

<i>Preface</i>	xi
CHAPTER 1 INTRODUCTION TO PLASMA PHYSICS	1
<i>Part One Plasma Concepts and Terminology</i>	3
1.1 Equilibrium and Metaequilibrium	3
1.2 Debye Length	3
1.3 Plasma Parameter	4
1.4 Distribution Function	5
1.5 Temperature and Other Moments of the Distribution Function	6
1.6 Magnetic Pressure	7
1.7 Particle Drifts	7
1.8 Plasma Frequency	9
1.9 Waves in Plasmas	9
1.10 Landau Damping	12
1.11 Plasma Stability and Controlled Thermonuclear Fusion	12
1.12 Shock Waves and Solitary Waves	16
1.13 Collisions	18
1.14 Diffusion and Bohm Diffusion	20
1.15 Plasma Radiation	23
<i>Part Two Plasma Production</i>	25
1.16 The Low-pressure Cold-cathode Discharge	27
1.17 The Thermionic-arc Discharge	27
1.18 Plasma Guns	30
1.19 Alkali Metal Vapor Plasma— Q Machines	33
1.20 RF-produced Plasmas	33
1.21 Dense-plasma Focus	37
1.22 The Solar Plasma	39
1.23 Laser-produced Plasmas	39
<i>Part Three Measurement of Plasma Properties</i>	41
1.24 Current and Voltage Measurements in Plasmas	42
1.25 Plasma Probes	45
1.26 Other Methods of Measurement of Plasma Properties	50
<i>References</i>	54

CHAPTER 2 THERMODYNAMICS AND STATISTICAL MECHANICS OF EQUILIBRIUM PLASMAS	55
2.1 The Plasma Parameter	57
2.2 Gibbs Distribution and Correlation Functions	58
2.3 Two-particle Correlations in an Equilibrium Plasma	60
2.4 Free Energy of a Plasma	63
2.5 Equation of State of a Plasma	65
2.6 The Plasma as a Fluid	66
2.7 The Ideal Plasma	67
2.8 Potential of a Test Particle in a Plasma	68
2.9 Other Examples of the Plasma as a Fluid	70
2.10 Coulomb Energy of a Plasma	74
2.11 Discussion	76
<i>References</i>	77
CHAPTER 3 MACROSCOPIC PROPERTIES OF PLASMAS	78
3.1 The Distribution Function and the Liouville Equation	79
3.2 Macroscopic Variables of a Plasma	82
3.3 Macroscopic Equations for a Plasma; Fluid Equations	84
3.4 Two-fluid Plasma Theory	88
3.5 One-fluid Plasma Theory; Magnetohydrodynamics	89
3.6 Approximations Commonly Used in One-fluid Theory	92
3.7 Simplified One-fluid Equations and the MHD Equation	95
3.8 Properties of the Plasma Described by the One-fluid and MHD Models	98
3.9 Dynamic Properties of a Plasma Described by the One-fluid and MHD Plasma Theories	104
3.10 Double-adiabatic Theory	118
3.11 The Dynamic Pinch	123
<i>References</i>	128
CHAPTER 4 WAVES IN THE FLUID PLASMA	129
4.1 Dielectric Constant of a Field-free Fluid Plasma $n(\mathbf{E}_0 = \mathbf{B}_0 = 0)$	130
4.2 Plasma Oscillations	133
4.3 Plasma Oscillations in a One-dimensional Drifting Plasma	136
4.4 Space-charge Waves in a Warm Plasma	143
4.5 Plane Waves in a Cold Plasma	147
4.6 Microwave Transmission Method of Measuring Plasma Properties	153
4.7 Plasma-column Resonances	157
4.8 Space-charge Waves in Finite Plasmas	167
4.9 Dielectric Constant of a Cold Magnetized Plasma ($\mathbf{E}_0 = 0, \mathbf{B}_0 = B_0 \hat{z}$)	178
4.10 Waves that Propagate Parallel to the Magnetic Field in a Cold Magnetized Plasma ($\mathbf{E}_0 = 0, \mathbf{B}_0 = B_0 \hat{z}$)	182
4.11 Waves that Propagate Perpendicular to the Magnetic Field in a Cold Magnetized Plasma ($\mathbf{E}_0 = 0, \mathbf{B}_0 = B_0 \hat{z}$)	196
4.12 Wave Frequencies in Typical Plasmas	200
4.13 Space-charge Waves in Cold Finite Plasmas in a Finite Magnetic Field	202
4.14 Low-frequency Drift Waves in Nonuniform Plasmas	206
<i>References</i>	213
CHAPTER 5 STABILITY OF THE FLUID PLASMA	215
<i>Part One The Plasma Stability Problem</i>	215
5.1 The Equilibrium Problem	217
5.2 Classification of Plasma Instabilities	218

5.3	Methods of Stability Analysis	220
5.4	Regions of Stability	221
	<i>Part Two Stability of Unconfined Plasma from Macroscopic Fluid Equations</i>	221
5.5	Two-stream Instabilities of Space-charge Waves	222
5.6	Fire-hose Instability of an Alfvén Wave	228
	<i>Part Three Stability of Magnetically Confined Plasma from Macroscopic Fluid Equations</i>	229
5.7	Stability of the Fluid Plasma Supported against Gravity by a Magnetic Field	230
5.8	Stability of Magnetically Confined Fluid Plasma from Thermodynamic Considerations; Interchange Instability	233
5.9	Macroscopic Equations for the Study of the Hydrodynamic Stability of Magnetically Confined Plasmas	241
5.10	Stability of the Fluid Plasma Supported against Gravity by a Magnetic-field: Normal-mode Analysis	246
5.11	Energy Principle	251
5.12	Stability of a Plane Plasma-Magnetic Field Interface: Energy Principle Analysis	257
5.13	Stability of Self-confined Plasma (B_θ Only): Energy Principle Analysis	261
5.14	Stabilizing Influence of Line Tying	263
	<i>Part Four Stability Theory and Controlled Thermonuclear Fusion Research</i>	265
5.15	Open Plasma-confinement Experiments	267
5.16	Closed Plasma-confinement Experiments	275
5.17	Other Plasma-confinement Experiments	281
	<i>References</i>	285
	CHAPTER 6 TRANSPORT PHENOMENA IN PLASMA	287
6.1	Binary Coulomb Collisions	289
6.2	Deflection of a Charged Particle by Multiple Coulomb Collisions	291
6.3	Fokker-Planck Theory for Transport in a Fully Ionized Plasma	295
6.4	Relaxation Times in a Fully Ionized Plasma	301
6.5	Transport Properties of a Fully Ionized Plasma	307
6.6	Boltzmann Transport Equation and the Lorentz Model for a Weakly Ionized Plasma	311
6.7	Modified Boltzmann Equation	315
6.8	Transport Coefficients in a Weakly Ionized Plasma	318
6.9	Ambipolar Diffusion	321
6.10	Transport Properties of a Weakly Ionized Plasma in a Steady and Homogeneous Magnetic Field	327
6.11	Ambipolar Diffusion of a Weakly Ionized Plasma across a Magnetic Field	333
6.12	MHD Power Generators	340
	<i>References</i>	347
	CHAPTER 7 KINETIC EQUATIONS FOR A PLASMA	349
7.1	The Microscopic Equations for a Many-body System	350
7.2	The Statistical Equations for a Many-body System	351
7.3	Statistical Equations for a Coulomb Plasma	355
7.4	Closing the Chain of Statistical Equations	357
7.5	Kinetic Equation in Zero Order—The Vlasov Equation	358
7.6	Kinetic Equation in First Order	359
7.7	Properties of the Vlasov Equation	360
7.8	Properties of the Kinetic Equation to Order g	366
	<i>References</i>	367

CHAPTER 8 THE VLASOV THEORY OF PLASMA WAVES	368
8.1 The Vlasov Equations	368
8.2 The Linearized Vlasov Equations	369
8.3 Solution of the Linearized Vlasov Equations for Electrostatic Perturbations of a Field-free Plasma Equilibrium	371
8.4 Time-asymptotic Solutions for $\phi_k(t)$	375
8.5 Simplified Derivation for Electrostatic Waves in a Plasma	381
8.6 The Vlasov Theory of Langmuir Waves, Ion-sound Waves, and Landau Damping ($E_0 = B_0 = 0$)	383
8.7 Perturbed Distribution Function for Plasma Waves	391
8.8 The Dispersion Relation for Waves in a General Plasma Equilibrium	395
8.9 The Vlasov Theory of Small-amplitude Waves in a Field-free Plasma Equilibrium—Electrostatic and Electromagnetic Waves [$E_0 = B_0 = 0, f_0 = f_0(v^2)$]	398
8.10 The Vlasov Theory of Small-amplitude Waves in a Uniformly Magnetized Plasma [$B_0 = B_0 \hat{z}, E_0 = 0, f_{a0} = f_{a0}(v_\perp^2, v_\parallel)$]	402
8.11 The Vlasov Theory of Waves in Cold Magnetized Plasma	407
8.12 Waves That Propagate Perpendicular to the Equilibrium Magnetic Field in a Hot Magnetized Plasma ($E_0 = 0, B_0 = \hat{z} B_0$)—Electromagnetic Waves and the Bernstein Modes	407
8.13 Waves That Propagate Parallel to the Equilibrium Magnetic Field in a Magnetized Hot Plasma—Electrostatic and Electromagnetic Waves ($E_0 = 0, B_0 = \hat{z} B_0$)	414
8.14 Electromagnetic Waves Propagating at an Arbitrary Angle with Respect to the Equilibrium Magnetic Field in a Magnetized Hot Plasma ($E_0 = 0, B_0 = \hat{z} B_0$)	417
8.15 Waves in an Inhomogeneous Magnetized Hot Plasma [$E_0 = 0, B_0 = \hat{z} B_0(x), n_0 = n_0(x)$]	418
8.16 Low-frequency Electrostatic Waves in an Inhomogeneous Magnetized Plasma	427
8.17 Nonlinear Electrostatic Waves (BGK Waves)	432
8.18 Fluid Waves vs. Vlasov Waves	437
8.19 Summary of Wavelike Vlasov States	439
<i>References</i>	441
CHAPTER 9 THE VLASOV THEORY OF PLASMA STABILITY	442
9.1 Introduction	442
9.2 Stability of Monotone-decreasing Distribution; The Newcomb-Gardner Theorem	445
9.3 Stability of Multi peaked Distributions—The Two-stream Instability	449
9.4 Stability of Multi peaked Distribution in Warm Plasmas—Gentle-bump Instability	458
9.5 Mechanism of the Two-stream Instability	463
9.6 The Nyquist Method and the Penrose Criterion for Stability	464
9.7 Ion-acoustic Instability	476
9.8 Applications of Two-stream-instability Theory	478
9.9 Instabilities in Anisotropic Plasmas	482
9.10 Electromagnetic Pinching Instabilities	483
9.11 Discussion of Pinching Instabilities	494
9.12 Stability of Anisotropic Magnetized Plasma	495
9.13 Loss-cone Instability	497
9.14 Other Instability Mechanisms	505
9.15 Thermodynamic Bounds on Field Levels and Growth Rates in Unstable Plasma	506
<i>References</i>	511
CHAPTER 10 THE NONLINEAR VLASOV THEORY OF PLASMA WAVES AND INSTABILITIES	512
10.1 The Need for a Nonlinear Theory of Plasmas	512

10.2	Quasilinear Equations for Changes in a Plasma Distribution	514
10.3	Conservation of Particles, Momentum, and Energy in Quasilinear Theory	518
10.4	Landau Damping in Quasilinear Theory	520
10.5	The Gentle-bump Instability in Quasilinear Theory	527
10.6	Quasilinear Theory of the Two-stream Instability	532
10.7	Electron Trapping in a Single Plasma Wave	536
10.8	Plasma Wave Echoes	539
10.9	Nonlinear Wave-particle Interactions (Weak Turbulence)	549
	<i>References</i>	554
CHAPTER 11 FLUCTUATIONS, CORRELATIONS, AND RADIATION		556
11.1	Shielding of a Moving Test Charge	557
11.2	Electric Field Fluctuations in a Plasma	563
11.3	Electric Field Fluctuations in a Nonmaxwellian Plasma	568
11.4	Drag on a Test Particle; Emission of Electrostatic Waves	570
11.5	Electromagnetic Fluctuations and Radiation	573
11.6	Scattering of Incoherent Radiation from a Plasma Density Fluctuations	575
11.7	Emission of Radiation from a Plasma; Kirchhoff's Law	588
11.8	Blackbody Radiation from and in a Plasma	590
11.9	Cyclotron (Synchrotron) Radiation from a Plasma in a Magnetic Field	592
11.10	Test Source Theory of Radiation from a Plasma	595
11.11	Kinetic Equations Including Collisional Relaxation of a Plasma	599
	<i>References</i>	605
APPENDIX I PARTICLE MOTION		607
I.1	The Equations of Motion	608
I.2	Particle Motion in Static Homogeneous Electric and Magnetic Fields	608
I.3	Particle Motion in Slowly Varying Homogeneous Electric and Magnetic Fields	613
I.4	Particle Motion in a Static Homogeneous Magnetic Field and a Rapidly Varying, Small-amplitude, Electric Field [$\mathbf{B}_0 \neq 0$, $\mathbf{E}(t) \neq 0$]	615
I.5	Particle Motion in a Homogeneous Large-amplitude Plane Electromagnetic Wave	618
I.6	Particle Motion in Static Inhomogeneous Magnetic Fields	621
I.7	Adiabatic Invariants	627
I.8	Plasma Properties from Orbit Theory	628
	<i>References</i>	630
APPENDIX II SUMMARY OF SOME PROPERTIES OF VECTORS AND TENSORS, SOME INTEGRAL THEOREMS, AND CURVILINEAR COORDINATES		631
APPENDIX III SYSTEMS OF UNITS, CONVERSION FACTORS, AND FREQUENTLY USED SYMBOLS		644
III.1	Systems of Units	644
III.2	Conversion Factors and Physical Constants	647
III.3	Frequently used Symbols	647
APPENDIX IV SELECTED ADDITIONAL READINGS FOR THE ADVANCED STUDENT		654
IV.1	Turbulent Heating	654
IV.2	Plasma Shock Waves	655
IV.3	Transport Theory	655
IV.4	Hydromagnetic and Fluid Models	656

IV.5	Computer Techniques in Plasma Physics	656
IV.6	Normal Modes of a Plasma and Landau Damping	657
IV.7	Plasma Equilibrium	657
IV.8	Plasma Oscillations and Stability	658
IV.9	Solid State Plasmas	659
IV.10	Applications of Plasma Physics to Space Physics	659
	<i>Name Index</i>	661
	<i>Subject Index</i>	666

INTRODUCTION TO PLASMA PHYSICS

Plasma physics is the study of charged particles collected in sufficient number so that the long-range Coulomb force is a factor in determining their statistical properties, yet low enough in density so that the force due to a near-neighbor particle is much less than the long-range Coulomb force exerted by the many distant particles. It is the study of low-density ionized gases. The term "plasma" was first used to describe a collection of charged particles by Tonks and Langmuir,¹ in 1929, in their studies of oscillations in electric discharges. However, the most characteristic aspect of the plasma state, the fact that because of the long range of the Coulomb force the charged particles exhibit a collective behavior, was known much earlier, and was probably first described by Lord Rayleigh,² in 1906, in his analysis of electron oscillations in the Thomson model of the atom.

The term "fourth state of matter," often used to describe the plasma state, was coined by W. Crookes³ in 1879 to describe the ionized medium created in a gas discharge. The term fourth state of matter follows from the idea that as heat is added to a solid, it undergoes a phase transition to a new state, usually liquid.

¹ L. Tonks and I. Langmuir, Oscillations in Ionized Gases, *Phys. Rev.*, **33**:195 (1929).

² Lord Rayleigh, *Phil. Mag.*, **11**:117 (1906).

³ W. Crookes, *Phil. Trans.*, **1**:135 (1879).

If heat is added to a liquid, it undergoes a phase transition to the gaseous state. The addition of still more energy to the gas results in the ionization of some of the atoms. At a temperature above $100,000^{\circ}\text{K}$ most matter exists in an ionized state; this ionized state of matter is called the *fourth state*. A plasma state can exist at temperatures lower than $100,000^{\circ}\text{K}$ provided there is a mechanism for ionizing the gas, and if the density is low enough so that recombination is not rapid.

Although 99.9 percent of the apparent universe exists in a plasma state, there is very little in the way of natural plasma here on earth because the low temperature and high density of the earth and its near atmosphere preclude the existence of plasma. This means that plasma must be created by experimental means to study its properties. However, in the upper atmosphere (ionosphere), plasma does exist, created by photoionization of the tenuous atmosphere. Farther out from the earth, plasma is trapped in the earth's magnetic field in the near vacuum of space. Plasma streams toward the earth from the sun (the solar wind), and fills many regions of interstellar space, forming the medium through which outer space is viewed.

Plasma physics generally involves the well-known physics of classical mechanics, electromagnetism, and nonrelativistic statistical mechanics. The challenge of plasma physics comes from the fact that many plasma properties result from the long-range Coulomb interaction, and therefore are collective properties that involve many particles interacting simultaneously.

In its simplest form, a plasma is a collection of protons and electrons at sufficiently low density so that binary (short-range) interactions are negligible. Many-body theory, or the many-body problem, is the study of the properties of such a medium. When a collection of protons and electrons coexist in an equilibrium state, the properties of this state are described by equilibrium statistical mechanics with the appropriate Gibbs ensemble. However, most of the interesting features of plasmas occur for nonequilibrium situations.

Revived interest in plasma physics in the United States began in 1952 with the attempts of a program, then classified, known as Project Sherwood,¹ to develop a controlled thermonuclear fusion reactor. Similar programs were started in England, France, and the U.S.S.R. at about the same time. These programs have grown substantially since that time, and now there are many nations with major research programs in this field. Although the development of a controlled fusion reactor is one of the more challenging practical applications of plasma physics, it is only one of the many areas in which plasma physics plays a role. Plasma physics has played a major role in the development of much

¹ A. S. Bishop, "Project Sherwood," Addison-Wesley, Reading, Mass., 1958.

of contemporary physics, and it is important in the study of problems in such areas as astrophysics, atomic physics, chemistry, life sciences, molecular physics, magnetohydrodynamic power generation, and atmospheric physics.

Plasma physics has its own vocabulary and set of ideas. The main purpose of this chapter is to review plasma physics on an elementary level, provide a background sketch of the familiar concepts of the field, identify many of the terms used repeatedly in discussing the plasma state, review some of the schemes by which plasma is produced in the laboratory, and review some of the methods by which plasma properties are measured.

PART ONE: PLASMA CONCEPTS AND TERMINOLOGY

1.1 EQUILIBRIUM AND META-EQUILIBRIUM

The term "equilibrium" is often loosely used in plasma physics to describe a quasi-steady-state condition that persists only until the plasma particles collide with each other. Frequently, plasma studies are made by investigating small perturbations about such a *metaequilibrium* state.

Thermodynamic equilibrium means that the ions and the electrons are each described by a maxwellian distribution characterized by the same single parameter, the temperature. In this situation, the medium is in equilibrium with its surroundings, and it radiates and absorbs energy at the same rate. The spectrum of emitted radiation is *blackbody*.

In many of the theoretical and experimental situations of interest in plasma physics, the ions and electrons are neither at the same temperature nor in thermodynamic equilibrium with their surroundings. The term *metaequilibrium* is used to describe situations that eventually will be altered by binary collisions.

1.2 DEBYE LENGTH

The electrostatic potential of an isolated particle of charge q is

$$\phi = \frac{q}{r} \quad (1.2.1)$$

In a plasma, electrons are attracted to the vicinity of an ion and shield its electrostatic field from the rest of the plasma. Similarly, an electron at rest repels other electrons and attracts ions. This effect alters the potential in the vicinity of a charged particle. The potential of a charge at rest in a plasma is given by

$$\phi = \frac{q}{r} e^{-r/\lambda_D} \quad (1.2.2)$$

where λ_D is the Debye length originally defined in the Debye-Hückel theory of electrolytes. For an electron-proton plasma

$$\lambda_D = \left(\frac{\kappa T}{8\pi n e^2} \right)^{1/2} = 4.9 \left(\frac{T}{n} \right)^{1/2} \quad (1.2.3)$$

where n = density of electrons (or ions), cm^{-3}

T = temperature, $^{\circ}\text{K}$

κ = Boltzmann's constant ($= 1.38 \times 10^{-16}$ ergs/ $^{\circ}\text{K}$)

The Debye length is a measure of the sphere of influence of a given test charge in a plasma. In general, the Debye length depends on the speed of the test charge with respect to the plasma.

1.3 PLASMA PARAMETER

The plasma parameter g indicates the number of plasma particles in a Debye sphere, and is defined by

$$g = \frac{1}{n\lambda_D^3} \quad (1.3.1)$$

For Debye shielding to occur, and for the description of a plasma to be statistically meaningful, the number of particles in a Debye sphere must be large; that is, $g \gg 1$. The assumption $g \ll 1$ is called the *plasma approximation*. The plasma parameter is also a measure of the ratio of the mean interparticle potential energy to the mean plasma kinetic energy. An ideal gas corresponds to *zero* potential energy between the particles. In many situations the plasma parameter is small and the plasma is treated as an ideal gas of charged particles, that is, a gas that can have a charge density and electric field but in which no two *discrete* particles interact.

To ensure that $n\lambda_D^3$ be large, the density must be *low*, since

$$g = \frac{1}{n\lambda_D^3} \propto \frac{n^{1/2}}{T^{3/2}}$$

Because the collision frequency decreases with density n , and also decreases with increasing temperature T , the condition $g \rightarrow 0$ corresponds to a decreasing collision frequency.

Problem 1.3.1 Show that, if ratio of mean kinetic energy to mean interparticle potential energy is much greater than 1, that is, if

$$\frac{\langle KE \rangle}{\langle PE \rangle} \gg 1$$

the number of particles per Debye sphere $n\lambda_D^3$ must also be much larger than 1. ////

The plasma parameter g is one of the more important dimensionless parameters associated with a plasma, and may be interpreted as a measure of the degree to which plasma or collective effects dominate over single-particle behavior. The plasma state is described by equations obtained from an expansion of the exact many-body equations in powers of g .

A distinctive contrast between the statistical mechanics of a plasma and that of a neutral gas is that in a plasma the expansion parameter g is small when *many* particles interact at the same time, since λ_D^3 is essentially the volume of the interaction region; for a neutral gas, the atomic radius R is a measure of the interaction region and nR^3 ($\ll 1$) is the expansion parameter. The plasma behaves as a nearly ideal gas *in spite of* the presence of many interacting particles; the reason is that the strength of the interaction between individual particles is so weak, as shown in Prob. 1.3.1.

1.4 DISTRIBUTION FUNCTION

The most detailed description of a plasma gives the location and velocity of each plasma particle as a function of time. It is impossible to obtain such a description of a real plasma, except in some recent "experiments" which involve the use of digital computers to follow the position and velocity of a large number of ions and electrons. Therefore it is customary to use the distribution function f to describe a plasma. The distribution function is the number of particles per unit volume in six-dimensional velocity-configuration phase space. From the Boltzmann H theorem¹ it is known that under the action of binary collisions an ideal gas relaxes to a maxwellian distribution of velocities.

$$\bar{n}f(v) = \bar{n} \left(\frac{m}{2\pi\kappa T} \right)^{3/2} e^{-mv^2/2\kappa T} \quad (1.4.1)$$

where $\bar{n} = N/V$, with N the number of particles of a certain type (e.g., ions or electrons) in the system and V the volume of the system. Although laboratory plasmas probably never achieve exactly a maxwellian distribution, they may approach it closely, and it is useful in many theoretical treatments to assume that the plasma is described by a maxwellian velocity distribution.

¹ S. Chapman and T. G. Cowling, "The Mathematical Theory of Non-uniform Gases," Cambridge, London, 1952.

Because ions and electrons behave rather differently on a time scale less than ion-electron relaxation times (a binary collision process), it is usually necessary to define a separate distribution function for each charge species α , so that

$$\bar{n}_\alpha f_\alpha(\mathbf{x}, \mathbf{v}) d\mathbf{x} d\mathbf{v} = \text{number of particles of type } \alpha \text{ in a volume element } d\mathbf{x} d\mathbf{v} \text{ centered about } \mathbf{x}, \mathbf{v}$$

with the normalization

$$\int \bar{n}_\alpha f_\alpha d\mathbf{x} d\mathbf{v} = N_\alpha = \text{total number of particles of type } \alpha \text{ in the system}$$

1.5 TEMPERATURE AND OTHER MOMENTS OF THE DISTRIBUTION FUNCTION

In the maxwellian distribution, the parameter T , which defines the distribution function, is the temperature of the plasma.

$$\frac{3}{2} n \kappa T = \frac{1}{2} \int \bar{n} m v^2 f d\mathbf{v} \quad (1.5.1)$$

Even in cases where laboratory plasmas are not accurately described by a maxwellian, it is customary to relate the experimental distribution function to the maxwellian distribution which best fits the data, by describing the system as a plasma at temperature T , with T defined by (1.5.1). If the distribution function has no relation to a maxwellian distribution, it is poorly described by merely specifying the temperature. Instead, a complete description must either specify the distribution function itself or specify *all* the moments of the distribution.

$$n_\alpha = \bar{n}_\alpha \int f_\alpha d\mathbf{v} \quad \text{density} \quad (1.5.2)$$

$$\mathbf{V}_\alpha = \frac{\int \mathbf{v} f_\alpha d\mathbf{v}}{\int f_\alpha d\mathbf{v}} \quad \text{mean velocity} \quad (1.5.3)$$

$$\mathbf{P}_\alpha = \frac{n_\alpha m_\alpha \int (\mathbf{v} - \mathbf{V}_\alpha)(\mathbf{v} - \mathbf{V}_\alpha) f_\alpha d\mathbf{v}}{\int f_\alpha d\mathbf{v}} \quad \text{pressure tensor} \quad (1.5.4.)$$

⋮

These moments are associated with the macroscopic parameters of the system such as the density, mean velocity, pressure, etc. Note that for an isotropic system at rest, the pressure tensor reduces to a scalar,

$$\mathbf{P}_\alpha = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}$$

where $p = n_\alpha \kappa T_\alpha$.

1.6 MAGNETIC PRESSURE

Frequently plasma is imbedded in, or surrounded by, a magnetic field. In a static magnetic field there is a pressure force $B^2/8\pi$ dynes/cm² across a surface tangent to the magnetic flux surface, at a point. Characteristically, a plasma is diamagnetic, and thus it excludes a magnetic field suddenly applied to its boundary. This means that a magnetic pressure force $B^2/8\pi$ can balance the kinetic pressure $p = nkT$ of the plasma at a boundary between plasma and magnetic field. If $B^2/8\pi > p$, the plasma may be compressed to a higher density and temperature.

In situations where plasma and magnetic field coexist, a frequently used parameter is the local ratio of plasma pressure to magnetic pressure, given the name *plasma beta*; i.e.,

$$\beta = \frac{n_i kT_i + n_e kT_e}{B^2/8\pi} \quad (1.6.1)$$

This β should not be confused with the relativistic $\beta = v/c$. At a plane plasma magnetic field interface in equilibrium, pressure balance requires

$$\nabla \left(nkT + \frac{B^2}{8\pi} \right) = 0 \quad (1.6.2)$$

Problem 1.6.1 What is the magnetic pressure in atmospheres of a 100,000-G magnetic field? What magnetic field strength is required to confine a plasma of density $n = 10^{12} \text{ cm}^{-3}$ at a kinetic temperature of 100 keV, if $\beta = 1.0$ percent? ////

1.7 PARTICLE DRIFTS

Most of the interesting plasma properties result from collective effects, as previously mentioned. However, some plasma properties can be obtained by considering the motion of individual charged particles in various combinations of electric, magnetic, and gravitational fields, which may be time-varying and inhomogeneous. In a uniform magnetic field in the absence of collisions, a charged particle moves on a helical trajectory. The particle rotates at the cyclotron frequency $\omega_c = eB/mc$ perpendicular to the magnetic field with a

radius of rotation $a_L = v_\perp/\omega_c$, called the *Larmor*¹ *radius*, or the *gyro radius*. Its motion along the magnetic field is unimpeded. The cyclotron frequency is given in radians per second. For example,

$$\omega_c = \frac{eB}{mc} = 1.76 \times 10^7 B \text{ (Gauss)} \quad \text{rad/s, for electrons} \quad (1.7.1)$$

Problem 1.7.1 Find the orbit of a particle of charge q and mass m started from rest in a crossed electric and magnetic field. Write the orbit in terms of a cyclotron orbit and a drift, and show that the drift is $V_{DE} = cE/B$. Show that one half of this particle's kinetic energy is in drift motion and one half is orbital energy. ///

This cyclotron rotation is the basic motion of a charged particle perpendicular to a magnetic field, but there are departures from this motion when other fields are involved. For instance, an $\mathbf{E} \times \mathbf{B}$ drift occurs when a steady electric and magnetic field both act on a charged particle. The combined action of these two fields causes, in addition to the cyclotron rotation, a drift motion perpendicular to both the electric and magnetic fields, at a velocity given by

$$\mathbf{V}_{DE} = c \frac{\mathbf{E} \times \mathbf{B}}{B^2} \quad (1.7.2)$$

This drift motion is independent of either the mass or charge of the particle involved, and is superposed on the cyclotron rotation. Frequently the cyclotron motion is much more rapid than the drift motion, and the motion is well described by specifying the orbit of the *guiding center*,² defined as the center of the particle's circular orbit. Thus \mathbf{V}_{DE} is referred to as a drift velocity of the guiding center.

If instead of an electric field acting on the particle there is a gravitational field, the drift velocity is

$$\mathbf{V}_{DG} = \frac{mc}{q} \frac{\mathbf{g} \times \mathbf{B}}{B^2} \quad (1.7.3)$$

Another drift motion that is important in plasma problems is the gradient B drift that occurs when the magnetic field is inhomogeneous. The gradient drift is

¹ This radius of gyration should be called the "cyclotron radius," to be consistent with conventional terminology; however, it is referred to as the Larmor radius in most of the contemporary plasma physics literature.

² The concept of a guiding center for charged-particle motions was introduced in 1940 by H. Alfvén, who used the guiding-center motions in a theory of magnetic storms and aurora. He collected his works and ideas in "Cosmical Electrodynamics," Oxford, New York, 1950, and in 1970 he was awarded the Nobel prize in physics for his pioneering work in plasma physics.

also superposed on the cyclotron rotation. For weak gradients ($\nabla B/B < 1/a_L$), the drift speed of the guiding center (perpendicular to \mathbf{B} and to ∇B) is

$$V_{D,\text{grad}} = \frac{w_{\perp} c}{qB^2} \nabla_{\perp} B \quad (1.7.4)$$

where $w_{\perp} (= \frac{1}{2} m v_{\perp}^2)$ is the kinetic energy perpendicular to \mathbf{B} . The details of particle-orbit theory are given in Appendix I, where curvature drift and other drift motions are treated.

1.8 PLASMA FREQUENCY

Because of the long-range forces between plasma particles, a plasma behaves in some situations as a system of coupled oscillators. One basic characteristic oscillator frequency of the plasma state is the plasma frequency ω_p , defined as

$$\omega_p = 2\pi f_p = \left(\frac{4\pi n e^2}{m} \right)^{1/2} \text{ rad/s}$$

$$f_p \approx 10^4 \sqrt{n} \text{ Hz, for electrons} \quad (1.8.1)$$

where n is the number of particles per cubic centimeter, and m is the electron mass. It is customary to call ω_p the plasma "frequency," even though it has units of radians per second.

The plasma frequency is often used as a means of specifying the electron density in a plasma. It is also a measure of the length of time required for an electron or ion moving with thermal speed to travel a Debye length.

If a group of electrons in a two-component plasma are displaced slightly from their equilibrium position \mathbf{x}_0 , they will experience a force that seeks to return them to \mathbf{x}_0 . When they arrive at the equilibrium position, they will have a kinetic energy equal to the potential energy of their initial displacement, and will continue past \mathbf{x}_0 until they have reconverted their kinetic energy back to potential energy. The frequency of this simple period harmonic motion will be at ω_p , the plasma frequency. This phenomenon is known as a *plasma oscillation*.

1.9 WAVES IN PLASMAS

There are a large number of possible oscillations in a system of coupled oscillators with as many degrees of freedom as a plasma, and wavelike disturbances, e.g., electric fields $\mathbf{E} = \mathbf{E}_0 \exp [i(\mathbf{k} \cdot \mathbf{x} - \omega t)]$, will propagate in a plasma. The frequency ω and the wave number k are functionally related to one another by a

dispersion relation obtained from the plasma equations. A knowledge of the dispersion characteristics $\omega(k)$ for the propagating waves is certainly necessary for an understanding of the plasma state. The phase velocity of a wave is $v_p = \omega/k$, and the group velocity of a wave is $v_g = \partial\omega/\partial k$.

A plasma can propagate both linear and nonlinear waves. *Linear* refers to the simplifying approximations that are possible for small-amplitude waves, and *nonlinear* refers to shock waves and large-amplitude phenomena not predicted by linear models.

Four examples of linear waves (ion-sound waves, electromagnetic waves, drift waves, and plasma waves) are discussed below.

1.9.1 Ion-sound Waves

Sound waves in an ideal gas are dispersionless longitudinal waves that propagate below the gas collision frequency with speed $V_s = (\gamma\kappa T/m)^{1/2}$. A plasma also propagates a low-frequency acoustic, or *ion-sound wave* by a mechanism in which the perturbation charge-separation electric field provides the coupling between the electrons and the ions. These waves propagate with speed

$$C_s = \left(\frac{Z\gamma_e \kappa T_e + \gamma_i \kappa T_i}{m_i} \right)^{1/2} \quad (1.9.1)$$

The symbols γ_e and γ_i denote the usual ratio of specific heats, and Z is the ionic charge expressed in electron units. These waves are strongly damped (unlike sound waves in an ordinary gas) unless the electron temperature is much higher than the ion temperature and the wave frequency is much higher than the collision frequency. In this latter case these longitudinal waves propagate as acoustic waves. The electron pressure provides the restoring force, and the ion mass provides the inertial effect.

1.9.2 Electromagnetic Waves

Homogeneous plane electromagnetic waves propagate through a dielectric medium with dispersion given by

$$k^2 = \frac{\omega^2 \epsilon}{c^2} \quad (1.9.2)$$

For a cold homogeneous isotropic plasma of mobile electrons and immobile ions, the high-frequency dielectric constant is

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \quad (1.9.3)$$

Problem 1.9.1 Derive the dielectric constant as given in (1.9.3) for a plasma of cold electrons and stationary ions by computing the field arising from a displacement of electrons. ////

The corresponding dispersion relation for electromagnetic waves in a plasma is

$$k^2 = \frac{\omega^2 - \omega_p^2}{c^2} \quad (1.9.4)$$

If the wave frequency ω is less than the plasma frequency ω_p , the wave number is imaginary and the waves are evanescent. Above the plasma frequency ($\omega > \omega_p$), waves propagate, and at very high frequencies, the free electrons of the plasma only slightly influence the electromagnetic wave. Effects such as finite size, a steady magnetic field, or plasma inhomogeneities modify this picture considerably.

1.9.3 Waves in Magnetized or Nonuniform Plasmas

In a homogeneous steady magnetic field many additional wave motions are possible. For example, a slow dispersionless electromagnetic wave or magneto-hydrodynamic wave, called an *Alfvén wave*, propagates through the plasma at frequencies below the ion-cyclotron frequency with speed

$$V_A = \frac{B}{\sqrt{4\pi\rho_m}} \quad (1.9.5)$$

where $\rho_m = nm_i$ is the mass density.

As a second example, drift waves propagate in inhomogeneous plasmas as a result of particle drifts and plasma currents associated with plasma density gradients. This extra freedom introduced by the density gradient allows a new wave motion in which

$$\omega \approx \frac{\kappa T}{m} \frac{1}{\omega_c} \frac{\nabla n_0}{n_0} k \quad (1.9.6)$$

where κ is the Boltzmann constant, and T is the temperature.

1.9.4 Plasma Waves

These waves are also known as *space-charge waves*, *electrostatic waves*, or *Langmuir waves*. They propagate only if there is a distribution of electron velocities or if the electrons have an average velocity in the observer's frame.