to guide the reader through a sequence of increasingly sharp mathematical methods for the classification of complex behaviour. We propose a systematic, critical, ordering of the available complexity measures, relating them to well-established physical theories, such as statistical mechanics and ergodic theory, and to mathematical models, such as measure-preserving transformations and discrete automata. The object (usually a pattern generated by some unknown rule) is investigated in the infinite-time or in the infinite-resolution limit, or in both, as appropriate. The difficulty of describing or reproducing its scaling properties shall be interpreted as an evidence of complexity.

In Chapter 1, we introduce the scientific background in which the concept of complexity arises, mentioning physical systems and models which will be more thoroughly illustrated in the following two chapters. Our survey is intended to discriminate those phenomena that are actually relevant to complexity. In Chapter 4, we review the fundamentals of symbolic dynamics, the most convenient framework to achieve a common treatment of otherwise heterogeneous systems. Chapters 5, 6, and 7 deal with probability, ergodic theory, information, thermodynamics, and automata theory. These fields form the basis for the discussion of complexity. Although the concepts and quantities they deal with may not all look modern or fashionable, they do provide, in a broad sense, a classification of complexity. We stress the complementarity of different indicators (power spectrum, degree of mixing, entropy, thermodynamic functions, automaton representation of a language) in the specification of a complex system. Physical and mathematical aspects of this analysis are illustrated with several paradigmatic examples, used throughout the book to compare various complexity measures with each other. Chapters 5, 6, and 7 are therefore essential for the understanding of Chapter 8, where we introduce the principal "classical" definitions of complexity and some of the most recent ones. We show that only a few of them actually bring a novel and useful contribution to the characterization of complexity. It will also appear that these definitions cannot be ordered by elementary inclusion relations, in such a way that simplicity of an object according to the most "liberal" measure implies simplicity according to the strictest one and vice versa for a complex object. In many cases, the results will take the form of existence or non-existence statements rather than being expressed by numbers or functions.

Chapter 9 deals with complexity measures that explicitly refer to a hierarchical organization of the dynamical rules underlying symbolic patterns produced by unknown systems. In this context, complexity is related to a particularly strong condition: namely, the lack of convergence of a hierarchical approach. In Chapter 10, the main results presented in this book are summarized and directions for future research are pointed out.

The study of complexity is a new subject in so far as it is just beginning to encompass different fields of research. Therefore, we have necessarily neglected a great number of interesting topics and mentioned others only in passing (for all these, we refer the reader to the suggested bibliography).

We have tried to give all necessary definitions in as rigorous a way as allowed, without sacrificing clarity and readability. In the effort to be concise, we have concentrated on providing a guide through the areas of research that are essential for the understanding of the novel subjects treated in this book. We hope that the uniformity of notation and the relegation of technical mathematical notions to the appendices will help the reader follow the main course of the discussion without much need for consulting standard textbooks or the original research papers.

Our primary concern has been to stimulate the reader to explore the rich and beautiful world of complexity, to create and work out examples and, perhaps, to propose his/her own complexity measure.

We apologize for any errors and passages of weak or unclear exposition, as well as for misjudgment about the importance of experiments or mathematical tools for the characterization of complexity. We shall be happy to receive comments from all who care to note them.

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Contents

Part 1

Phenomenology and models

Chapter 1

Introduction

The scientific basis of the discussion about complexity is first exposed in general terms, with emphasis on the physical motivation for research on this topic. The genesis of the "classical" notion of complexity, born in the context of the early computer science, is then briefly reviewed with reference to the physical point of view. Finally, different methodological questions arising in the practical realization of effective complexity indicators are illustrated.

1.1 Statement of the problem

The success of modern science is the success of the experimental method. Measurements have reached an extreme accuracy and reproducibility, especially in some fields, thanks to the possibility of conducting experiments under well controlled conditions. Accordingly, the inferred physical laws have been designed so as to yield nonambiguous predictions. Whenever substantial disagreement is found between theory and experiment, this is attributed either to unforeseen external forces or to an incomplete knowledge of the state of the system. In the latter case, the procedure so far has followed a reductionist approach: the system has been observed with an increased resolution in the search for its "elementary" constituents. Matter has been split into molecules, atoms, nucleons, quarks, thus reducing reality to the assembly of a huge number of bricks, mediated by only three fundamental forces: nuclear, electro-weak and gravitational interactions.

The discovery that everything can be traced back to such a small number of different types of particles and dynamical laws is certainly gratifying. Can one thereby say, however, that one *understands* the origin of earthquakes, weather variations, the growing of trees, the evolution of life? Well, in principle, yes. One has just to fix the appropriate initial conditions for each of the elementary particles and insert them into the dynamical equations to determine the solution[1]. Without the need of giving realistic numbers, this undertaking evidently appears utterly vain, at least because of the immense size of the problem. An even more fundamental objection to this attitude is that a real understanding implies the achievement of a synthesis from the observed data, with the elimination of information about variables that are irrelevant for the "sufficient" description of the phenomenon. For example, the equilibrium state of a gas is accurately specified by the values of only three macroscopic observables (pressure, volume and temperature), linked by a closed equation. The gas is viewed as a collection of essentially independent subregions, where the "internal" degrees of freedom can be safely neglected. The change of descriptive level, from the microscopic to the macroscopic, allows recognition of the inherent simplicity of this system.

This extreme synthesis is no longer possible when it is necessary to study motion at a mesoscopic scale as determined, e.g., by an impurity. In fact, the trajectory of a Brownian particle (e.g., a pollen grain) in a fluid can be exactly accounted for only with the knowledge of the forces exerted by the surrounding molecules. Although the problem is once more intractable, in the sense mentioned above, a partial resolution in this case has been found in the passing from the description of *single items* to that of *ensembles*: instead of tracing an individual orbit, one evaluates the probability for the Brownian particle to be in a given state, which is equivalent to considering a family of orbits with the same initial conditions but experiencing different microscopic configurations of the fluid. Although less detailed, this new level of description in principle involves evaluation and manipulation of a much larger amount of information: namely, the time evolution of a continuous set of initial conditions. This difficulty has been overcome in equilibrium statistical mechanics by postulating the equiprobability of the microscopic states. This constitutes a powerful short-cut towards a compact model for Brownian motion in which knowledge of the macroscopic variables again suffices. The fluid is still at equilibrium but the Brownian particle constitutes an *open* subsystem that evolves in an erratic way, being subject to random fluctuations on one side and undergoing frictional damping on the other. In addition to this, deterministic drift may be present.

These examples introduce two fundamental problems concerning physical modelling: the practical feasibility of predictions, given the dynamical rules, and the relevance of a minute compilation of the system's features. The former question entails both the inanity of the effort of following the motion of a huge number of particles and the impossibility of keeping the errors under

1. Excluding the possible existence of other unknown forces.

control. In fact, as the study of nonlinear systems has revealed, arbitrarily small uncertainties about the initial conditions are exponentially amplified in time in the presence of deterministic chaos (as in the case of a fluid). This phenomenon may already occur in a system specified by three variables only. The resulting limitation on the power of predictions is not to be attributed to the inability of the observer but arises from an intrinsic property of the system. The second observation points out that the elimination of the particles' coordinates in favour of a few macroscopic variables does not imply, in many cases, a reduced ability to perform predictions for quantities of interest. The success of statistical mechanics in explaining, e.g., specific heats, electric conductivity, and magnetic susceptibility demonstrates the significance of this approach. As long as it affects just irrelevant degrees of freedom, chaotic motion does not downgrade coarse representations of the dynamics but may even accelerate their convergence.

Nature provides plenty of patterns in which coherent macroscopic structures develop at various scales and do not exhibit elementary interconnections: for instance, the often cited biological organisms or, more simply, vortices in the atmosphere or geological formations (sand dunes, rocks of volcanic origin). They immediately suggest seeking a compact description of the spatio-temporal dynamics based on the relationships *among* macroscopic elements rather than lingering on their inner structure. In a word, it is useful and possible to condense information. This is not a mere technical stratagem to cope with a plethora of distinct unrelated patterns. On the contrary, similar structures evidently arise in different contexts, which indicates that universal rules are possibly hidden behind the evolution of the diverse systems that one tries to comprehend. Hexagonal patterns are found in fluid dynamics, as well as in the spatial profile of the electric field of laser sources. Vortices naturally arise in turbulent fluids, chemically interacting systems, and toy models such as cellular automata.

Systems with a few levels of coherent formation, while interesting and worth studying, are incomparably simpler than systems characterized by a *hierarchy* of structures over a *range* of scales. The most striking evidence of this phenomenon comes from the ubiquity of *fractals* (Mandelbrot, 1982), objects exhibiting nearly scale-invariant geometrical features which may be nowhere differentiable. A pictorial representation of this can be obtained by zooming in on, e.g., a piece of a rugged coastline: tinier and tinier bays and peninsulae are revealed when the resolution is increased, while gross features are progressively smeared out. Nonetheless, one perceives an almost invariant structure during the magnification process. Among the several examples of fractals discussed in Mandelbrot (1982), we recall cauliflowers, clouds, foams, galaxies, lungs, pumice-stone, sponges, trees. If the coarse-graining is interpreted as a change in the level of description, an exact scale-invariance, whenever it is observed, testifies to the simplicity of the system. Although such *self-similar* objects can be assimilated to translationally invariant patterns, by interchanging the operations of shift and magnification, there is a fundamental difference. The dynamical rules that support a given pattern are, in general, invariant under some symmetry group (e.g., translation). Hence, it is not surprising that the pattern exhibits the same symmetry (e.g., periodicity, as in a crystal). This is not the case of fractals, the "symmetry" of which is not built in the generation mechanism. The puzzling issue is that the same physical laws account for both types of behaviour as well as for the astounding variety of forms that we habitually experience.

A hierarchical organization in nested subdomains is particularly evident in the vicinity of a (continuous) phase transition, as occurring, e.g., in magnetic materials or superconductors. The coarse-graining procedure (Kadanoff, 1966) has led to the formulation of the renormalization-group theory (Wilson, 1971) which, in spite of its conceptual simplicity, has explained the observed phenomenology with high precision. Phase transitions, however, occur at special parameter values (e.g., melting points) whereas hierarchical structures appear to be a much more general characteristic of nature. As an example, we cite $1/f$ noise, which is the result of signals showing self-similar properties upon rescaling of the time axis. This phenomenon, although one of the commonest in nature, has so far withstood any global theoretical approach. Many other systems exhibit various levels of organization which are neither too strict, as in a crystal, nor too loose, as in a gas, nor are they amenable to any known theoretical modelling. The difficulty of obtaining a concise description may arise from "fuzziness" of the subsystems, which prevents a univocal separation of scales, or from substantial differences in the interactions at different levels of modelling.

Summarizing this introductory section, we remark that the concept of complexity is closely related to that of *understanding*, in so far as the latter is based upon the accuracy of *model* descriptions of the system obtained using a condensed information about it. Hence, a "theory of complexity" could be viewed as a theory of modelling, encompassing various reduction schemes (elimination or aggregation of variables, separation of weak from strong couplings, averaging over subsystems), evaluating their efficiency and, possibly, suggesting novel representations of natural phenomena. It must provide, at the same time, a definition of complexity and a set of tools for analysing it: that is, a system is not complex by some abstract criterion but because it is intrinsically hard to model, no matter which mathematical means are used. When defining complexity, three fundamental points ought to be considered (Badii, 1992):

1. Understanding implies the presence of a *subject* having the task of describing the *object*, usually by means of model predictions. Hence, complexity is a "function" of both the subject and the object.
2. The object, or a suitable representation of it, must be conveniently

divided into *parts* which, in turn, may be further split into subelements, thus yielding a *hierarchy*. Notice that the hierarchy need not be manifest in the object but may arise in the construction of a model. Hence, the presence of an actual hierarchical structure is not an infallible indicator of complexity.

3. Having individuated a hierarchical encoding of the object, the subject is faced with the problem of studying the *interactions* among the subsystems and of incorporating them into a model. Consideration of the interactions at different levels of resolution brings in the concept of *scaling*. Does the increased resolution eventually lead to a stable picture of the interactions or do they escape any recognizable plan? And if so, can a different model reveal a simpler underlying scheme?

1.2 Historical perspective

Although the inference of concise models is the primary aim of all science, the first formalization of this problem is found in discrete mathematics. The object is represented as a sequence of integers which the investigator tries to reproduce exactly by detecting its internal rules and incorporating them into the model, also a sequence of integers. The procedure is successful if a size reduction is obtained. For example, a periodic sequence, such as 011011011..., is readily specified by the "unit cell" (011 in this case) and by the number of repetitions.

This approach has given rise to two disciplines: computer science and mathematical logic. In the former, the model is a computer program and the object sequence is its output. In the latter, the model consists of the set of rules of a formal system (e.g., the procedure to extract square roots) and the object is any valid statement within that system (as, e.g., $\sqrt{4} = 2$). Compression means that knowledge of the whole formal system permits the deduction of all theorems automatically, without any external information. In this view, the complexity of symbol strings is called *algorithmic* and is defined as the size of the minimal program which is able to reproduce the input string (Solomonoff, 1964; Kolmogorov 1965; Chaitin 1966). As a consequence, completely random objects have maximal complexity because no compression is possible for them. This characterization of clearly structureless patterns makes the definition unsuitable in a physical context. Actually, algorithmic complexity coincides, in most cases, with entropy and is therefore a measure of disorder.

A detailed analysis allowed the models to be regrouped in computational classes with qualitatively and quantitatively different ability in the manipulation of symbolic objects. The main four classes form a hierarchy, named after Chomsky (1956, 1959), which culminates in the Turing machine (Turing, 1936), the prototype of a universal computer: that is, of an automaton which is able