А · С · КОМПАНЕЕЦ

ТЕОРЕТИЧЕСКАЯ ФИЗИКА

ИЗДАТЕЛЬСТВО ЛИТЕРАТУРЫ НА ИНОСТРАННЫХ ЯЗЫКАХ
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FROM THE PREFACE TO THE FIRST EDITION

This book is intended for readers who are acquainted with the course of general physics and analysis of nonspecializing institutions of higher education. It is meant chiefly for engineer-physicists, though it may also be useful to specialists working in fields associated with physics—chemists, physical chemists, biophysicists, geophysicists, and astronomers.

Like the natural sciences in general, physics is based primarily on experiment, and, what is more, on quantitative experiment. However, no series of experiments can constitute a theory until a rigorous logical relationship is established between them. Theory not only allows us to systematize the available experimental material, but also makes it possible to predict new facts which can be experimentally verified.

All physical laws are expressed in the form of quantitative relationships. In order to interrelate quantitative laws, theoretical physics appeals to mathematics. The methods of theoretical physics, which are based on mathematics, can be fully mastered only by those who have acquired a very considerable volume of mathematical knowledge. Nevertheless, the basic ideas and results of theoretical physics are readily comprehensible to any reader who has an understanding of differential and integral calculus, and is acquainted with vector algebra. This is the minimum of mathematical knowledge required for an understanding of the text that follows.

At the same time, the aim of this book is not only to give the reader an idea about what theoretical physics is, but also to furnish him with a working knowledge of the basic methods of theoretical physics. For this reason it has been necessary to adhere, as far as possible, to a rigorous exposition. The reader will more readily agree with the conclusions reached if their inevitability has been made obvious to him. In order to activize the work of the student, some of the applications of the theory have been shifted into the exercises, in which the line of reasoning is not so detailed as in the basic text.

In compiling such a relatively small book as this one it has been necessary to cut down on the space devoted to certain important sections of theoretical physics, and omit other branches entirely. For instance, the mechanics of solid media is not included at all since to set out this branch, even in the same detail as the rest of the text, would mean doubling the size of the book. A few results from the mechanics of continuous media are included in the exercises as illustrations in thermodynamics. At the same time, the mechanics and electrodynamics of solid media are less related to the fundamental, gnosiological problems of physics than microscopic electrodynamics, quantum theory, and statistical physics. For this reason, very little space is devoted to macroscopic electrodynamics: the material has been selected in such a way as to show the reader how the transition is made from microscopic electrodynamics to the theory of quasi-stationary fields and the laws of the propagation of light in media. It is assumed that the reader is familiar with these problems from courses of physics and electricity.

On the whole, the book is mainly intended for the reader who is interested in the physics of elementary processes. These considerations have also dictated the choice of material; as in all nonencyclopaedic manuals, this choice is inevitably somewhat subjective.

In compiling this book, I have made considerable use of the excellent course of theoretical physics of L. D. Landau and E. M. Lifshits. This comprehensive course can be recommended to all those who wish to obtain a profound understanding of theoretical physics.

I should like to express my deep gratitude to my friends who have made important observations: Ya. B. Zeldovich, V. G. Levich, E. L. Feinberg, V. I. Kogan and V. I. Goldansky.

A. Kompaneyets

PREFACE TO THE SECOND EDITION

In this second edition I have attempted to make the presentation more systematic and rigorous without adding any difficulties. In order to do this it has been especially necessary to revise Part III, to which I have added a special section (Sec. 30) setting out the general principles of quantum mechanics; radiation is now considered only with the aid of the quantum theory of the electromagnetic field, since the results obtained from the correspondence principle do not appear sufficiently justified.

Gibbs' statistics are included in this edition, which has made it necessary to divide Part IV into something in the nature of two cycles: Sec. 39-44, where only the results of combinatorial analysis are set out, and Sec. 45-52, an introduction to the Gibbs' method, which is used as background material for a discussion of thermodynamics. A phenomenological approach to thermodynamics would nowadays appear an anachronism in a course of theoretical physics.

In order not to increase the size of the book overmuch, it has been necessary to omit the theory of beta decay, the variational properties of eigenvalues, and certain other problems included in the first edition.

I am greatly indebted to A. F. Nikiforov and V. B. Uvarov for pointing out several inaccuracies in the first edition of the book.

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PART I

MECHANICS

Sec. 1. Generalized Coordinates

Frames of reference. In order to describe the motion of a mechanical system, it is necessary to specify its position in space as a function of time. Obviously, it is only meaningful to speak of the relative position of any point. For instance, the position of a flying aircraft is given relative to some coordinate system fixed with respect to the earth; the motion of a charged particle in an accelerator is given relative to the accelerator, etc. The system, relative to which the motion is described, is called a frame of reference.

Specification of time. As will be shown later (Sec. 20), specification of time in the general case is also connected with defining the frame of reference in which it is given. The intuitive conception of a universal, unique time, to which we are accustomed in everyday life, is, to a certain extent, an approximation that is only true when the relative speeds of all material particles are small in comparison with the velocity of light. The mechanics of such slow movements is termed Newtonian, since Isaac Newton was the first to formulate its laws.

Newton's laws permit a determination of the position of a mechanical system at an arbitrary instant of time, if the positions and velocities of all points of the system are known at some initial instant, and also if the forces acting in the system are known.

Degrees of freedom of a mechanical system. The number of independent parameters defining the position of a mechanical system in space is termed the number of its degrees of freedom.

The position of a particle in space relative to other bodies is defined with the aid of three independent parameters, for example, its Cartesian coordinates. The position of a system consisting of N particles is determined, in general, by $3N$ independent parameters.

However, if the distribution of points is fixed in any way, then the number of degrees of freedom may be less than $3N$. For example, if two points are constrained by some form of rigid nondeformable coupling, then, upon the six Cartesian coordinates of these points, $x_1, y_1, z_1, x_2, y_2, z_2$, is imposed the condition

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = R_{12}^2 , \quad (1.1)$$

where R_{12} is the given distance between the points. It follows that the Cartesian coordinates are no longer independent parameters: a relationship exists between them. Only five of the six values $x_1, \ldots, z_2$ are now independent. In other words, a system of two particles, separated by a fixed distance, has five degrees of freedom. If we consider three particles which are rigidly fixed in a triangle, then the coordinates of the third particle must satisfy the two equations:

$$(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2 = R_{31}^2 , \quad (1.2)$$

$$(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2 = R_{32}^2 . \quad (1.3)$$

Thus, the nine coordinates of the vertices of the rigid triangle are defined by the three equations (1.1), (1.2) and (1.3), and hence only six of the nine quantities are independent. The triangle has six degrees of freedom.

The position of a rigid body in space is defined by three points which do not lie on the same straight line. These three points, as we have just seen, have six degrees of freedom. It follows that any rigid body has six degrees of freedom. It should be noted that only such motions of the rigid body are considered as, for example, the rotation of a top, where no noticeable deformation occurs that can affect its motion.

Generalized coordinates. It is not always convenient to describe the position of a system in Cartesian coordinates. As we have already seen, when rigid constraints exist, Cartesian coordinates must satisfy supplementary equations. In addition, the choice of coordinate system is arbitrary and should be determined primarily on the basis of expediency. For instance, if the forces depend only on the distances between particles, it is reasonable to introduce these distances into dynamical equations explicitly and not by means of Cartesian coordinates.

In other words, a mechanical system can be described by coordinates whose number is equal to the number of degrees of freedom of the system. These coordinates may sometimes coincide with the Cartesian coordinates of some of the particles. For example, in a system of two rigidly connected points, these coordinates can be chosen in the following way: the position of one of the points is given in Cartesian coordinates, after which the other point will always be situated on a sphere whose centre is the first point. The position of the second point on the sphere may be given by its longitude and latitude.

Together with the three Cartesian coordinates of the first point, the latitude and longitude of the second point completely define the position of such a system in space.

For three rigidly bound points, it is necessary, in accordance with the method just described, to specify the position of one side of the triangle and the angle of rotation of the third vertex about that side.

The independent parameters which define the position of a mechanical system in space are called its generalized coordinates. We will represent them by the symbols q_α, where the subscript α signifies the number of the degree of freedom.

As in the case of Cartesian coordinates, the choice of generalized coordinates is to a considerable extent arbitrary. It must be chosen so that the dynamical laws of motion of the system can be formulated as conveniently as possible.

Sec. 2. Lagrange's Equation

In this section, equations of motion will be obtained in terms of arbitrary generalized coordinates. In such form they are especially convenient in theoretical physics.

Newton's Second Law. Motion in mechanics consists in changes in the mutual configuration of bodies in time. In other words, it is described in terms of the mutual distances, or lengths, and intervals of time. As was shown in the preceding section, all motion is relative; it can be specified only in relation to some definite frame of reference.

In accordance with the level of knowledge of his time, Newton regarded the concepts of length and time interval as absolute, which is to say that these quantities are the same in all frames of reference. As will be shown later, Newton's assumption was an approximation (see Sec. 20). It holds when the relative speeds of all the particles are small compared with the velocity of light; here Newtonian mechanics is based on a vast quantity of experimental facts.

In formulating the laws of motion a very convenient concept is the material particle, that is, a body whose position is completely defined by three Cartesian coordinates. Strictly speaking, this idealization is not applicable to any body. Nevertheless, it is in every way reasonable when the motion of a body is sufficiently well defined by the displacement in space of any of its particles (for example, the centre of gravity of the body) and is independent of rotations or deformations of the body.

If we start with the concept of a particle as the fundamental entity of mechanics, then the law of motion (Newton's Second Law) is formulated thus:

$$m\frac{d^2\mathbf{r}}{dt^2}=\mathbf{F}. \tag{2.1}$$

Here, F is the resultant of all the forces applied to the particle (the vector sum of the forces) $\frac{d^2\mathbf{r}}{dt^2}$ is the vector acceleration, the Cartesian components of which are

$$\frac{d^2x}{dt^2},\ \frac{d^2y}{dt^2},\ \frac{d^2z}{dt^2}.$$

The quantity m involved in equation (2.1) characterizes the particle and is called its mass.

Force and mass. Equality (2.1) is the definition of force. However, it should not be regarded as a simple identity or designation, because (2.1) establishes the form of the interaction between bodies in mechanics and thereby actually describes a certain law of nature. The interaction is expressed in the form of a differential equation that includes only the second derivatives of the coordinates with respect to time (and not derivatives, say, of the fourth order).

In addition, certain limiting assumptions are usually made in relation to the force. In Newtonian mechanics it is assumed that forces depend only on the mutual arrangement of the bodies at the instant to which the equality refers and do not depend on the configuration of the bodies at previous times. As we shall see later (see Part II), this supposition about the character of interaction forces is valid only when the speeds of the bodies are small compared with the velocity of light.

The quantity m in equality (2.1) is a characteristic of the body, its mass. Mass may be determined by comparing the accelerations which the same force imparts to different bodies; the greater the acceleration, the less the mass. In order to measure mass, some body must be regarded as a standard. The choice of a standard body is completely independent of the choice of standards of length and time. This is what makes the dimension (or unit of measurement) of mass a special dimension, not related to the dimensions of length and time.

The properties of mass are established experimentally. Firstly, it can be shown that the mass of two equal quantities of the same substance is equal to twice the mass of each quantity. For example, one can take two identical scale weights and note that a stretched spring gives them equal accelerations. If we join two such weights and subject them to the action of the same spring, which has been stretched by the same amount as for each weight separately, the acceleration will be found to be one half what it was. It follows that the overall mass of the weights is twice as great, since the force depends only on the tension of the spring and could not have changed.

Thus, mass is an *additive* quantity, that is, one in which the whole is equal to the sum of the quantities of each part taken separately. Experiment shows that the principle of additivity of mass also applies to bodies consisting of different substances.

In addition, in Newtonian mechanics, the mass of a body is a constant quantity which does not change with motion.

It must not be forgotten that the additivity and constancy of masses are properties that follow only from experimental facts which relate to very specific forms of motion. For example, a very important law, that of the conservation of mass in chemical transformations involving rearrangement of the molecules and atoms of a body, was established by M. V. Lomonosov experimentally.

Like all laws deduced from experiment, the principle of additivity of mass has a definite degree of precision. For such strong interactions as take place in the atomic nucleus, the breakdown of the additivity of mass is apparent (for more detail see Sec. 21).

We may note that if instead of subjecting a body to the force of a stretched spring it were subjected to the action of gravity, then the acceleration of a body of double mass would be equal to the acceleration of each body separately. From this we conclude that the force of gravity is itself proportional to the mass of a body. Hence, in a vacuum, in the absence of air resistance, all bodies fall with the same acceleration.

Inertial frames of reference. In equation (2.1) we have to do with the acceleration of a particle. There is no sense in talking about acceleration without stating to which frame of reference it is referred. For this reason there arises a difficulty in stating the cause of the acceleration. This cause may be either interaction between bodies or it may be due to some distinctive properties of the reference frame itself. For example, the jolt which a passenger experiences when a carriage suddenly stops is evidence that the carriage is in nonuniform motion relative to the earth.

Let us consider a set of bodies not affected by any other bodies, that is, one that is sufficiently far away from them. We can suppose that a frame of reference exists such that all accelerations of the set of bodies considered arise only as a result of the interaction between the bodies. This can be verified if the forces satisfy Newton's Third Law, i.e., if they are equal and opposite in sign for any pair of particles (it is assumed that the forces occur instantaneously, and this is true only when the speeds of the particles are small compared with the speed of transmission of the interaction).

A frame of reference for which the acceleration of a certain set of particles depends only on the interaction between these particles is called an *inertial* frame (or inertial coordinate system). A free particle, not subject to the action of any other body, moves, relative to such a reference frame, uniformly in a straight line or, in everyday language, by its own momentum. If in a given frame of reference Newton's Third Law is not satisfied we can conclude that this is not an inertial system.

Thus, a stone thrown directly downwards from a tall tower is deflected towards the east from the direction of the force of gravity. This direction can be independently established with the aid of a suspended weight. It follows that the stone has a component of acceleration which is not caused by the force of the earth's attraction. From this we conclude that the frame of reference fixed in the earth is noninertial. The noninertiality is, in this case, due to the diurnal rotation of the earth.

On the forces of friction. In everyday life we constantly observe the action of forces that arise from direct contacts between bodies. The sliding and rolling of rigid bodies give rise to forces of friction. The action of these forces causes a transition of the macroscopic motion of the body as a whole into the microscopic motion of the constituent atoms and molecules. This is perceived as the generation of heat. Actually, when a body slides an extraordinarily complex process of interaction occurs between the atoms in the surface layer. A description of this interaction in the simple terms of frictional forces is a very convenient idealization for the mechanics of macroscopic motion, but, naturally, does not give us a full picture of the process. The concept of frictional force arises as a result of a certain averaging of all the elementary interactions which occur between bodies in contact.

In this part, which is concerned only with elementary laws, we shall not consider averaged interactions where motion is transferred to the internal, microscopic, degrees of freedom of atoms and molecules. Here, we will study only those interactions which can be completely expressed with the aid of elementary laws of mechanics and which do not require an appeal to any statistical concepts connected with internal, thermal, motion.

Ideal rigid constraints. Bodies in contact also give rise to forces of interaction which can be reduced to the kinematic properties of rigid constraints. If rigid constraints act in a system they force the particles to move on definite surfaces. Thus, in Sec. 1 we considered the motion of a single particle on a sphere, at the centre of which was another particle.

This kind of interaction between particles does not cause a transition of the motion to the internal, microscopic, degrees of freedom of bodies. In other words, motion which is limited by rigid constraints is completely described by its own macroscopic generalized coordinates q_α.

If the limitations imposed by the constraints distort the motion, they thereby cause accelerations (curvilinear motion is always accelerated motion since velocity is a vector quantity). This acceleration can be formally attributed to forces which are called reaction forces of rigid constraints.

Reaction forces change only the direction of velocity of a particle but not its magnitude. If they were to alter the magnitude of the velocity, this would produce a change also in the kinetic energy of the particle. According to the law of conservation of energy, heat would then be generated. But this was excluded from consideration from the very start.

To summarize, the reaction forces of ideally rigid constraints do not change the kinetic energy of a system. In other words, they do not perform any work on it, since work performed on a system is equivalent to changing its kinetic energy (if heat is not generated).

In order that a force should not perform work, it must be perpendicular to the displacement. For this reason the reaction forces of constraints are perpendicular to the direction of particle velocity at each given instant of time.

However, in problems of mechanics, the reaction forces are not initially given, as are the functions of particle position. They are determined by integrating equations (2.1), with account taken of constraint conditions. Therefore, it is best to formulate the equations of mechanics so as to exclude constraint reactions entirely. It turns out that if we go over to generalized coordinates, the number of which is equal to the number of degrees of freedom of the system, then the constraint reactions disappear from the equations. In this section we shall make such a transition and will obtain the equations of mechanics in terms of the generalized coordinates of the system.

The transformation from rectangular to generalized coordinates. We take a system with a total of $3N \equiv n$ Cartesian coordinates of which ν are independent. We will always denote Cartesian coordinates by the same letter x_i, understanding by this symbol all the coordinates x, y, z; this means that i varies from 1 to $3N$, that is, from 1 to n. The generalized coordinates we denote by q_α $(1 \leqslant \alpha \leqslant \nu)$. Since the generalized coordinates completely specify the position of their system, x_i are their unique functions:

$$x_i = x_i(q_1, q_2, \ldots q_\alpha, \ldots, q_\nu). \tag{2.2}$$

From this it is easy to obtain an expression for the Cartesian components of velocity. Differentiating the function of many variables $x_i(\ldots q_\alpha)$ with respect to time, we have

$$\frac{dx_i}{dt} = \sum_{\alpha=1}^{\nu} \frac{\partial x_i}{\partial q_\alpha} \frac{dq_\alpha}{dt}.$$

In the subsequent derivation we shall often have to perform summations with respect to all the generalized coordinates q_α,