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# 统计机器学习导论

[日] 杉山将 (Masashi Sugiyama) 著

(英文版)

INTRODUCTION TO  
**STATISTICAL MACHINE  
LEARNING**



Masashi Sugiyama



机械工业出版社  
China Machine Press

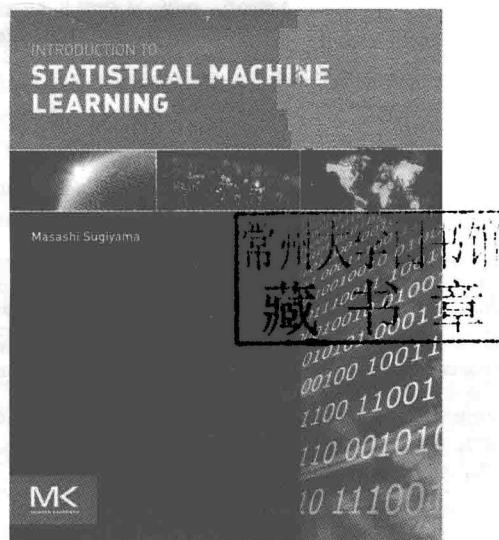
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*Introduction to Statistical  
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## 图书在版编目 (CIP) 数据

统计机器学习导论 (英文版) / (日) 杉山将 (Masashi Sugiyama) 著. —北京: 机械工业出版社, 2018.1

(经典原版书库)

书名原文: Introduction to Statistical Machine Learning

ISBN 978-7-111-58678-4

I. 统… II. 杉… III. 机器学习 - 英文 IV. TP181

中国版本图书馆 CIP 数据核字 (2017) 第 309485 号

本书版权登记号: 图字 01-2017-7344

Elsevier (Singapore) Pte Ltd.  
3 Killiney Road, #08-01 Winsland House I, Singapore 239519  
Tel: (65) 6349-0200; Fax: (65) 6733-1817

Introduction to Statistical Machine Learning  
Masashi Sugiyama  
Copyright © 2016 Elsevier Inc. All rights of reproduction in any form reserved.  
ISBN-13: 978-0-12-802121-7

This reprint of Introduction to Statistical Machine Learning by Masashi Sugiyama was undertaken by China Machine Press and is published by arrangement with Elsevier (Singapore) Pte Ltd.

Introduction to Statistical Machine Learning by Masashi Sugiyama 由机械工业出版社进行影印，并根据机械工业出版社与爱思唯尔（新加坡）私人有限公司的协议约定出版。

ISBN: 978-7-111-58678-4

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出版发行: 机械工业出版社 (北京市西城区百万庄大街 22 号 邮政编码: 100037)

责任编辑: 曲 煜

责任校对: 殷 虹

印 刷: 北京市兆成印刷有限责任公司

版 次: 2018 年 1 月第 1 版第 1 次印刷

开 本: 186mm × 240mm 1/16

印 张: 32

书 号: ISBN 978-7-111-58678-4

定 价: 119.00 元

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本书法律顾问: 北京大成律师事务所 韩光 / 邹晓东

# 出版者的话

文艺复兴以来，源远流长的科学精神和逐步形成的学术规范，使西方国家在自然科学的各个领域取得了垄断性的优势；也正是这样的优势，使美国在信息技术发展的六十多年间名家辈出、独领风骚。在商业化的进程中，美国的产业界与教育界越来越紧密地结合，计算机学科中的许多泰山北斗同时身处科研和教学的最前线，由此而产生的经典科学著作，不仅擘划了研究的范畴，还揭示了学术的源变，既遵循学术规范，又自有学者个性，其价值并不会因年月的流逝而减退。

近年，在全球信息化大潮的推动下，我国的计算机产业发展迅猛，对专业人才的需求日益迫切。这对计算机教育界和出版界都既是机遇，也是挑战；而专业教材的建设在教育战略上显得举足轻重。在我国信息技术发展时间较短的现状下，美国等发达国家在其计算机科学发展的几十年间积淀和发展的经典教材仍有许多值得借鉴之处。因此，引进一批国外优秀计算机教材将对我国计算机教育事业的发展起到积极的推动作用，也是与世界接轨、建设真正的世界一流大学的必由之路。

机械工业出版社华章公司较早意识到“出版要为教育服务”。自1998年开始，我们就将工作重点放在了遴选、移译国外优秀教材上。经过多年的不懈努力，我们与Pearson, McGraw-Hill, Elsevier, MIT, John Wiley & Sons, Cengage等世界著名出版公司建立了良好的合作关系，从他们现有的数百种教材中甄选出Andrew S. Tanenbaum, Bjarne Stroustrup, Brian W. Kernighan, Dennis Ritchie, Jim Gray, Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman, Abraham Silberschatz, William Stallings, Donald E. Knuth, John L. Hennessy, Larry L. Peterson等大师名家的一批经典作品，以“计算机科学丛书”为总称出版，供读者学习、研究及珍藏。大理石纹理的封面，也正体现了这套丛书的品位和格调。

“计算机科学丛书”的出版工作得到了国内外学者的鼎力相助，国内的专家不仅提供了中肯的选题指导，还不辞劳苦地担任了翻译和审校的工作；而原书的作者也相当关注其作品在中国的传播，有的还专门为本书的中译本作序。迄今，“计算机科学丛书”已经出版了近两百个品种，这些书籍在读者中树立了良好的口碑，并被许多高校采用为正式教材和参考书籍。其影印版“经典原版书库”作为姊妹篇也被越来越多实施双语教学的学校所采用。

权威的作者、经典的教材、一流的译者、严格的审校、精细的编辑，这些因素使我们的图书有了质量的保证。随着计算机科学与技术专业学科建设的不断完善和教材改革的逐渐深化，教育界对国外计算机教材的需求和应用都将步入一个新的阶段，我们的目标是尽善尽美，而反馈的意见正是我们达到这一终极目标的重要帮助。华章公司欢迎老师和读者对我们的工作提出建议或给予指正，我们的联系方法如下：

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# Preface

*Machine learning* is a subject in computer science, aimed at studying theories, algorithms, and applications of systems that learn like humans. Recent development of computers and sensors allows us to access a huge amount of data in diverse domains such as documents, audio, images, movies, e-commerce, electric power, medicine, and biology. Machine learning plays a central role in analyzing and benefiting from such *big data*.

This textbook is devoted to presenting mathematical backgrounds and practical algorithms of various machine learning techniques, targeting undergraduate and graduate students in computer science and related fields. Engineers who are applying machine learning techniques in their business and scientists who are analyzing their data can also benefit from this book.

A distinctive feature of this book is that each chapter concisely summarizes the main idea and mathematical derivation of particular machine learning techniques, followed by compact MATLAB programs. Thus, readers can study both mathematical concepts and practical values of various machine learning techniques simultaneously. All MATLAB programs are available from

[“<http://www.ms.k.u-tokyo.ac.jp/software/SMLbook.zip>”](http://www.ms.k.u-tokyo.ac.jp/software/SMLbook.zip).

This book begins by giving a brief overview of the field of machine learning in Part 1. Then Part 2 introduces fundamental concepts of *probability and statistics*, which form the mathematical basis of statistical machine learning. Part 2 was written based on

Sugiyama, M.  
Probability and Statistics for Machine Learning,  
Kodansha, Tokyo, Japan, 2015. (in Japanese).

Part 3 and Part 4 present a variety of practical machine learning algorithms in the *generative* and *discriminative* frameworks, respectively. Then Part 5 covers various advanced topics for tackling more challenging machine learning tasks. Part 3 was written based on

Sugiyama, M.  
Statistical Pattern Recognition: Pattern Recognition Based on Generative  
Models,  
Ohmsha, Tokyo, Japan, 2009. (in Japanese),

and Part 4 and Part 5 were written based on

Sugiyama, M.  
An Illustrated Guide to Machine Learning,  
Kodansha, Tokyo, Japan, 2013. (in Japanese).

The author would like to thank researchers and students in his groups at the University of Tokyo and Tokyo Institute of Technology for their valuable feedback on earlier manuscripts.

Masashi Sugiyama  
The University of Tokyo

# Biography

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## MASASHI SUGIYAMA

**Masashi Sugiyama** received the degrees of Bachelor of Engineering, Master of Engineering, and Doctor of Engineering in Computer Science from Tokyo Institute of Technology, Japan in 1997, 1999, and 2001, respectively. In 2001, he was appointed Assistant Professor in the same institute, and he was promoted to Associate Professor in 2003. He moved to the University of Tokyo as Professor in 2014. He received an Alexander von Humboldt Foundation Research Fellowship and researched at Fraunhofer Institute, Berlin, Germany, from 2003 to 2004. In 2006, he received a European Commission Program Erasmus Mundus Scholarship and researched at the University of Edinburgh, Edinburgh, UK. He received the Faculty Award from IBM in 2007 for his contribution to machine learning under non-stationarity, the Nagao Special Researcher Award from the Information Processing Society of Japan in 2011 and the Young Scientists' Prize from the Commendation for Science and Technology by the Minister of Education, Culture, Sports, Science and Technology Japan for his contribution to the density-ratio paradigm of machine learning. His research interests include theories and algorithms of machine learning and data mining, and a wide range of applications such as signal processing, image processing, and robot control.



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