Encyclopedia of Fluid Mechanics

VOLUME 10

Surface and Groundwater Flow Phenomena

Nicholas P. Cheremisinoff, Editor

in collaboration with—

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PREFACE

This volume of the *Encyclopedia* has been prepared as an introduction to advanced topics in environmental flow problems, dealing with surface and groundwater flow phenomena. It is intended as an orientation volume on major principles and topics concerning hydrology and surface-water flow behavior, which will be expanded upon later in this series. The present work comprises 21 chapters, the first 14 of which concern surface and related topics. Chapters 1 through 7 cover general principles and observations of flow dynamics. Chapter 1 discusses turbulence modeling of incompressible mixtures, which is appropriate for a theoretical foundation for stream flows as well as slurry systems. Chapter 2 covers velocity profile analysis for open-channel flow configurations. Chapter 3 covers the mixing process and transport properties encountered in natural streams. Discussions are extended to pollutant dispersion in streams and other surface waterways. Thermodynamic principles are covered in Chapter 5 with modeling approaches to predicting river water temperatures from hydrodynamic parameters. The importance of this chapter extends to ecological impacts from thermal gradients in surface-water flows. Chapter 6 returns to the dynamics of mixing with topic coverage pertaining to interactions between water currents and sedimented effluents. Chapter 7 is devoted to the topic of jet outfalls entering shallow tailwaters.

Chapters 8 through 11 cover various aspects of turbulence, stratification in surface-water bodies and surface-wave phenomena. The first of these chapters concerns modeling the hydraulic effects of aquifer folds. Numerical modeling principles are extended to lake eutrophication and subsequent impacts on aquatic life in Chapter 9. Chapter 10 covers depth-varying eddy viscosity in tidal flows, and Chapter 11 provides an extensive review of global wave statistics.

Chapter 12 departs from flow dynamics with a discussion of toxicology testing and methodology related to prioritization of toxicological effects on waterbodies. Chapter 13 provides review of numerical techniques by addressing the dam breach problem, and Chapter 14 covers rainfall runoff.

The balance of the volume is devoted to groundwater flow phenomena and ecological impacts derived from contaminant introductions. Chapter 15 discusses waste leachate migration. Chapter 16 is concerned with methodology and test methods for estimating phytotoxicity limits, which impact on groundwater quality monitoring. Chapter 17 covers modeling of the transport of contaminants in groundwater. Chapter 18 reviews the principles of groundwater quality monitoring. Chapter 19 covers colloidal-suspension migration through porous media, and Chapter 20 extends these discussions to include heat transfer. Finally Chapter 21 covers computer techniques for groundwater simulation.

This volume represents the efforts of thirty-two experts who devoted time and effort to the production of this work. Each individual is to be considered responsible for the statements made in his or her respective chapters. Heartfelt gratitude is extended to the contributors and to Gulf Publishing Company for the fine production of this series.

Nicholas P. Cheremisinoff
Editor

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ADVANCED NUMERICAL FLOW MODELING

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CHAPTER 1

ONE-EQUATION TURBULENCE MODELING OF INCOMPRESSIBLE MIXTURES

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CONTENTS

INTRODUCTION

Two-phase particulate flow is encountered in rivers and geotechnical phenomena and is widely applied in mechanical, mining, chemical, and other industrial processes The hydraulic handling of bulk materials (coal, sand, phosphate, limestone, etc) is a typical case for industrial applications

This chapter is based on a short course given at the Nihon University, Japan

and a good case study for modeling. A specific set of governing equations is derived in this chapter for fluid-particle mixture flow. By using the double-time averaging approach, the phase interaction and temporal nonuniformity terms are derived in the differential equations. Two alternatives for the one-equation turbulence modeling of incompressible multiphase flow are presented: kinetic energy and eddy viscosity models. Their application is illustrated for noncolloidal, dilute and dense slurry flows. This chapter will focus on the following aspects of two-phase particulate flow modeling:

1. First, a classification of the slurry flow patterns after their microstructure is presented. The classification shows the area of applicability of different models [69].
2. The continuum multiphase flow equations are obtained by mass weighted local time averaging. The composed local time-turbulence time averaging yields a specific formulation of the turbulent flow equations, which includes the particle–fluid (drag, lift, turbulence) and particle–particle (Coulombic frictional, Bagnold collisional, inertial) interactions [1, 76, 77, 82].
3. The turbulence modulation by solid particles in the Stokes range is incorporated in the double-time averaged kinetic energy equation without any additional empirical constants. The power spectrum of the carrier fluid is divided into three regions, which interact in different ways and at different rates with the suspended particles as a function of the particle-eddy ratio and density ratio. The predictions are illustrated for dilute liquid-solid jets [1].
4. The kinetic energy (k-model) and eddy-viscosity (ν_t-model) one-equation turbulence models for dense slurry flow are presented, and their predictions compared for a set of data [70, 73]. The particle–particle interactions and turbulence modification by the presence of solids are incorporated in the model. An explicit numerical scheme for the concentration, solid and mixture velocities, eddy viscosity, and kinetic energy distributions is applied. The numerical predictions for velocity and concentration distributions are compared to a large set of experimental data on slurry flows with concentrations of up to 70% of their maximum packing concentration, considering one to five species of solid particles (of size $d_s < 13$ mm) in pipes of various diameters (D = 40 to 500 mm). The approach leads to results of practical interest: headloss-velocity correlations, concentration profiles of single solid species and broad size distribution, critical velocity in pipes, effect of pipe inclination, as well as other data of engineering relevance such as pipe wear rate. In our previous work we presented four models for dense liquid-solid turbulent flow: (a) An algebraic eddy-viscosity turbulence model [72], (b) an algebraic phenomenological model [76], (c) a one-equation turbulence two-phase eddy viscosity model for single size particles [70], and (d) a one-equation turbulence kinetic energy (k-model) and eddy-viscosity (ν_t-model) for multi-species particle flow [73].

The present analysis is limited to incompressible flows. The applications refer to liquid-solid flows, as well as to some gas-solid flows where the fluid compressibility does not play an important role.

To simulate the turbulence of single-phase flow one needs 1 to 2 (for algebraic models), 2 to 3 (for one-equation models), 5 to 6 (for two-equation models), or more (for higher order models) empirical coefficients. The two-equation turbulence models appear to be best suited for single-fluid flow. However, one needs more than 50 coefficients to completely describe dilute two-phase flow with a model of the same order [27]. It would be quite difficult to fit the coefficients unless analytical studies replaced some empiricism. At the same time, the two-phase algebraic models do not reflect the convective and diffusive processes of turbulence. Hence it is reasonable to apply, for engineering as well as other purposes, the *one-equation turbulence models*, in which the kinetic energy and mixture eddy viscosity are determined from a transport equation. This may be regarded as a compromise between a rigorous scientific analysis considering in detail of all micromechanical mechanisms (which presently cannot lead to results of engineering relevance for dense slurry flow) and a computational approach for applications. Previous work on turbulent single-fluid flow [84] suggests the one-equation models give similar accuracy to the two-equation models when applied to plane boundary layer problems, with and without separation.

A large number of publications have suggested empirical correlations between the mean flow indices in pipes, pumps, and other slurry equipment, and numerous articles have proposed theoretical developments. This chapter does not intend to review those techniques, but to summarize four modeling aspects: a classification of slurry flow patterns according to their microstructure; a double-time averaging to formulate the two-fluid flow governing equations; a one-equation turbulence approach for dilute slurry flow in which the particle-fluid drag interaction plays the prevalent role; and a one-equation turbulence model for dense slurry flow, which includes the particle-particle interaction terms.

DIAGRAM TO CLASSIFY LIQUID-SOLID MIXTURE FLOWS

Before applying any model, it is essential to identify the two-phase flow pattern as a function of the particle-fluid and particle-particle interaction mechanisms. A classification of slurry flow patterns based on the flow microstructure is presented in this section. The solid particle motion is determined by a combination of colloidal and hydrodynamical effects, such as Brownian diffusion, electrostatic repulsion, mass attraction, fluid drag, hydrodynamical lift, lubrication, interparticle friction, and collisions, as well as inertial effects. Their relative importance determines the flow structure (spatial distributions, time evolution, dynamic interactions) that is used to identify a flow pattern. Open questions are: Which criteria should be used to compare those patterns, and under which conditions can the experimental and computational results be extrapolated to other flow conditions? Mixture flow patterns are fairly complex, and idealizations are necessary to define simple patterns. Such idealizations have been adopted here to stress the main phenomenological trends and classify liquid-solid particle flow into a diagram. Three criteria of classification according to the flow microstructure have been selected for the slurry flow diagram: particle dispersion in fluid, particle-particle interaction mechanisms, and two-phase flow stability in a given domain.

Generally, in technical literature, the slurry flow patterns have been classified according to the specific trends observed at four levels of flow description:

1. micromechanical view (after the particle and fluid motion, and phase interactions).
2. macroscopic view (after the constitutive equations or rheograms).
3. probabilistic view (after the spatial and temporal fluctuations of the velocity, solid concentration, and other flow indices).
4. computational view (after the elliptic, parabolic or hyperbolic character of the governing equations).

Other specific classifications of liquid-solid mixture flows have frequently been used as a function of application:

1. according to the particle size of a solid material when the flow conditions are given; for instance, in industrial slurry pipelines, a particle size less than 10 μm generally means a non-Newtonian flow, between 10 μm and 200 μm a quasi-homogeneous flow, and over 200 μm a heterogeneous flow.
2. according to the concentration and velocity distributions for given equipment.
3. according to the stress tensor composition (The mixture flow stress tensor may be decomposed into three parts: fluid stress, τ_f; additional stress due to the fluid-particle interaction, τ_{fs}; and supplementary stress caused by the particle-particle interactions, τ_s.)

The micromechanical view offers the most comprehensive flow description and is used in this paper to classify slurry flows into a flow diagram. For the sake of simplicity, only uniform-size particles are considered here. The balance of forces acting on solid particles determine the flow time and length scales, which can be related to the degree of dispersion of solids in fluid and the particle interaction mechanisms.

Forces Acting on Single Particles

The force (F) and torque (T) balances acting on a single particle are, respectively

$$\mathbf{F}_i + \mathbf{F}_o + \mathbf{F}_f + \mathbf{F}_s + \mathbf{F}_w = 0 \quad (1)$$

$$T_i + T_b + T_f + T_s + T_w = 0 \tag{2}$$

where the subscript i denotes inertial, b body force, f fluid-particle interaction, s solid-solid interaction, and w wall-solid interaction. The index s denoting the forces attached to the solid phase is omitted for simplicity (for instance, $\mathbf{F}_i$ is used instead of $\mathbf{F}_{s,i}$).

Generally, the particle-fluid interaction force for a single rigid particle (without wall effects) has the Brownian ($\mathbf{F}_{BD}$), frictional ($\mathbf{F}_d + \mathbf{F}_l$), and inertial components ($\mathbf{F}_{app} + \mathbf{F}_{his}$):

$$\mathbf{F}_f = \mathbf{F}_{BD} + \underbrace{(\mathbf{F}_d + \mathbf{F}_l)}_{\text{(frictional effects)}} + \underbrace{(\mathbf{F}_{app} + \mathbf{F}_{his})}_{\text{(inertial effects)}} \tag{3}$$

where $\mathbf{F}_d$ = drag force,
$\mathbf{F}_l$ = lift force acting on freely (Saffman force) and forced (Magnus force) rotating particles in shear flow
$\mathbf{F}_{app}$ = apparent mass force
$\mathbf{F}_{his}$ = (Basset) history force

The interaction force with other solid particles, $\mathbf{F}_s$, has colloidal and hydrodynamical components. The London van der Waal force ($\mathbf{F}_{MA}$ = mass attraction force) and the surface electrostatic force ($\mathbf{F}_{ER}$ = electrostatic repulsion force) are the principal colloidal forces and are considered here. The hydrodynamical components are caused by the interstitial fluid ($\mathbf{F}_{cur}$ by induced secondary currents, and $\mathbf{F}_{lub}$ by lubrication), inter-particle collision $\mathbf{F}_{col}$, and friction by long-term (persistent) contacts $\mathbf{F}_{fr}$. The interparticle force $\mathbf{F}_s$ is

$$\mathbf{F}_s = \underbrace{(\mathbf{F}_{MA} + \mathbf{F}_{ER})}_{\text{colloidal}} + \underbrace{(\mathbf{F}_{cur} + \mathbf{F}_{lub})}_{\text{by interstitial fluid}} + \underbrace{(\mathbf{F}_{fr} + \mathbf{F}_{col})}_{\text{by direct contact}} \tag{4}$$

General background on the colloidal and single-particle fluid interaction forces can be found elsewhere [21, 53, 87, 97].

Time and Length Scales for the Flow Microstructure

The frequency of occurrence of a micromechanical interaction mechanism between particles or between particles and fluid is a function of the time scale (t_m), which is determined from the force balance. The time scales for the particle interaction mechanisms in slurry flows are given in Table 1 (for typical colloidal and noncolloidal interactions). The time scales for particle interaction in dense slurry flow by lubrication and collisions were obtained in [68]. From the ratios of the time scales one can define

- the relative particle interaction number $N_{i,m}$. A micromechanical interaction mechanism that is characterized by the time scale t_m is more frequent and therefore has a larger contribution to the momentum transfer than another interaction mechanism characterized by the time scale t_n, if

$$(1/t_m)/(1/t_n) = t_n/t_m = N_{i,m} > 1 \tag{5}$$

If $N_{i,m} > 1$, the m-th mechanism is prevalent comparative to the n-th mechanism.

- the deformation reduced time scale t_m/t_{def} (a Stokes number). If t_m/t_{def} is larger than unity, then the micromechanical mechanism characterized by the time scale t_m is affected or modified by the mean rate of deformation. The deformation time scale for single fluids and dilute suspensions is defined by $t_{def} = 1/\dot{\gamma}$, where $\dot{\gamma} = du/dn$ = mean rate of deformation. At higher concentrations, where the interparticle distance λ is smaller than d_s, the frequency of particle interactions is affected by the interparticle distance. We define

$$t_{def} = \min(\lambda, d_s)/(d_s\dot{\gamma}) = 1/\dot{\gamma}_a \tag{6}$$

Table 1
Time Scales for Particle Interactions

Type of Particle Interaction (m)	Time Scale (t_m)	Comments
	In the Colloidal Range	
Brownian Diffusion (BD)	$t_{BD} \approx \frac{3\pi}{4} \cdot \frac{\mu_f d_s^3}{k_B T}$	$t_{BD} \dot{\gamma} = t^*_{BD}$ is the Peclet number for microstructure
Electrostatic Repulsion (ER)	$t_{ER} \approx \frac{\mu_f d_s^2}{\epsilon_f \psi_0^2} \cdot \frac{e^{\kappa\lambda}}{\kappa}$	For the order of magnitude analysis: $e^{\kappa\lambda}/\kappa \approx 1$
Mass Attraction (MA)	$t_{MA} \approx \frac{\mu_f d_s \lambda^2}{A}$	For the order of magnitude analysis: $\lambda \approx d_s$
	In the Hydrodynamical Range	
Viscous Liquid-Solid Interaction (drag)	$t_d = \frac{4}{3} \frac{s d_s}{C_D \lvert u_f - u_s \rvert}$	where $C_D = f(Re_s)$
	$= s d_s^2 / 18 \nu_f$	for $Re_s < 0.1$
	$= \frac{4}{3} \frac{s d_s}{0.44 \lvert u_f - u_s \rvert}$	for $Re_s > 10^3$
Lubrication (lub)	$t_{lub} = \frac{18}{s d_s} \frac{\lambda}{\dot{\gamma}}$	$\lambda / d_s = \frac{1 - (\alpha^*)^{0.33}}{(\alpha^*)^{0.33}}$ where $\alpha^* = \alpha/\alpha_{max}$
Collisions (col)	$t_{col} = \frac{4.5}{s\dot{\gamma}} \log_e(\lambda/2k)$	k = particle-surface roughness

where $\dot{\gamma}_\alpha = \beta \cdot \dot{\gamma}$ = modified deformation rate
$\beta = \max(1, \alpha_{lin})$
$\alpha_{lin} = d/\lambda = [(\alpha_{max}/\alpha)^{0.33} - 1]^{-1}$ = linear concentration
α, α_{max} = volumetric concentration, and its maximum packing value

Three typical length scales can be defined in a particulate two-phase flow: the particle length scale, d_s; the dispersion length scale, ℓ_{dis} (for instance, the turbulence mixing length is the dispersion length scale in turbulent flow); and the flow domain length scale, L. The dispersion length scales frequently encountered in the colloidal and hydrodynamical range are listed in Table 2.

Criteria for Diagram Construction

Three criteria are used for the slurry flow classification and diagram construction: the solids dispersion in fluid, the dominant particle interaction mechanism, and the mixture flow stability.

The particles are dispersed in the carrier fluid if the dispersion energy is at least equal to the agglomeration energy. The dispersion energy is obtained from the work performed by the dispersion forces, for instance, by the electrical repulsion force in the colloidal zone and turbulence drag force

Table 2
Dispersion Length Scales

Type of Interactions ($F_{dispersion}/F_{agglomeration}$)	Length Scale (ℓ_{dis})	Comments
F_{BD}/F_{MA}	$\ell_{BD} = \frac{k_B T d_s}{A}$	Colloidal range
F_{ER}/F_{MA}	$\ell_{ER} = \frac{\epsilon_f \psi_0 d_s^2}{2A}$	Colloidal range
F_h/F_p	$\ell_h = \frac{\zeta u_*^2}{a\,(s - s_m)}$	$u_s \approx u_* =$ friction velocity
	$\zeta = 1$ for passive solids	a = acceleration of the external potential field
	$\zeta = s$ for interactive solids	

in the hydrodynamical zone The agglomeration energy is calculated from the work done by the agglomeration forces, for instance by the mass attraction in the colloidal zone and gravitational potential field in the hydrodynamical zone The dispersion index I_{dis} is defined by the ratio between the dispersion and agglomeration energy per unit time Since the agglomeration energy can be expressed as a function of the agglomeration force multiplied by the particle length scale (d_s) the dispersion index in turn can be expressed as the ratio between the dispersion length scale and particle length scale

$$I_{dis} = \ell_{dis}/d_s \tag{7}$$

where $\ell_{dis} = E_{dispersion}/E_{agglomeration}$ = dispersion length scale (Table 2) If I_{dis} is less than unity particles will agglomerate, otherwise ($I_{dis} > 1$) they will be dispersed within fluid If $I_{dis} > 10$ the dispersion can be classified as quasihomogeneous

The importance of a particle interaction mechanism is determined by its time scale (t_m) relative to other characteristic time scales in the flow The frequency of interaction is inversely proportional to its time scale and is denoted here as the threshold interaction index for the mechanism m, $I_{i\,m} = 1/t_m$ The interaction index is defined as a function of the deformation time scale

$$I_i = 1/t_{def} = \gamma_\alpha \tag{8a}$$

In a previous work, γ_α (1/s) was normalized by the drag time scale, t_d (s) [69] Here, for the sake of simplicity, the modified deformation rate (γ_α) is used without normalization The interaction index I_i is compared successively to the threshold interaction indices If

$$I_{i\,m-1} < I_i < I_{i\,m} \tag{8b}$$

then the interaction mechanism (m − 1) is affected by the mean rate of deformation while the m-th mechanism remains prevalent in the flow

The flow stability depends on the flow conditions in the whole domain The ratios between the inertial and viscous macroscopic forces, i e, the domain Re number, gives an indication on the hydrodynamical stability of the flow The stability index is defined by

$$I_s = Re/Re_{cr} \tag{9}$$