ELEMENTS OF VIBRATION ANALYSIS

Second Edition

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To My Wife and to the
Memory of My Parents

ELEMENTS OF VIBRATION ANALYSIS
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PREFACE

In the last several decades, impressive progress has been made in vibration analysis, prompted by advances in technology. On the one hand, the requirement for the analysis of increasingly complex systems has been instrumental in the development of powerful computational techniques. On the other hand, the development of fast digital computers has provided the means for the numerical implementation of these techniques. Indeed, one of the most significant advances in recent years is the finite element method, a method developed originally for the analysis of complex structures. The method has proved to be much more versatile than conceived originally, finding applications in other areas, such as fluid mechnics and heat transfer. At the same time, significant progress was being made in linear system theory, permitting efficient derivation of the response of large-order systems. *Elements of Vibration Analysis* was written in recognition of these advances.

The second edition of *Elements of Vibration Analysis* differs from the first edition in several respects. In the first place, the appeal of the first few chapters has been broadened by the inclusion of more applied topics, as well as additional explanations, examples, and homework problems. Advanced material has been transferred to later chapters. The chapter on the finite element method, Chap. 8, has been rewritten almost entirely so as to reflect the more current thinking on the subject, as well as to include more recent developments. The section on the Routh-Hurwitz criterion in Chap. 9 has been expanded. On the other hand, some advanced material in Chap. 10 has been deleted. The chapter on random vibrations, Chap. 11, has been enlarged by absorbing material on Fourier transforms from Chap. 2 and by expanding the discussion of narrowband processes. Chapter 12 represents an entirely new chapter, devoted to techniques for the computation of the response on digital computers. The chapter includes material indispensable in a modern course in vibrations. Finally, some of the material in App. C has been rewritten by placing the emphasis on physical implications. As a result of these revisions, the first part of the second edition is more accessible to juniors and should have broader appeal than the first edition. Moreover, the material in later chapters makes this second edition an up-to-date book on vibration analysis.

The book contains material for several courses on vibrations. The material covers a broad spectrum of subjects, from the very elementary to the more advanced, and is arranged in increasing order of difficulty. The first five chapters of the book are suitable for a beginning course on vibrations, offered at the junior or senior level. The material in Chaps. 6–12 can be used selectively for courses on dynamics of structures, nonlinear oscillations, random vibrations, and advanced vibrations, either at the senior, or first-year graduate level. To help the instructor in tailoring the material to his or her needs, the book is reviewed briefly:

Chapter 1 is devoted to the free vibration of single-degree-of-freedom linear systems. This is standard material for a beginning course on vibrations.

Chapter 2 discusses the response of single-degree-of-freedom linear systems to external excitation in the form of harmonic, periodic, and nonperiodic forcing functions. The response is obtained by the classical and Laplace transformation methods. A large number of applications is presented. If the response by Laplace transformation is not to be included in a first course on vibrations, then Secs. 2.17 and 2.18 can be omitted.

Chapter 3 is concerned with the vibration of two-degree-of-freedom systems. The material is presented in a way that makes the transition to multi-degree-of-freedom systems relatively easy. The subjects of beat phenomenon and vibration absorbers are discussed. The material is standard for a first course on vibrations.

Chapter 4 presents a matrix approach to the vibration of multi-degree-of-freedom systems, placing heavy emphasis on modal analysis. The methods for obtaining the system response are ideally suited for automatic computation. The material is suitable for a junior level course. Sections 4.11 through 4.13 can be omitted on a first reading.

Chapter 5 is devoted to exact solutions to response problems associated with continuous systems, such as strings, rods, shafts, and bars. Again the emphasis is on modal analysis. The intimate connection between discrete and continuous mathematical models receives special attention. The material is suitable for juniors and seniors.

Chapter 6 provides an introduction to analytical dynamics. Its main purpose is to present Lagrange's equations of motion. The material is a prerequisite for later chapters, where efficient ways of deriving the equations of motion are necessary. The chapter is suitable for a senior-level course.

Chapter 7 discusses approximate methods for treating the vibration of continua for which exact solutions are not feasible. Discretization methods based on series solutions, such as the Rayleigh-Ritz method, and lumped methods are presented. The material is suitable for seniors.

Chapter 8 is concerned with the finite element method. The earlier material is presented in a manner that can be easily understood by seniors. Later material is more suitable for beginning graduate students.

Chapter 9 is the first of two chapters on nonlinear systems. It is devoted to such qualitative questions as stability of equilibrium. The emphasis is on geometric description of the motion by means of phase plane techniques. The material is suitable for seniors or first-year graduate students, but Secs. 9.6 and 9.7 can be omitted on a first reading.

Chapter 10 uses perturbation techniques to obtain quantitive solutions to response problems of nonlinear systems. Several methods are presented, and phenomena typical of nonlinear systems are discussed. The material can be taught in a senior or a first-year graduate course.

Chapter 11 is devoted to random vibrations. Various statistical tools are introduced, with no prior knowledge of statistics assumed. The material in Secs. 11.1 through 11.12 can be included in a senior-level course. In fact, its only prerequisites are Chaps. 1 and 2, as it considers only the response of single-degree-of-freedom linear systems to random excitation. On the other hand, Secs. 11.13 through 11.18 consider multi-degree-of-freedom and continuous systems and are recommended only for more advanced students.

Chapter 12 is concerned with techniques for the determination of the response on a digital computer. Sections 12.2 through 12.5 discuss the response of linear systems in continuous time by the transition matrix and Sec. 12.6 presents discrete-time techniques. Section 12.7 is concerned with the response of nonlinear systems. All this material is intended for a senior, or first-year graduate course. Sections 12.8 through 12.12 are concerned with frequency-domain techniques and in particular with aspects of implementation on a digital computer. The material is suitable for a graduate course.

Appendix A presents basic concepts involved in Fourier series expansions, App. B is devoted to elements of Laplace transformation, and App. C presents certain concepts of linear algebra, with emphasis on matrix algebra. The appendixes can be used for acquiring an elementary working knowledge of the subjects, or for review if the material was studied previously.

It is expected that the material in Chaps. 1 through 3 and some of that in Chap. 4 will be used for a one-quarter, elementary course, whether at the junior or senior level. For a course lasting one semester, additional material from Chap. 4 and most of Chap. 5 can be included. A second-level course on vibrations has many options. Independent of these options, however, Chap. 6 must be regarded as a prerequisite for further study. The choice among the remaining chapters depends on the nature of the intended course. In particular, Chaps. 7 and 8 are suitable for a course whose main emphasis is on deterministic structural dynamics. Chapters 9 and 10 can form the core for a course on nonlinear oscillations. Chapter 11 can be used for a course on random vibrations. Finally, Chap. 12 is intended for an advanced, modern course on vibration analysis, with emphasis on numerical results obtained on a digital computer.

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Leonard Meirovitch

CONTENTS

CHAPTER

ONE

FREE RESPONSE OF SINGLE-DEGREE-OF-FREEDOM LINEAR SYSTEMS

1.1 GENERAL CONSIDERATIONS

As mentioned in the Introduction, systems can be classified according to two distinct types of mathematical models, namely, discrete and continuous. Discrete models possess a finite number of degrees of freedom, whereas continuous models possess an infinite number of degrees of freedom. The number of degrees of freedom of a system is defined as the number of independent coordinates required to describe its motion completely (see also Sec. 4.2). Of the discrete mathematical models, the simplest ones are those described by a first-order or a second-order ordinary differential equation with constant coefficients. A system described by a single second-order differential equation is commonly referred to as a *single-degree-of-freedom system.* Such a model is often used as a very crude approximation for a generally more complex system, so that one may be tempted to regard its importance as being only marginal. This would be a premature judgment, however, because in cases in which a technique known as modal analysis can be employed, the mathematical formulation associated with many linear multi-degree-of-freedom discrete systems and continuous systems can be reduced to sets of *independent* second-order differential equations, each similar to the equation of a single-degree-of-freedom system. Hence, a thorough study of single-degree-of-freedom linear systems is amply justified. Unfortunately, the same technique cannot be used for nonlinear multi-degree-of-freedom discrete and continuous systems. The reason is that the above reduction is based on the principle of superposition, which applies only to linear systems (see Sec. 2.11). Nonlinear systems are treated in Chaps. 9 and 10 of this text and require different methods of analysis than do linear systems.

The primary objective of this text is to study the behavior of systems subjected to given excitations. The behavior of a system is characterized by the motion caused by these excitations and is commonly referred to as the system *response*. The motion is generally described by displacements, and less frequently by velocities or accelerations. The excitations can be in the form of initial displacements and velocities, or in the form of externally applied forces. The response of systems to initial excitations is generally known as *free response*, whereas the response to externally applied forces is known as *forced response*.

In this chapter we discuss the free response of single-degree-of-freedom linear systems, whereas in Chap. 2 we present a relatively extensive treatment of forced response. No particular distinction is made in this text between damped and undamped systems, because the latter can be regarded merely as an idealized limiting case of the first. The response of both undamped and damped systems to initial excitations is presented.

1.2 CHARACTERISTICS OF DISCRETE SYSTEM COMPONENTS

The elements constituting a discrete mechanical system are of three types, namely, those relating forces to displacements, velocities, and accelerations, respectively.

The most common example of a component relating forces to displacements is the *spring* shown in Fig. 1.1*a*. Springs are generally assumed to be massless, so that a force F_s acting at one end must be balanced by a force F_s acting at the other end, where the latter force is equal in magnitude but opposite in direction. Due to the force F_s, the spring undergoes an elongation equal to the difference between the displacements x_2 and x_1 of the end points. A typical curve depicting F_s as a function

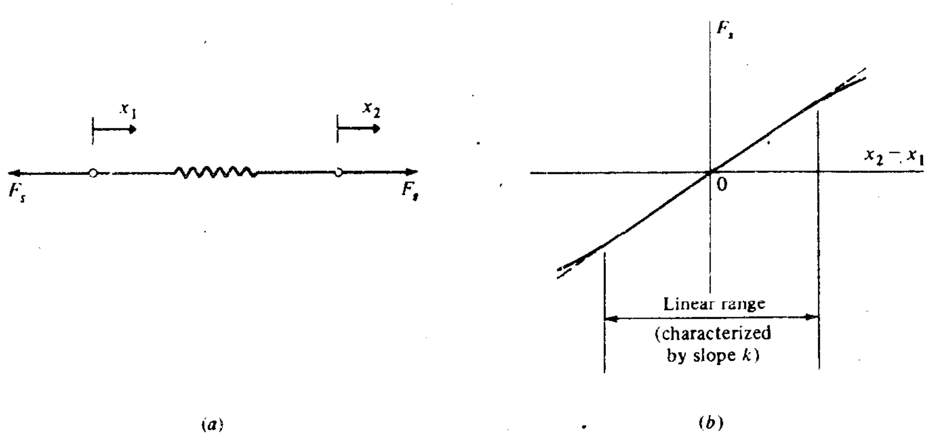

Figure 1.1

of the elongation $x_2 - x_1$ is shown in Fig. 1.1*b*; it corresponds to a so-called "softening spring," because for increasing elongations $x_2 - x_1$ the force F_s tends to increase at a diminishing rate. If the force F_s tends to increase at a growing rate for increasing elongations $x_2 - x_1$, the spring is referred to as a "stiffening spring." The force-elongation relation corresponding to Fig. 1.1*b* is clearly nonlinear. For small values of $x_2 - x_1$, however, the force can be regarded as being proportional to the elongation, where the proportionality constant is the slope k. Hence, in the range in which the force is proportional to the elongation the relation between the spring force and the elongation can be written in the form

$$F_s = k(x_2 - x_1) \tag{1.1}$$

A spring operating in that range is said to be *linear*, and the constant k is referred to as the *spring constant*, or the *spring stiffness*. It is customary to label the spring, when it operates in the linear range, by its stiffness k. Note that the units of k are pounds per inch (lb/in) or newtons per meter (N/m). The force F_s is an *elastic force* known as the *restoring force* because, for a stretched spring, F_s is the force that tends to return the spring to the unstretched configuration. In many cases the unstretched configuration coincides with the static equilibrium configuration (see Sec. 1.4).

The element relating forces to velocities is generally known as a damper; it consists of a piston fitting loosely in a cylinder filled with oil or water so that the viscous fluid can flow around the piston inside the cylinder. Such a damper is known as a *viscous damper* or a *dashpot* and is depicted in Fig. 1.2*a*. The damper is also assumed to be massless, so that a force F_d at one end must be balanced by a corresponding force at the other end. If the forces F_d cause smooth shear in viscous fluid, the curve F_d versus $\dot{x}_2 - \dot{x}_1$ is likely to be linear, as shown in Fig. 1.2*b*, where dots designate time derivatives. Hence, the relation between the damper force and the velocity of one end of the damper relative to the other is

$$F_d = c(\dot{x}_2 - \dot{x}_1) \tag{1.2}$$

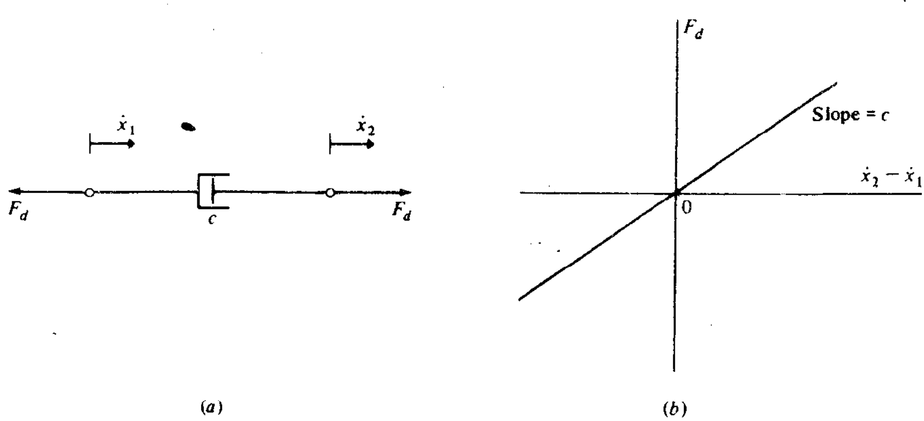

Figure 1.2

The constant of proportionality c, which is merely the slope of the curve F_d versus $\dot{x}_2 - \dot{x}_1$, is called the *coefficient of viscous damping*. We shall refer to such dampers by their viscous damping coefficients c. The units of c are pound · second per inch (lb · s/in) or newton · second per meter (N · s/m). The force F_d is a damping force because it resists an increase in the relative velocity $\dot{x}_2 - \dot{x}_1$.

The element relating forces to accelerations is clearly the *discrete mass* (Fig. 1.3*a*). This relation has the form

$$F_m = m\ddot{x} \tag{1.3}$$

Equation (1.3) is a statement of Newton's second law of motion, according to which the force F_m is proportional to the acceleration $\ddot{x}$, measured with respect to an inertial reference frame, where the proportionality constant is simply the mass m (see Fig. 1.3*b*). The units of m are pound · second^2 per inch (lb · s^2/in) or kilograms (kg). Note that in SI units the kilogram is a basic unit and the newton is a derived unit.

The physical properties of the components are recognized as being described in Figs. 1.1*b*, 1.2*b*, and 1.3*b* with the constants k, c, and m playing the role of parameters. It should be reiterated that, unless otherwise stated, springs and dampers possess no mass. On the other hand, masses are assumed to behave like rigid bodies.

The preceding discussion is concerned exclusively with translational motion, although there are systems, such as those in torsional vibration, that undergo rotational motion. There is complete analogy between systems in axial and torsional vibration, with the counterparts of springs, viscous dampers, and masses being torsional springs, torsional viscous dampers, and disks possessing mass moments of inertia. Indeed, denoting the angular displacements at the two end points of a torsional spring by θ_1 and θ_2, and the restoring torque in the spring k by M_s, the curve M_s versus $\theta_2 - \theta_1$ is similar to that given in Fig. 1.1*b*. Moreover, denoting the damping torque by M_d and the damping coefficient of the torsional viscous damper by c, the curve M_d versus $\dot{\theta}_2 - \dot{\theta}_1$ is similar to that of Fig. 1.2*b*.

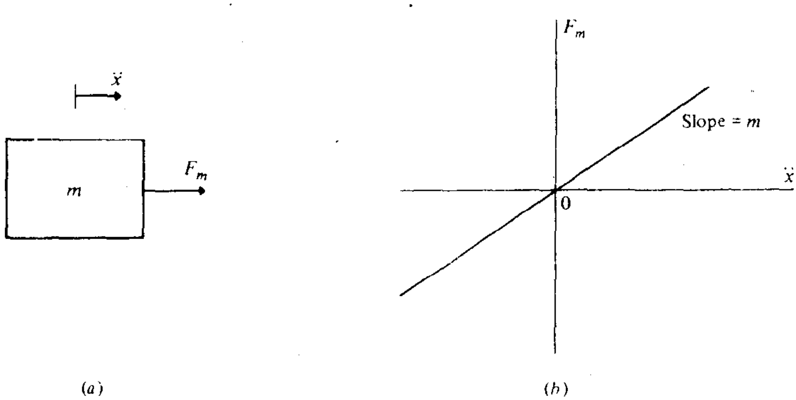

Figure 1.3

Finally, if the torsional system contains a disk of polar mass moment of inertia I, and the disk undergoes the angular displacement θ, then the curve M_I versus $\ddot{\theta}$ is similar to that of Fig. 1.3*b*, where M_I is the inertia torque. Of course, the moment of inertia I is simply the slope of that curve. Note that the units of the torsional spring k are pound · inch per radian (lb · in/rad) or newton · meter per radian (N · m/rad), etc.

On occasions, certain dynamical systems consisting of distributed elastic members and lumped rigid masses can be approximated by strictly lumped systems. The approximation is based on the assumption that the mass of the distributed elastic member is sufficiently small, relative to the lumped masses, that it can be ignored. In this case, the fact that the elastic member is distributed loses all meaning, so that the elastic member can be replaced by an equivalent spring. The equivalent spring constant is determined by imagining a spring yielding the same displacement as the elastic member when subjected to the same force, or torque. The procedure is illustrated in Example 1.1 for a member in torsion and in Example 1.2 for a member in bending.

At times several springs are used in various combinations. Of particular interest are *springs connected in parallel* and *springs connected in series*, as shown in Figs. 1.4*a* and *b*, respectively. We shall be concerned here with linear springs. For the springs in parallel of Fig. 1.4*a*, the force F_s divides itself into the forces F_{s1} and F_{s2} in the corresponding springs k_1 and k_2. Because the springs are linear, we have the relations

$$F_{s1} = k_1(x_2 - x_1) \qquad F_{s2} = k_2(x_2 - x_1) \tag{1.4}$$

But the forces F_{s1} and F_{s2} must add up to the total force F_s, or $F_s = F_{s1} + F_{s2}$, from which it follows that

$$F_s = k_{eq}(x_2 - x_1) \qquad k_{eq} = k_1 + k_2 \tag{1.5}$$

where k_{eq} denotes the stiffness of an *equivalent spring* representing the combined effect of k_1 and k_2. If a number n of springs of stiffnesses k_i $(i = 1, 2, \ldots, n)$ are arranged in parallel, then it is not difficult to show that

$$k_{eq} = \sum_{i=1}^{n} k_i \tag{1.6}$$

For springs in series, as shown in Fig. 1.4*b*, we can write the relations

$$F_s = k_1(x_0 - x_1) \qquad F_s = k_2(x_2 - x_0) \tag{1.7}$$

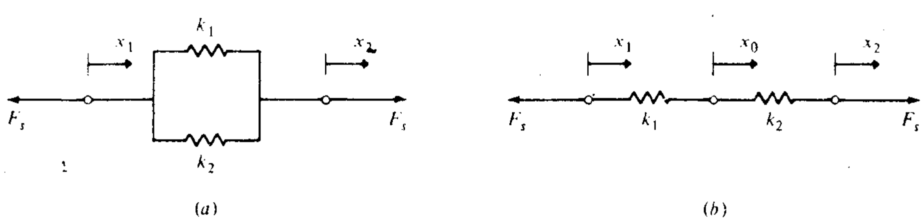

Figure 1.4