周光炯　编注

力学与工程科学专业英语

北京大学出版社

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前　言

目前，大学本科生在学完大学英语课程之后，要想顺利地阅读专业书刊，尚存在不少困难。这主要是由于学生尚未掌握足够多科技书刊中所常用的词汇，词组和语法结构知识所致。编注本书的目的，即在于提供这方面的学习材料，以便学生能在较短时间内迅速提高阅读专业书刊的能力。

本书是在原《力学专业英语讲义》的基础上经过适当加工而成的。该讲义曾在北京大学力学与工程科学系连续使用过不下十余次，中间曾经多次修改，并由不同的教师使用，反映效果良好。1994 年，编注者再一次对该讲义作了全面修改与更新，并增加了不少新内容，以便能适用于更多的理、工科专业、如数学、物理、力学和各工程专业等。

本书编选的文章共 38 篇，分为两大部分，基本上按由浅入深的原则编排。第一部分包括文章 28 篇，主要是简明、扼要、系统、全面地介绍理工科中一些专业基础课程的内容。第二部分共 10 篇。大多数是介绍工科各系的文章。所以，本书不仅是一本基础专业英语教材，也是一本讲述有关学科和学系的中级英语科普读物。

本书是为已通过大学 4 级英语考试的学生编写的。《大学英语教学大纲通用词汇表(1—4)》(1995 年 2 月，上海外语教育出版社、高等教育出版社)中的一切词汇、词组，除了极少数有特殊含义的以外，一般均不再加注解。此外，每篇文章的注解和注释是完全独立的，互不相干。这样，既便于任意选择教学内容，也有利于通过重复学习以加深对词汇、词组的理解与记忆。

使用本书时，可根据不同专业的需要，要求和授课时数，由教师选取书中的若干篇文章作为教材。并建议采用自学——提问——讨论的教学方式，即课前由学生独立自学所指定的内容，课堂上由学生或教师提问并开展讨论。这样，不仅有利于迅速提高学生的阅读能力，而且也可更好地理解和掌握教学内容。

本书中的大部分文章摘选自书末所附书目(Bibliography)中的各种书刊。多数文章的末尾均注有撰写该文的著者姓名和刊载该文的书目编号。少数文章的根源及著者不详，或由编注者自己编辑而成的均未加注。在此，仅向文章被摘用的全体著者和出版社表示衷心的感谢。

由于编注者的业务和外语水平有限，在本书的注解与译文中，不妥与谬误之处在所难免，恳请广大读者和专家给予批评指正。

编注者

1995 年 11 月

CONTENTS

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PART B

PART A

1. THIS IS MATHEMATICS

Why has mathematics become so important in recent years? Why is our government spending millions of dollars to educate more mathematicians? Can the new electronic brains solve our mathematical problems faster and more accurately than a person and eliminate the need for mathematicians?(1)

To answer these questions, we need to know what mathematics is and how it is used. Mathematics is much more than arithmetic, which is the science of numbers and computation. It is more than algebra, which is the language of symbols, operations, and relations. It is much more than geometry, which is the study of shapes, sizes, and spaces. It is more than statistics, which is the science of interpreting data and graphs. It is more than calculus, which is the study of change, limits, and infinity. Mathematics is all of these—and more.

Mathematics is a way of thinking, a way of reasoning. Mathematics can be used to determine whether or not an idea is true, or, at least, whether it is probably ture. Mathematics is a field of exploration and invention, where new ideas are being discovered every day. It is a way of thinking that is used to solve all kinds of problems in the sciences, government, and industry. It is a language of symbols that is understood in all civilized nations of the world. It has even been suggested that mathematics would be the language that would be understood by the inhabitants of Mars (if there are any)! It is an art like music, with symmetry, pattern, and rhythm that can be very pleasing.

Mathematics has also been described as the study of patterns, where a pattern is any kind of regularity in form or idea.(2) This study of patterns has been very important for science because pattern, regularity and symmetry occur so often in nature. For example, light, sound, magnetism, electric currents, waves of the sea, the flight of a plane, the shape of a snowflake, and the mechanics of the atom all have patterns that can be classified by mathematics.

词　汇

mathematician[mæθimaˈtiʃən]n. 数学家
electronic brain[ilekˈtrɔnik brein]电脑
accurately[ˈækˈjuritli]a. 准确地
computation[kɔmpjuˈteiʃən]n. 计算
algebra[ˈældʒibrə]n. 代数(学)
statistics[stəˈtistiks]n. 统计(学)
calculus[ˈkælkjuləs]n. 微积分(学)
limit[ˈlimit]n. 极限
infinity[inˈfiniti]n. 无限
thinking[ˈθiŋkiŋ]n. 思考
reasoning[ˈriːzəniŋ]n. 推理
exploration[eksplɔːˈreiʃən]n. (1)探索;(2)研究
civilized nation[ˈsivilaizd ˈneiʃən]文明国家
Mars[maːz]n. 火星
symmetry[ˈsimitri]n. 对称(性)
regularity[regjuˈlæriti]n. 规律性
magnetism[ˈmægnitizəm]n. 磁(学,性)
electric current[iˈlektrik ˈkʌrənt]电流

snowflake['snoufleik]n. 雪花(片)

词　组

to eliminate the need for (or of) 对……不需要
to need to + 不定式　必须;需要
to be (much) more than … 比……更多; 远非……可比
to be used to + 不定式　用来
to be being + 过去分词　正被……; 为现在进行时的被动态
It is suggested that　有人提议,它的现在完成时被动态为 It has been suggested that
to be described as 被描述为; 被说成是
in nature 性质上; 事实上

注　释

(1) Can the New electronic brains……for mathematicans?

此句为一并列句,主语为 the new electronic brains, 谓语为 solve 与 eliminate 二者用 and 连接。全句可译为:

新的电脑能较人更快更准确地解决我们的数学问题吗? 它能消除对数学家的需要吗?

(2) Mathematics has also……or idea

此句中的 where……idea 为一定语从句。全句可译为:

数学也被说成是对模式的研究。这里的模式系指任何一种形状或概念上的规律性。

2. MATHEMATICAL SYMBOLS AND EXPRESSIONS

Below are some of the more common symbols and expressions used in mathematics.

$\frac{1}{2}$	a (or one) half	$\frac{3}{4}$	three fourths (or quarters)
$\frac{1}{3}$	a (or one) third	$\frac{4}{5}$	four fifths or four over five
$\frac{1}{4}$	a (or one) fourth (or quarter)	$\frac{113}{300}$	one hundred and thirteen over three hundred
$\frac{1}{10}$	a (or one) tenth	$2\frac{1}{2}$	two and a half
$\frac{1}{100}$	a (or one) hundredth	$2\frac{7}{8}$	two and seven over eight or two and seven eighths
$\frac{1}{1000}$	a (or one) thousandth	$3\frac{1}{8}$	three and one eighth
$\frac{1}{1234}$	a (or one) over one (or a) thousand two hundred and thirty-four	$4\frac{1}{3}$	four and a third
$\frac{2}{3}$	two thirds or two over three	$125\frac{3}{4}$	a (or one) hundred twenty-five and three fourths (or quarters)

0.1	o point (or decimal) one; zero point (or decimal) one; nought point (or decimal) one
0.25	nought point (or decimal) two five or point (or decimal) two five.
2.35	two point (or decimal) three five
45.67	forty-five point (or decimal) sixty-seven
$X+Y=Z$	X plus Y is (or are; equals; is equal to) Z
$X-Y=Z$	X minus Y is (or equals; is equal to) Z
$X\pm Y=Z$	X plus or minus Y is (or equals; is equal to) Z
$X\times Y=Z$	X times (or multiplied by) Y equals (or are; is equal to; make(s)) Z
$\frac{X}{Y}=Z$	X over Y is (or equals; is equal to) Z or X divided by Y is (or equals) Z
$1\times 1=1$	once one is one
$2\times 2=4$	twice two is four
$X:Y$	the ratio of X to Y
$a:b=c:d$	the ratio of a to b equals the ratio of c to d or a is to b as c is to d.
$X=Y$	X is equal to or equals Y
$X\neq Y$	X is not equal to or does not equal Y
$X\approx Y$	X is approximately equal to Y or X approximately equals Y
$X<Y$	X is less than Y
$X\nless Y$	X is not less than Y
$X\leqslant Y$	X is less than or is equal to Y
$X>Y$	X is greater (or more) than Y
$X\ngtr Y$	X is not greater (or more) than Y
$X\geqslant Y$	X is greater (or more) than or is equal to Y
$X<<Y$	X is much less than Y
$X>>Y$	X is much (or far) greater than Y.
$X\propto Y$	X is (directly) proportional to Y or varies (directly) as Y
$X\%$	X per cent
$\|X\|$	the absolute value of X or modulus of X
X^2	X square; X squared; the square of X; the second power of X; X to the second (power) or to the power two
X^3	X cube; X cubed; the cube of X; the third power of X; X to the third (power) or to the power three
X^4	the fourth power of X; X to the fourth (power) or to the power four
X^n	the nth power of X; X to the nth (power) or to the power n
X^{-n}	X to the minus nth (power)
$\sqrt{X}$	the square root of X or root X
$\sqrt[3]{X}$	the cube root of X
$\sqrt[n]{X}$	the nth root of X
$\sqrt[5]{X^2}$	the fifth root of X square
$n!$	n factorial
$\angle$	angle
$\perp$	right angle
$\triangle$	triangle
∞	infinity
$A/\!/B$	A is parallel to B

$A \perp B$	A is perpendicular to B
( ),〔 〕,{ },《 》	brackets
( )	parentheses; round brackets or curves
〔 〕	square (angular) brackets
{ }	braces or curly brackets
$< \ >$	angle brackets
《 》	double brackets
$(X+Y)$	bracket $X+Y$ bracket closed
$\bar{X}$	X bar; the mean value of X
X'	X prime
X''	X double prime
X'''	X triple prime
$\dot{X}$	X dot
$\ddot{X}$	X two dots
Xm	X sub m
X''_m	X double prime, sub m
X^m	X super m
$\log_{10}X$	log$X$ (or log of X) to the base 10 (common logarithm)
$\ln X$	log X to the base e (natural logarithm)
$\sin^{-1}X$	aro sine X
$\sinh X$	the hyperbolic sine X
$\sum$	sigma; summation of; the sum of the terms indicated
Π	the product of the terms indicated
(x,y,z)	set of elements (or members) x,y,z
(a,b) 〕a,b〔	open interval
〔a,b〕	closed interval
〔a,b) 〔a,b〔	half-open interval (open at the right)
$\cap$	intersection
$\cup$	union
$\exists$	there is a or there exists
$p \equiv q$ $p \leftrightarrow q$ $p \Leftrightarrow q$	p is equivalent to or is identical with q; p is identically equal to or is identity to q
$p \supset q$ $p \rightarrow q$ $p \Rightarrow q$	p implies q or if p then q
$A \subset X$	A is a subset of the set X or is contained in the set X
$A \not\subset X$	A is not a subset of the set X or is not contained in the set X
$A \in X$	A is a member (or an element) of the set X; A belongs to X
$A \notin X$	A is not a member (or an element) of the set X; A does not belong to X
$f(x)$	function f of x
$y=f(x)$	y is a function of x

$\lim\limits_{x\to a} f(x)=b$ $\lim\limits_{x=a} f(x)=b$ $f(x)\to b$ as $x\to a$	b is the limit of $f(x)$ as x approaches a
$\lim\limits_{x=a^-} f(x)$ $f(a^-)$	limit of $f(x)$ as x approaches a from the left
$f(x)\nearrow b$ $f(x)\uparrow b$	$f(x)$ increase, approaching the limit b
$f(x)\searrow b$ $f(x)\downarrow b$	$f(x)$ decrease, appoaching the limit b
$\triangle$	(finite difference or increment) delta
$\triangle x$ or δx	(the increment of x) delta x
dx	dee x; differential x
$\frac{dy}{dx}$	the first derivative of y with respect to x
$\frac{d^2y}{dx^2}$	the second derivative of y with respect to x
$\frac{d^ny}{dx^n}$	the nth derivative of y with respect to x
$\frac{\partial y}{\partial x}$	the first partial derivative of y with respect to x
$\bar{F}$	vector F
∇	$i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}$ del; nabla; vector differential operator
∇^n	nth del (nabla)
∇f grad f	gradient of f
∇V div V	divergence of V
$\nabla\times V$ curl V rot V	curl of V
∇^2 $\triangle$ div. grad.	Laplacian
$\int_a^b$	integral between limits a and b
$\int_E f(x)\,dV(x)$	the integral of function f over set E with respect to measure V

词　汇

decimal['desiməl]a. 十进(小数)的；n. 小数
nought[nɔːt]n. 零
approximately[ə'prɔksimitli]ad. 近似地
absolute value['æbsəluːt'væljuː]绝对值
modulus['mɔdjulas]n. 模数
factorial[fæk'ɔːriəl]a. 阶乘的；n. 阶乘
right angle[rait'æŋgl]直角
infinity[in'finiti]n. 无限

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brackets[ˈbrækits]n. 括号
parentheses[pəˈrenθises]n. 括号;园括号（复数）
round brackets　园括号
square brackets　方括号
braces[breises]n. 大括号
curly brackets　波形括号
angle brackets　角括号
double brackets　双括号
prime[praim]n. (1)负数;(2)字码右上角的撇号
triple[ˈtripl]a. 三次(倍)的
sub subscript[ˈsʌbskript]的简写;n. 下标
super superscript[ˈsjuːpəskript]的简写;n. 上标
base[beis]n. 基数
logarithm[ˈlɔgəriθəm]n. 对数
common logarithm　普通对数
natural logarithm　自然对数
arc sine　反正弦
hyperbolic sine　双曲线正弦
sigma[ˈsigmɔ]n. 希腊字母 $\sum$ 或 σ
summation[sʌˈmeiʃən]. 求和
term　[təːm]n. 项
set[set]n. 集(合)
open interval　开区间
closed interval　闭区间
half－open interval　半开区间
intersection[intəˈsekʃən]n. 相交
union[ˈjuːnjən]n. (1)结(联)合;(2)并(集)
identity[aiˈdentiti]n. (1)恒等式;(2)完全相同
subset[ˈsʌbset]n. 子集(合)
limit[ˈlimit]n. 极限
finite difference　有限差分
increment[ˈinkrimənt]n. 增量
delta[ˈdeltə]n. 希腊字母△或 δ
dee[diː]n. D 字
differential[difəˈrenʃəl]a. 微分(差别)的;n. 微分
derivative[diˈrivətiv]a. 导出的;n. 导数;微商
partial derivative　偏导数
vector[ˈvektə]n. 矢(向)量
del　n. 倒三角形▽
nabla[ˈnæblɔ]n. 劈形算符▽
operator[ˈɔpəretiə]n. (1)算子;(2)操作人员
gradient[ˈgreidiənt]n. 梯度
divergence[daiˈvəːdʒəns]n. 散度
curl[kəːl]n. 旋度
Laplacian[ləˈplaːsiən]拉普拉斯算子
integral[ˈintigrəl]a. 积分的;n. 积分
measure[ˈmeʒə]n. 量度,测度

词　组

to be equal to　等于
(to be) multiplied by　乘以
(to be) divided by　除以
to be(much) less than　小于
to be(much) greater (more) than　大于
to be (directly or inversely) proportional to　与……成正或反比
to vary(directly or inversely) as　与……成正或反比(变化)
to be parallel to　平行于
to be perpendicular to　垂直于
to be equivalent to　等于,与……等效
to be indentical with　与……完全相等

3. SETS: A USEFUL MATHEMATICAL IDEA

One of the greatest new ideas in mathematics developed in the past century is SET THEORY, invented by the German mathematician Georg Cantor (1845—1918). It has been a means of finding new facts and of proving old facts.

The idea of a set is very simple. A set is a collection of objects, numbers, persons, or ideas. You are already familiar with sets such as a set of books, a set of dishes, or a set of tools. You are even a member of a set of people such as your mathematics class, your scout troop, or your family.

Set ideas are used in many fields of mathematics. In arithmetic, we talk about sets of numbers. For example, the set of prime numbers less than 10 is (2,3,5,7).

In algebra, we talk about the solution sets for sentences[1]. For example, the solution set for $X-5=7$ is the number 12.

In geometry, we talk about the set of points that meet certain conditions. The set of points on line AB and also on the boundary of the circle is the points P and Q.

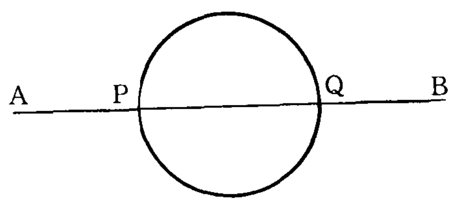

Fig. 1

In statistics, we talk about sets of data. For example, the set of scores on a weekly math quiz was (7,10,5,6,9,7,8,4,6,8,5,7).

In probability we talk about the set of all possible events. For example, the set of ways three letters, A,B,C, can be arranged is (ABC, BAC, BCA, CAB, CBA).

In measurement, we talk about a set of units of measure. A set of metric units of length is (kilometer, hectometer, decameter, meter, decimeter, centimeter, millimeter).

In everyday situations, we talk about sets of persons or objects with certain characteristics. The set of all students in the algebra class and also on the basketball team, for example, might be (John, Bill, Steve).[2]

Set theory has been the basis for a new philosophy about mthematics. It has freed mathematics from working with individual numbers, permitting it to consider sets of numbers[3]. Set ideas have also made it possible to solve many problems that involve relationships rather than numbers. And sets have been proved useful in setting up problems for electronic computers.

词　汇

set theory　集(合)论
scout [skaut] troop　童子军
prime number　素(质)数
solution set　解答组
sentence['sentəns]n. 句子,表示一完整思想的一组字,此处指代数方程
math=mathematics
quiz[kwiz]n. 测验
measure['meʒə]测度,量度
hectometer['hektoumi:tə]n. 百米

decameter['dekəmi:tə]n. 十米
decimeter['desimitə]分米(十分之一米)
basket ball team　兰球队
electronic computer　电子计算机

词　组

a collection of　一批(群,堆)
to be familir with　熟习,精通
a set of　一套(组,群,批)
to talk about　谈论
less than　小于
to free M from N　使 M 摆脱 N
to work with　与……一道工作;同……打交道

注　释

(1) In algebra, …… for sentences.

此句中的 sentence 其原意为语法中的句子或逻辑中的命题,这里可解释为代数中的方程式。全句可译为:

在代数学中,我们讨论代数方程的解答组。

(2) The set of all students …… might be (John, Bill, Steve)

此句的主语为 The set of all students, 谓语为 might be。全句可译为:

例如,既上代数课又参加兰球队的全部学生可能是 John,Bill 和 Steve。

(3) It has freed …… of numbers.

此句中的现在分词短语 permitting……numbers 为结果状语,而 to consider……numbers 为宾语 it 的补语。主句 it has …… 中的 it 指集(合)论,而 permitting it …… 中的 it 则指数学。全句可译为:

集(合)论使得数学摆脱了与单个数打交道,而容许它(指数学)考虑数的集(合)。

4. AXIOMS, DEFINITIONS AND THEOREMS

The claim has been made for mathematics that it is the best and most precise language known for the statement of complicated ideas. (1) We would like now to examine the structure of this language and analyze the rather extravagant claims we have made for it.

One of the bases for the claim to precision is that careful definitions are made for most of the words used in mathematics. (2) For example, in geometry a set of points is collinear if and only if there is a line which contains all the points of the set, or a set S is called convex if for every two points P and Q belonging to S, the entire segment PQ lies in S. However, you will note that these definitions for collinearity and convexity are given in terms of other words which are assumed to be known, in this case "set", "point", "line", "belongs to", and "segment". Some of these may have been defined previously. If so, they must have been defined in terms of other words. If you have ever tried to look up an unfamiliar word in the dictionary, you will grasp the idea. A dictionary is helpful only if you already know a good many words. The same is true in mathematics. "Collinear" is made clear by the definition given above if you already understand what "set", "point", "line", and "contains" mean. Where, then, is our precise language? One of the differences between mathematics and other disciplines is that in mathematics we state clearly at the beginning that we realize it is impossible to define every term. Consequently, we state which terms we shall not try to define. These terms are called, reasonably enough, undefined terms. This does not mean that we are operating completely in the dark. We talk about how these undefined terms will be used, and we try to explain what we assume concerning their properties, but we recognize their logical status at the outset. This is what Bertrand Russell[3] meant by, "…… mathematics is the subject in which we never know what we are talking about."

Once the set of undefined terms has been agreed upon, definitions of other terms are then made in terms of these. We make no pretense that all words will be defined in our mathematical system. By a mathematical definition, we simply mean an agreement on the way in which certain words or symbols will be used.

If we stopped here, our language would be pretty useless. The next step is to state precisely what properties of our undefined terms we plan to use in this mathematical structure. These statements about the undefined terms are called axioms or postulates. Axioms were once said to be "self-evident truths"; however, modern devlopments in mathematics have shown this description to be at best inadequate and usually meaningless". Instead of claiming that axioms are obviously true, we simply assume that the undefined terms have the properties given by the axioms, and investigate the consequences of these assumptions. This set of undefined terms, together with the set of statements about these terms (axioms of the system), and a special set of statements which give the rules of logic used in developing the structure constitute the basis for the mathematical theory being studied.[4] The theory is then developed by making new definitions and proving statements (called theorems) which are logical consequences of the axioms of the system.

Hence when we say that a theorem in a mathematical system is true, we are not making a claim about the physical world or about any absolute standard of truth, but simply that it is a consequence of the axioms and rules of logic which are the basis for the theory. That is, we claim only "If …,then…". This is perhaps the most characteristic feature of mathematics.

词　汇

axiom['æksiəm]n. 公理
theorem['θiərəm]n. 定理
complicate['kɔmplikeit]v. (使)变复杂
extravagant[iks'trævigant]a. 过份的
collinear[kɔ'linjə]a. 共线的
convex['kɔn'veks]a. 凸的;n. 凸状
segment['segmənt]n. (1)片段;(2)部分
collinearity[kɔ'linjari'ti]n. 共线性
convexity[kɔn'veksiti]n. 凸状,凸性(度)
term[tə:m]n. (1)术语;(2)项
pretense[pri'tens]或 pretence n. (1)假装;(2)自吹
postulate['pɔstjuleit]v. 假定(说)
self-evident a. 不言而喻的
inadequate[in'ædikwit]a. 不适当的,不充足的
characteristic feature　独有的特征

词　组

to make a claim for (about, to, on)断言;要求
known for…　已知…的
a set of　一批(套,群,组)
if and only if　当且仅当
to lie in　处于
in terms of　用…表示
to be assumed to be　假定为
only if　仅当
at the begining　在开始
in the dark　在黑暗中
to talk about　谈论
at the outset　开始
to agree upon (on)　对……意见一致
to make no pretense　不虚 谛

together with 连同，和……一起

注　释

(1) The claim has been made for …… complicated ideas.

此句中的 that it is …… complicated ideas 是主语 The claim 的同位语从句，其中 it 系指 mathematics。全句可译为：

有人已对数学断言过，它是已知的，陈述复杂概念的最好的和最精密的语言。

(2) One of the bases …… in mathematics

此句中的 that …… in mathematics 为一表语从句。全句可译为：

断言数学精密的根据之一是在数学中所使用的大部分词汇都是经过仔细定义的。

(3) Bertrand Russell (1874－1970)英国哲学家，著有“数学原理”(1903，1910)与“西方哲学史”(1946)等书。

(4) This set of undefined terms, …… being studied.

此句的主语是 This set of undefined terms, together with …… 和 a special set of statements，谓语为 constitute。Which give …… the structure 为 a special set of statements 的定语从句，being studied 为现在分词被动态，用以说明 theory。全句可译为：

这套未加定义的术语，连同一套与这些术语有关的陈述(即这系统的公理)和一组特殊的陈述——它们给出了在发展这结构中所用的逻辑法则——构成了目前正在进行研究的数学理论的基础。

5. LIMITS OF FUNCTIONS

5.1 Limits from the left (right)

Suppose that f is defined on an interval (a,b). We say that $f(x)$ tends (or converges) to a limit l as x tends to b (a) from the left (right) and write

$$f(x) \to l \text{ as } x \to b- \quad (a+)$$

or, alternatively,

$$\lim_{x \to b-\ (a+)} f(x) = l$$

if the following criterion is satisfied.

Given any $\varepsilon>0$, we can find a $\delta>0$ such that $\lvert f(x)-l\rvert<\varepsilon$ provided that $b-\delta<x<b$. $(a<x<a+\delta)$

The number $|f(x)-l|$ is the distance between $f(x)$ and l. We can think of it as the error in approximating to l by $f(x)$. The definition of the statement $f(x)\to l$ as $x\to b-$ $(a+)$ then amounts to the assertion that we can make the error in approximating to l by $f(x)$ as small as we like by taking x sufficiently close to b (a) on the left (right). [1]

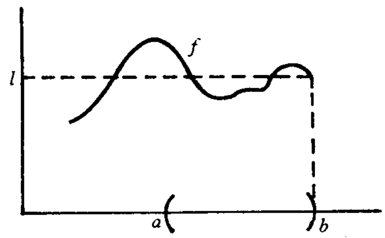

Fig. 2
(limit from the left)

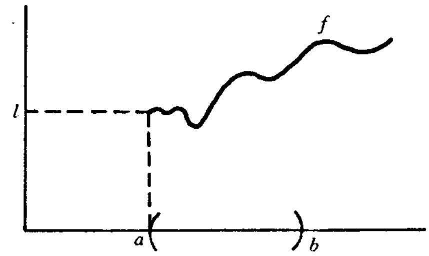

Fig. 3
(limit from the right)

5.2 $f(x)\to x$ as $x\to\xi$

Suppose that f is defined on an interval (a, b) except possibly for some point $\xi\in(a, b)$. We say that $f(x)$ tends (or converges) to a limit l as x tends to ξ and write

$$f(x)\to t \text{ as } x\to\xi$$

or, alternatively,

$$\lim_{x\to\xi} f(x)=l$$

if the following criterion is satisfied.

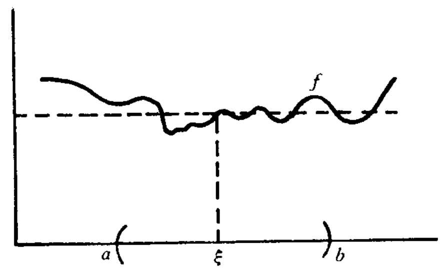

Fig. 4

Given any $\varepsilon>0$, we can find a $\delta>0$ such that $\lvert f(x)-l\rvert<\varepsilon$ provided that $0<\lvert x-\xi\rvert<\delta$

If α and β are real numbers, it is often useful to note that the inequality $\lvert\alpha\rvert<\lvert\beta\rvert$ is equivalent to $-\beta<\alpha<\beta$, or, what is the same thing, $-\beta<-\alpha<\beta$ [(see 5.14 exercise (1)]

Thus, in the definitions above, the condition $\lvert f(x)-l\rvert<\varepsilon$ can be replaced throughout by $-\varepsilon<f(x)-l<\varepsilon$ or, alternatively, by $-\varepsilon<l-f(x)<\varepsilon$.

Similarly, the condition $0<\lvert x-\xi\rvert<\delta$ in the last definition is equivalent to the assertion $-\delta<x-\xi<\delta$ and $x\neq\xi$。Thus to say that $0<\lvert x-\xi\rvert<\delta$ is to say that x satisfies one of the two inequalities $\xi-\delta<x<\xi$ or $\xi<x<\xi+\delta$

With the help of the last remark it is easy to prove the following result.

5.3 Proposition

Let f be defined on an interval (a,b) except possibly at a point $\xi\in(a,b)$. Then $f(x)\to l$ as $x\to\xi$ if and only if $f(x)\to l$ as $x\to\xi-$ and $f(x)\to l$ as $x\to\xi+$.

Example Let f be the function from $\mathfrak{R}(2)$ to itself defined by

$$f(x)=\begin{cases}1-x & (x\leqslant 1)\\ 2x & (x>1)\end{cases}$$

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