积分表
和
积分变换表

（附：常用数学公式选）

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目　录

一、不定积分表 ……1
二、定积分表 ……48
三、积分变换表 ……57
I. 富里埃变换表 ……57
II. 拉普拉斯变换表 ……60
四、常用数学公式选 ……66
I. 代数公式 ……66
II. 几何公式 ……72
III. 三角公式 ……76
IV. 解析几何公式 ……83

一、不定积分表

下面，所有公式中的积分常数都没有写出，当积分得出绝对值的对数（例如，$\int\frac{dx}{1+x}=\ln|1+x|$）时，绝对值符号也省略了．$x$, y, t 等表示自变量，f, u, v 表示它们的函数．

I. 基本积分

1. $d\int f(x)dx=f(x)dx$.　　2. $\int df(x)=f(x)$.

3. $\int af(x)dx=a\int f(x)dx$.　　4. $\int 0\cdot dx=C$.

5. $\int(u\pm v)dx=\int u\,dx\pm\int v\,dx$.

6. $\int udv=uv-\int v\,du$　(分部积分公式).

7. $\int\frac{udv}{dx}dx=uv-\int v\frac{du}{dx}dx$.

8. $\int f(y)dx=\int\frac{f(y)dy}{\frac{dy}{dx}}$.

9. $\int f(x)dx=\int f[\varphi(y)]\varphi'(y)dy$　$(x=\varphi(y))$.

10. $\int x^n dx=\frac{x^{n+1}}{n+1}$　$(x\neq -1)$.

11. $\int\frac{dx}{x}=\ln x$.　　12. $\int e^x dx=e^x$.

13. $\int a^x dx=\frac{a^x}{\ln a}$.　　14. $\int\sin x\,dx=-\cos x$.

15. $\int \cos x\,dx = \sin x$.

16. $\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x$.

17. $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x$.

18. $\int \frac{\sin x}{\cos^2 x}\,dx = \sec x$.

19. $\int \frac{\cos x}{\sin^2 x}\,dx = -\csc x$.

20. $\int \operatorname{tg} x dx = -\ln\cos x$.

21. $\int \operatorname{ctg} x dx = \ln \sin x$.

22. $\int \frac{dx}{\sin x} = \ln \operatorname{tg} \frac{x}{2}$.

23. $\int \frac{dx}{\cos x} = \ln \operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)$.

24. $\int \frac{dx}{1+x^2} = \operatorname{arc\,tg} x = -\operatorname{arc\,ctg} x$.

25. $\int \frac{dx}{1-x^2} = \operatorname{Arth} x = \frac{1}{2}\ln\frac{1+x}{1-x}$.

26. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x = -\arccos x$.

27. $\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{Arsh} x = \ln(x+\sqrt{x^2+1})$.

28. $\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{Arch} x = \ln(x+\sqrt{x^2-1})$.

29. $\int \operatorname{sh} x\,dx = \operatorname{ch} x$.

30. $\int \operatorname{ch} x\,dx = \operatorname{sh} x$.

31. $\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x$.

32. $\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x$.

33. $\int \operatorname{th} x\,dx = \ln \operatorname{ch} x$.

34. $\int \operatorname{cth} x\,dx = \ln \operatorname{sh} x$.

35. $\int \frac{dx}{\operatorname{sh} x} = \ln \operatorname{th} \frac{x}{2}$.

II. 含二项式 $a+bx$ 的积分

36. $\int (a+bx)^m dx = \frac{(a+bx)^{m+1}}{b(m+1)}$ $(m \neq -1)$.

37. $\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$.

38. $\int \frac{xdx}{a+bx} = \frac{x}{b} - \frac{a}{b^2}\ln(a+bx)$.

39. $\int \frac{x^2dx}{a+bx} = \frac{x^2}{2b} - \frac{ax}{b^2} + \frac{a^2}{b^3}\ln(a+bx)$.

40. $\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$.

41. $\int \frac{xdx}{(a+bx)^2} = -\frac{x}{b(a+bx)} + \frac{1}{b^2}\ln(a+bx)$

$= \frac{a}{b^2(a+bx)} + \frac{1}{b^2}\ln(a+bx)$.

42. $\int \frac{x^2dx}{(a+bx)^2} = \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a}{b^3}\ln(a+bx)$.

43. $\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}$.

44. $\int \frac{xdx}{(a+bx)^3} = -\left[\frac{x}{b} + \frac{a}{2b^2}\right]\frac{1}{(a+bx)^2}$.

45. $\int \frac{x^2dx}{(a+bx)^3} = \left[\frac{2ax}{b^2} + \frac{3a^2}{2b^3}\right]\frac{1}{(a+bx)^2} + \frac{1}{b^3}\ln(a+bx)$.

46. $\int \frac{x^ndx}{(a+bx)^m} = \frac{x^n}{(a+bx)^{m-1}(n+1-m)b}$

$- \frac{na}{(n+1-m)b}\int \frac{x^{n-1}dx}{(a+bx)^m}$.

当 $n=m-1$ 时，

47. $\int \frac{x^ndx}{(a+bx)^m} = -\frac{x^n}{(a+bx)^{m-1}(m-1)b}$

$+ \frac{n}{(m-1)b}\int \frac{x^{n-1}dx}{(a+bx)^{m-1}}$.

48. $\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\ln\frac{a+bx}{x}$.

49. $$\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2} \ln \frac{a+bx}{x}.$$

50. $$\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2} \ln \frac{a+bx}{x}.$$

51. $$\int \frac{dx}{x^2(a+bx)^2} = -\left[\frac{1}{ax} + \frac{2b}{a^2}\right] \frac{1}{a+bx} + \frac{2b}{a^3} \ln \frac{a+bx}{x}.$$

52. $$\int \frac{dx}{x^n(a+bx)^m} = -\frac{1}{(n-1)ax^{n-1}(a+bx)^{m-1}} + \frac{b(2-n-m)}{a(n-1)} \int \frac{dx}{x^{n-1}(a+bx)^m}.$$

53. $$\int \frac{dx}{x(a+bx)^m} = \frac{1}{(a+bx)^{m-1}a(m-1)} + \frac{1}{a} \int \frac{dx}{x(a+bx)^{m-1}}.$$

54. $$\int \frac{a+bx}{a'+b'x}\, dx = \frac{bx}{b'} + \frac{ab'-a'b}{b'^2} \ln(a'+b'x).$$

55. $$\int \frac{dx}{(a+bx)(a'+b'x)} = \frac{1}{ab'-a'b} \ln \frac{a'+b'x}{a+bx}.$$

56. $$\int \frac{xdx}{(a+bx)(a'+b'x)} = \frac{1}{ab'-a'b}\left[\frac{a}{b} \ln(a+bx) - \frac{a'}{b'} \ln(a'+b'x)\right].$$

57. $$\int \frac{dx}{(a+bx)^2(a'+b'x)} = \frac{1}{ab'-a'b}\left(\frac{1}{a+bx} + \frac{b'}{ab'-a'b} \ln \frac{a'+b'x}{a+bx}\right).$$

58. $$\int \frac{xdx}{(a+bx)^2(a'+b'x)} = \frac{-a}{b(ab'-a'b)(a+bx)} - \frac{a'}{(ab'-a'b)^2} \ln \frac{a'+b'x}{a+bx}.$$

III. 含有二项式 $a+bx^n$ 的积分

59. $\int \frac{dx}{a+bx^2}=\frac{1}{\sqrt{ab}}\operatorname{arc\,tg}\sqrt{\frac{b}{a}}x \quad (a>0,\ b>0)$,

或 $\frac{1}{2\sqrt{-ab}}\ln\frac{x\sqrt{b}-\sqrt{-a}}{x\sqrt{b}+\sqrt{-a}} \quad (a<0,\ b>0)$,

或 $\frac{1}{2\sqrt{-ab}}\ln\frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}} \quad (a>0,\ b<0)$.

60. $\int \frac{xdx}{a+bx^2}=\frac{1}{2b}\ln(a+bx^2)$.

61. $\int \frac{x^2dx}{a+bx^2}=\frac{x}{b}-\frac{a}{b}\int\frac{dx}{a+bx^2}$.

62. $\int \frac{dx}{x(a+bx^2)}=\frac{1}{2a}\ln\frac{x^2}{a+bx^2}$.

63. $\int \frac{dx}{x^2(a+bx^2)}=-\frac{1}{ax}-\frac{b}{a}\int\frac{dx}{a+bx^2}$.

64. $\int \frac{x^ndx}{ax^2+c}=\frac{x^{n-1}}{b(n-1)}-\frac{a}{b}\int\frac{x^{n-2}dx}{ax^2+1}$, $n\neq 1$.

65. $\int \frac{x^2dx}{(a+bx^2)^n}=-\frac{1}{2(n-1)b}\cdot\frac{x}{(a+bx^2)^{n-1}}+\frac{1}{2(n-1)b}\int\frac{dx}{(a+bx^2)^{n-1}}$.

66. $\int \frac{dx}{x^2(a+bx^2)^n}=\frac{1}{a}\int\frac{dx}{x^2(a+bx^2)^{n-1}}-\frac{b}{a}\int\frac{dx}{(a+bx^2)^n}$.

67. $\int \frac{dx}{(a+bx^2)^2}=\frac{x}{2a(a+bx^2)}+\frac{1}{2a}\int\frac{dx}{a+bx^2}$.

68. $\int \frac{dx}{(a+bx^2)^{m+1}}=\frac{1}{2ma}\cdot\frac{1}{(a+bx^2)^m}$

$$+\frac{2m-1}{2ma}\int\frac{dx}{(a+bx^2)^m}.$$

69. $$\int\frac{dx}{a+bx^3}=\frac{k}{3a}\left\{\frac{1}{2}\ln\frac{(x+k)^2}{x^2-kx+k^2}+\sqrt{3}\operatorname{arc\,tg}\frac{2x-k}{k\sqrt{3}}\right\}\quad\left(k^3=\frac{a}{b}\right).$$

70. $$\int\frac{xdx}{a+bx^3}=\frac{1}{3bk}\left\{\frac{1}{2}\ln\frac{(x+k)^2}{x^2-kx+k^2}-\sqrt{3}\operatorname{arc\,tg}\frac{2x-k}{k\sqrt{3}}\right\}\quad\left(k^3=\frac{a}{b}\right).$$

71. $$\int\frac{dx}{x(a+bx^3)}=\frac{1}{3k}\ln\frac{x^3}{a+bx^3},\quad k^3=\frac{a}{b}.$$

72. $$\int\frac{dx}{x^2(a+bx^3)}=-\frac{1}{kx}-\frac{b}{a}\int\frac{xdx}{a+bx^3},\quad k^3=\frac{a}{b}.$$

73. $$\int\frac{dx}{x(a+bx^n)}=\frac{1}{an}\ln\frac{x^n}{a+bx^n}.$$

IV. 含有二项式 $1\pm x^n$ 的积分

74. $$\int\frac{dx}{1+x^3}=\frac{1}{3}\ln\frac{1+x}{\sqrt{1-x+x^2}}+\frac{1}{\sqrt{3}}\operatorname{arc\,tg}\frac{x\sqrt{3}}{2-x}.$$

75. $$\int\frac{dx}{1+x^4}=\frac{1}{4\sqrt{2}}\ln\frac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2}+\frac{1}{2\sqrt{2}}\operatorname{arc\,tg}\frac{x\sqrt{2}}{1-x^2}.$$

76. $$\int\frac{dx}{1-x^3}=\frac{1}{3}\ln\frac{\sqrt{1+x+x^2}}{1-x}+\frac{1}{\sqrt{3}}\operatorname{arc\,tg}\frac{x\sqrt{3}}{2+x}.$$

77. $$\int\frac{dx}{1-x^4}=\frac{1}{4}\ln\frac{1+x}{1-x}+\frac{1}{2}\operatorname{arc\,tg}x=\frac{1}{2}(\operatorname{Ar\,th}x+\operatorname{arc\,tg}x).$$

78. $\int \frac{x\,dx}{1+x^3} = -\frac{1}{6}\ln\frac{(1+x)^2}{1-x+x^2} + \frac{1}{\sqrt{3}}\operatorname{arc\,tg}\frac{2x-1}{\sqrt{3}}.$

79. $\int \frac{x\,dx}{1+x^4} = \frac{1}{3}\operatorname{arc\,tg} x^2.$

80. $\int \frac{x\,dx}{1-x^3} = -\frac{1}{6}\ln\frac{(1-x)^2}{1+x+x^2} - \frac{1}{\sqrt{3}}\operatorname{arc\,tg}\frac{2x+1}{\sqrt{3}}.$

81. $\int \frac{x\,dx}{1-x^4} = \frac{1}{4}\ln\frac{1+x^2}{1-x^2}.$

82. $\int \frac{x^m dx}{(1+x^2)^n} = \frac{-1}{2n-m-1}\cdot\frac{x^{m-1}}{(1+x^2)^{n-1}} + \frac{m-1}{2n-m-1}\int\frac{x^{m-2}dx}{(1+x^2)^n}.$

83. $\int \frac{x^m dx}{(1-x^2)^n} = \frac{1}{2n-m-1}\cdot\frac{x^{m-1}}{(1-x^2)^{n-1}} - \frac{m-1}{2n-m-1}\int\frac{x^{m-2}dx}{(1-x^2)^n}.$

84. $\int \frac{dx}{x(1+x^2)} = \ln\frac{x}{\sqrt{1+x^2}}.$

85. $\int \frac{dx}{x(1+x^2)^n} = \frac{1}{2n-2}\cdot\frac{1}{(1+x^2)^{n-1}} + \int\frac{dx}{x(1+x^2)^{n-1}}.$

86. $\int \frac{dx}{x^m(1+x^2)^n} = -\frac{1}{m-1}\cdot\frac{1}{x^{m-1}(1+x^2)^{n-1}} - \frac{2n+m-3}{m-1}\int\frac{dx}{x^{m-1}(1+x^2)^n}.$

87. $\int \frac{dx}{x(1-x^2)} = \ln(x\sqrt{1-x^2}).$

88. $\int \frac{dx}{x(1-x^2)^n} = \frac{1}{2(n-1)(1-x^2)^{n-1}} + \int\frac{dx}{x(1-x^2)^{n-1}}.$

89. $\int \frac{dx}{x^m(1-x^2)^n} = -\frac{1}{(m-1)x^{m-1}(1-x^2)^{n-1}} + \frac{2n+m-3}{m-1}\int\frac{dx}{x^{m-2}(1-x^2)^n}.$

V. 含有三项式 $a+bx+cx^2$ 的积分

其中记 $R=a+bx+cx^2$, $\Delta=4ac-b^2$.

90. $$\int\frac{dx}{R}=\frac{1}{\sqrt{-\Delta}}\ln\frac{b+2cx-\sqrt{-\Delta}}{b+2cx+\sqrt{-\Delta}}$$
$$=\frac{-2}{\sqrt{-\Delta}}\operatorname{Arth}\frac{b+2cx}{\sqrt{-\Delta}}\quad(\Delta<0);$$
$$=\frac{-2}{b+2cx}\quad(\Delta=0);$$
$$=\frac{2}{\sqrt{\Delta}}\operatorname{arctg}\frac{b+2cx}{\sqrt{\Delta}}\quad(\Delta>0).$$

91. $$\int\frac{dx}{R^2}=\frac{b+2cx}{\Delta R}+\frac{2c}{\Delta}\int\frac{dx}{R}.$$

92. $$\int\frac{dx}{R^3}=\frac{b+2cx}{\Delta}\left[\frac{1}{2R^2}+\frac{3c}{\Delta R}\right]+\frac{6c}{\Delta^2}\int\frac{dx}{R}.$$

93. $$\int\frac{dx}{R^{n+1}}=\frac{b+2cx}{n\Delta R^n}+\frac{(4n-2)c}{n\Delta}\int\frac{dx}{R^n}.$$

94. $$\int\frac{xdx}{R}=\frac{1}{2c}\ln R-\frac{b}{2c}\int\frac{dx}{R}.$$

95. $$\int\frac{xdx}{R^2}=\frac{2a+bx}{\Delta R}-\frac{b}{\Delta}\int\frac{dx}{R}.$$

96. $$\int\frac{xdx}{R^3}=\frac{2a+bx}{2\Delta R^2}-\frac{3b(b+2cx)}{2\Delta^2 R}-\frac{3bc}{\Delta^2}\int\frac{dx}{R}.$$

97. $$\int\frac{x^2dx}{R}=\frac{x}{c}-\frac{b}{2c^2}\ln R+\frac{b^2-2ac}{2c^2}\int\frac{dx}{R}.$$

98. $$\int\frac{x^2dx}{R^2}=\frac{ab+(b^2-2ac)x}{c\Delta R}+\frac{2a}{\Delta}\int\frac{dx}{R}.$$

99. $$\int\frac{dx}{xR}=\frac{1}{2a}\ln\frac{x^2}{R}-\frac{b}{2a}\int\frac{dx}{R}.$$

100. $$\int\frac{dx}{xR^2}=\frac{1}{2a^2}\ln\frac{x^2}{R}+\frac{1}{2aR}\left[1-\frac{b(b+2cx)}{\Delta}\right]$$

$$-\frac{b}{2a^2}\left(1+\frac{2ac}{\Delta}\right)\int\frac{dx}{R}.$$

101. $$\int\frac{dx}{x^2R}=-\frac{b}{2a^2}\ln\frac{x^2}{R}-\frac{1}{ax}+\frac{b^2-2ac}{2a^2}\int\frac{dx}{R}.$$

102. $$\int\frac{dx}{x^2R^2}=-\frac{b}{a^2}\ln\frac{x^2}{R}-\frac{a+bx}{a^2xR}+\frac{(b^2-3ac)(b+2cx)}{a^2\Delta R}$$
$$-\frac{1}{\Delta}\left(\frac{b^4}{a^3}-\frac{6b^2c}{a^2}+\frac{6c^2}{a}\right)\int\frac{dx}{R}.$$

103. $$\int\frac{dx}{(a'+b'x)R}=\frac{1}{2(ab'^2-a'bb'+a'^2c)}$$
$$\times\left[b'\ln\frac{a'+b'x}{R}+(2a'c-bb')\int\frac{dx}{R}\right].$$

VI. 含有 $a+bx$ 与 $\sqrt{x}$ 的积分

104. $$\int\frac{dx}{(a+bx)\sqrt{x}}=\frac{2}{\sqrt{ab}}\operatorname{arc\,tg}\sqrt{\frac{bx}{a}}\quad(ab>0).$$

105. $$\int\frac{\sqrt{x}\,dx}{a+bx}=\frac{2\sqrt{x}}{b}-\frac{a}{b}\int\frac{dx}{(a+bx)\sqrt{x}}.$$

106. $$\int\frac{x\sqrt{x}\,dx}{a+bx}=\left(\frac{x}{3b}-\frac{a}{b^2}\right)2\sqrt{x}+\frac{a^2}{b^2}\int\frac{dx}{(a+bx)\sqrt{x}}.$$

107. $$\int\frac{dx}{(a+bx)^2\sqrt{x}}=\frac{\sqrt{x}}{a(a+bx)}+\frac{1}{2a}\int\frac{dx}{(a+bx)\sqrt{x}}.$$

108. $$\int\frac{\sqrt{x}\,dx}{(a+bx)^2}=-\frac{\sqrt{x}}{b(a+bx)}+\frac{1}{2b}\int\frac{dx}{(a+bx)\sqrt{x}}.$$

109. $$\int\frac{x\sqrt{x}\,dx}{(a+bx)^2}=\frac{2x\sqrt{x}}{b(a+bx)}-\frac{3a}{b}\int\frac{\sqrt{x}\,dx}{(a+bx)^2}.$$

VII 含有 $\sqrt{a+bx}$ 的积分

110. $$\int\sqrt{a+bx}\,dx=\frac{2}{3b}\sqrt{(a+bx)^3}.$$

111. $\int x\sqrt{a+bx}\,dx = \frac{6bx-4a}{15b^2}\sqrt{(a+bx)^3}$.

112. $\int x^2\sqrt{a+bx}\,dx = \frac{2(8a^2-12abx+15b^2x^2)}{105b^3}\sqrt{(a+bx)^3}$.

113. $\int x^n\sqrt{a+bx}\,dx = \frac{2}{b^{n+1}}\int u^2(u^2-a)^n du,\ u=\sqrt{a+bx}$.

114. $\int \frac{\sqrt{a+bx}}{x}dx = 2\sqrt{a+bx}+a\int \frac{dx}{x\sqrt{a+bx}}$.

115. $\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+bx}}{x}+\frac{b}{2}\int \frac{dx}{x\sqrt{a+bx}}$.

116. $\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b}\sqrt{a+bx}$.

117. $\int \frac{x\,dx}{\sqrt{a+bx}} = \frac{2(bx-2a)}{3b^2}\sqrt{a+bx}$.

118. $\int \frac{x^2dx}{\sqrt{a+bx}} = \frac{1}{15b^3}(8a^2-4ab\,x+3b^2x)\sqrt{a+bx}$.

119. $\int \frac{x^n dx}{\sqrt{a+bx}} = \frac{2x^n\sqrt{a+bx}}{2(n+1)b}-\frac{2na}{(2n+1)b}\int \frac{x^{n-1}dx}{\sqrt{a+bx}}$.

120. $\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}}\ln\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}$ $(a>0)$,

$= \frac{2}{\sqrt{-a}}\operatorname{arc\,tg}\sqrt{\frac{a+bx}{-a}}$ $(a<0)$.

121. $\int \frac{dx}{x^2\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax}-\frac{b}{2a}\int \frac{dx}{x\sqrt{a+bx}}$.

122. $\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}}$

$-\frac{(2n-3)b}{(2n-2)a}\int \frac{dx}{x^{n-1}\sqrt{a+bx}}$.

123. $$\int (a+bx)^{\pm\frac{n}{2}}dx = \frac{2(a+bx)^{\frac{2\pm n}{2}}}{b(2\pm n)}.$$

124. $$\int x(a+bx)^{\pm\frac{n}{2}}dx = \frac{2}{b^2}\left[\frac{(a+bx)^{\frac{2\pm n}{2}}}{4\pm n} - \frac{a(a+bx)^{\frac{2\pm n}{2}}}{2\pm n}\right].$$

125. $$\int \frac{dx}{\sqrt{(a+bx)^3}} = -\frac{2}{b\sqrt{a+bx}}.$$

126. $$\int \frac{xdx}{\sqrt{(a+bx)^3}} = (\sqrt{a+bx}+a)\frac{2}{b^2\sqrt{a+bx}}.$$

127. $$\int \frac{x^2dx}{\sqrt{(a+bx)^3}} = \left(\frac{a+bx}{3} - 2a\sqrt{a+bx} - a^2\right) \times \frac{2}{b^3\sqrt{a+bx}}.$$

128. $$\int \frac{\sqrt{(a+bx)^3}}{x}dx = \left(\frac{a+bx}{3}+a\right)2\sqrt{a+bx} + a^2\int \frac{dx}{x\sqrt{a+bx}}.$$

129. $$\int \frac{\sqrt{(a+bx)^3}}{x^2}dx = -\frac{\sqrt{(a+bx)^5}}{ax} + \frac{3b}{2a}\int \frac{\sqrt{(a+bx)^3}dx}{x}.$$

130. $$\int \frac{dx}{x\sqrt{(a+bx)^3}} = \frac{2}{a\sqrt{a+bx}} + \frac{1}{a}\int \frac{dx}{x\sqrt{a+bx}}.$$

131. $$\int \frac{dx}{x^2\sqrt{(a+bx)^3}} = \left(-\frac{1}{ax} - \frac{3b}{a^2}\right)\frac{1}{\sqrt{a+bx}} - \frac{3b}{2a^2}\int \frac{dx}{x\sqrt{a+bx}}.$$

132. $$\int \frac{(\alpha+\beta x)dx}{\sqrt{a+bx}} = \frac{2\alpha\sqrt{a+bx}}{b} + \beta\left(\frac{a+bx}{3} - a\right)\frac{2\sqrt{a+bx}}{b^2}.$$

133. $$\int \frac{dx}{(\alpha+\beta x)\sqrt{a+bx}}$$

$$= \frac{1}{\sqrt{\beta\Delta}} \ln \frac{\beta\sqrt{a+bx}-\sqrt{\beta\Delta}}{\beta\sqrt{a+bx}+\sqrt{\beta\Delta}} \quad (\beta\Delta>0),$$

$$= \frac{1}{\sqrt{-\beta\Delta}} \operatorname{arc\,tg} \frac{\beta\sqrt{a+bx}}{\sqrt{-\beta\Delta}} \quad (\beta\Delta<0)$$

$$= -\frac{2\sqrt{a+bx}}{b(\alpha+\beta x)} \quad (\Delta=0), \text{其中 } \Delta=a\beta-b\alpha,$$

134. $$\int \frac{(\alpha+\beta x)^n}{\sqrt{a+bx}}dx = \frac{1}{(2n+1)b}\left[\sqrt{a+bx}(\alpha+\beta x)^n - n\Delta \int \frac{(\alpha+\beta x)^{n-1}}{\sqrt{a+bx}}\,dx\right].$$

135. $$\int \frac{dx}{(\alpha+\beta x)^n\sqrt{a+bx}} = -\frac{1}{(n-1)\Delta}\left[\frac{\sqrt{a+bx}}{(\alpha+\beta x)^{n-1}} - \left(n-\frac{3}{2}\right)b \int \frac{dx}{(\alpha+\beta x)^{n-1}\sqrt{a+bx}}\right].$$

VIII. 含有 $\sqrt{x^2\pm a^2}$ 或 $\sqrt{a^2-x^2}$ 的积分

136. $$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x+\sqrt{x^2\pm a^2}\right)\right].$$

137. $$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \operatorname{arc\,sin}\frac{x}{a}\right).$$

138. $$\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2\pm a^2)^3}.$$

139. $$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3}.$$

140. $$\int x^2\sqrt{x^2\pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2\pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2\pm a^2} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2\pm a^2}\right).$$

141. $\int x^2\sqrt{a^2-x^2}\,dx=-\frac{x}{4}\sqrt{(a^2-x^2)^3}$

$$+\frac{a^2}{8}\left(x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}\right).$$

142. $\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln(x+\sqrt{x^2\pm a^2})$.

143. $\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}$ 或 $-\arccos\frac{x}{a}$.

144. $\int\frac{dx}{x\sqrt{a^2\pm x^2}}=-\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2\pm x^2}}{x}\right)$.

145. $\int\frac{dx}{x\sqrt{x^2-a^2}}=\frac{1}{a}\arccos\frac{a}{x}$.

146. $\int\frac{dx}{x^2\sqrt{x^2\pm a^2}}=\mp\frac{\sqrt{x^2\pm a^2}}{a^2x}$.

147. $\int\frac{dx}{x^2\sqrt{a^2-x^2}}=-\frac{\sqrt{a^2-x^2}}{a^2x}$.

148. $\int\frac{xdx}{\sqrt{a^2\pm x^2}}=\pm\sqrt{a^2\pm x^2}$.

149. $\int\frac{xdx}{\sqrt{x^2-a^2}}=\sqrt{x^2-a^2}$.

150. $\int\frac{x^2dx}{\sqrt{x^2\pm a^2}}=\frac{x}{2}\sqrt{x^2\pm a^2}\mp\frac{a^2}{2}\ln(x+\sqrt{x^2\pm a^2})$.

151. $\int\frac{x^2dx}{\sqrt{a^2-x^2}}=-\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}$.

152. $\int\frac{\sqrt{a^2\pm x^2}}{x}dx=\sqrt{a^2\pm x^2}-a\ln\frac{a+\sqrt{a^2\pm x^2}}{x}$.

153. $\int\frac{\sqrt{x^2-a^2}}{x}dx=\sqrt{x^2-a^2}-a\arccos\frac{a}{x}$.

154. $\int\frac{\sqrt{x^2\pm a^2}}{x^2}dx=-\frac{\sqrt{x^2\pm a^2}}{x}+\ln(x+\sqrt{x^2\pm a^2})$,

155. $\int \frac{\sqrt{a^2-x^2}}{x}dx=-\frac{\sqrt{a^2-x^2}}{x}-\arcsin\frac{x}{a}$.

156. $\int \sqrt{(x^2\pm a^2)^3}\,dx=\frac{1}{4}\left[x\sqrt{(x^2\pm a^2)^3}\right.$

$\left.\pm\frac{3a^2x}{2}\sqrt{x^2\pm a^2}+\frac{3a^4}{2}\ln(x+\sqrt{x^2+a^2})\right]$.

157. $\int \sqrt{(a^2-x^2)^3}dx=\frac{1}{4}\left[x\sqrt{(a^2-x^2)^3}\right.$

$\left.-\frac{3a^2x}{2}\sqrt{a^2-x^2}+\frac{3a^4}{2}\arcsin\frac{x}{a}\right]$.

158. $\int \frac{dx}{\sqrt{(x^2\pm a^2)^3}}=\pm\frac{x}{a^2\sqrt{x^2\pm a^2}}$.

159. $\int \frac{dx}{\sqrt{(a^2-x^2)^3}}=\frac{x}{a^2\sqrt{a^2-x^2}}$.

160. $\int \frac{xdx}{\sqrt{(x^2\pm a^2)^3}}=-\frac{1}{\sqrt{x^2\pm a^2}}$.

161. $\int \frac{xdx}{\sqrt{(a^2-x^2)^3}}=\frac{1}{\sqrt{a^2-x^2}}$.

162. $\int \frac{x^2dx}{\sqrt{(x^2\pm a^2)^3}}=-\frac{x}{\sqrt{x^2\pm a^2}}+\ln(x+\sqrt{x^2\pm a^2})$.

163. $\int \frac{x^2dx}{\sqrt{(a^2-x^2)^3}}=\frac{x}{\sqrt{a^2-x^2}}-\arcsin\frac{x}{a}$.

IX. 含有 $\sqrt{a+bx+cx^2}$ 的积分

其中记 $R=a+bx+cx^2$, $\Delta=4ac-b^2$.

164. $\int \frac{dx}{\sqrt{R}}=\frac{1}{\sqrt{c}}\ln(2\sqrt{cR}+2cx+b)\quad(c>0)$;

$=\frac{1}{\sqrt{c}}\operatorname{Arsh}\frac{2cx+b}{\sqrt{\Delta}}\quad(c>0,\ \Delta>0)$;

$=\frac{-1}{\sqrt{-c}}\arcsin\frac{2cx+b}{\sqrt{-\Delta}}\quad(c<0,\ \Delta<0)$;

$$= \frac{1}{\sqrt{c}} \ln(2cx+b) \quad (c>0,\ \Delta=0).$$

165. $$\int \frac{x\,dx}{\sqrt{R}} = \frac{\sqrt{R}}{c} - \frac{b}{2c}\int \frac{dx}{\sqrt{R}}.$$

166. $$\int \frac{x^2\,dx}{\sqrt{R}} = \left(\frac{x}{2c} - \frac{3b}{4c^2}\right)\sqrt{R} + \left(\frac{3b^2}{8c^2} - \frac{a}{2c}\right)\int \frac{dx}{\sqrt{R}}.$$

167. $$\int \frac{x^3\,dx}{\sqrt{R}} = \left(\frac{x^2}{3c} - \frac{5bx}{12c^2} + \frac{5b^2}{8c^3} - \frac{2a}{3c^2}\right)\sqrt{R} - \left(\frac{5b^3}{16c^3} - \frac{3ab}{4c^2}\right)\int \frac{dx}{\sqrt{R}}.$$

168. $$\int \frac{dx}{\sqrt{R^3}} = \frac{2(2cx+b)}{\Delta\sqrt{R}}.$$

169. $$\int \frac{x\,dx}{\sqrt{R^3}} = -\frac{2(2a+bx)}{\Delta\sqrt{R}}.$$

170. $$\int \frac{x^2\,dx}{\sqrt{R^3}} = -\frac{(\Delta-b^2)x-2ab}{c\Delta\sqrt{R}} + \frac{1}{c}\int \frac{dx}{\sqrt{R}}.$$

171. $$\int \sqrt{R}\,dx = \frac{(2cx+b)\sqrt{R}}{4c} + \frac{\Delta}{8c}\int \frac{dx}{\sqrt{R}}.$$

172. $$\int x\sqrt{R}\,dx = \frac{\sqrt{R^3}}{3c} - \frac{(2cx+b)b}{8c^2}\sqrt{R} - \frac{b\Delta}{16c^2}\int \frac{dx}{\sqrt{R}}.$$

173. $$\int x^2\sqrt{R}\,dx = \left(\frac{x}{4c} - \frac{5b}{24c^2}\right)\sqrt{R^3} + \left(\frac{5b^2}{16c^2} - \frac{a}{4c}\right)\frac{(2cx+b)\sqrt{R}}{4c} + \left(\frac{5b^2}{16c^2} - \frac{a}{4c}\right)\frac{\Delta}{8c}\int \frac{dx}{\sqrt{R}}.$$

174. $$\int \sqrt{R^3}\,dx = \left(\frac{R}{8c} + \frac{3\Delta}{64c^2}\right)(2cx+b)\sqrt{R}$$