国外实用金融统计丛书

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Computational Methods in Finance

（英文影印导读版）

金融中的计算方法

[美] 阿里·赫萨（Ali Hirsa） 著

机械工业出版社
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丁睿　注释

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本书主要讲述如何运用数值方法解决复杂函数方程。本书的第一部分描述了大量衍生品在各种模型中的定价方法，回顾了不同市场下常见的资产模型建模过程，并对多种衍生品定价的数值逼近方法进行了实验。这些方法包括转换技术，诸如快速傅里叶变换、分形快速傅里叶变换、Fourier-cosine 方法、鞍点法、扩散框架下的 PDE 以及带跳的 PIDE 的有限差分方法以及蒙特卡罗模拟等。第二部分侧重于实际市场中衍生品定价的基本步骤。作者讨论了如何通过调整模型参数使模型价格符合市场价格，其中还涵盖了各种滤波技术及其实现方法，并给出过滤技术和参数估计的例子。本书为读者准确模拟衍生品定价提供了有效的数值方法。

本书可作为金融工程专业高年级学生的教材，也可作为金融从业人员的参考书。

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前言

“无论意欲取得任何进展，都始终非常有必要引入近似技术，也就是数值计算。因此，对复杂函数方程的数值计算方法再一次成为我努力研究的一个主要方向。事实上我对数值分析从未产生过这样的兴趣，同我这一代大多数的数学家一样，我也曾认为这是一项功利的研究。对数值解的研究被认为是无能数学家最后的救命稻草。然而事实是，一旦从事这一领域的研究，就很快意识到得出数值解比建立一般的存在性和唯一性定理会要求更强的能力和更深刻的理解。获得一个有效的算法比证明一个定理要更有难度。任何科学理论的最终目标都是具体的数字推导。”这是节取自理查德·贝尔曼的著作《飓风之眼》185 页的一段话。考虑到量化金融的发展已经离不开计算（数值）技术以及近年来其在金融领域改革中的影响，引用这段话作为前言还是十分合适的。

在对大多数应用问题和物理现象的解释中，我们总是试图寻找一个接近真实解的近似值。因此，掌握一些计算方法或数值算法是必需的。在量化金融中，除了少数情况存在解析或半解析的解以外，我们通常使用近似值代替真实解。随着如今越来越复杂的金融产品的诞生，定量分析师、金融工程师和金融行业中其他的从业者特别需要稳健的数值解。计算金融研究领域已经在迅速发展，并且越来越复杂的金融产品和市场的发展也将会对数值方法提出更高的需求。

本书是基于我在哥伦比亚大学和纽约大学柯朗数学研究所使用的讲稿完善而成的。书中主题的选择受到了我在教学过程中学生和市场需求的影响。我的同事兼朋友 Rama Cont，建议我将这些笔记整合为一部教科书并出版。

我们的目的是编写一本有关金融中的数值计算方法的教科书，全方位介绍金融衍生品合约和相关产品的定价方法，同时介绍一些算法、模拟、模型校准和各类实用的参数估计的例子。本书是针对金融工程或金融数学方向第一或第二年级研究生、量化分析人员、研究人员、模型实现技术专家和对这一领域感兴趣的读者编写的，宗旨是保持本书的自包含性和观点的独立性。

总体来说，我们不会在理论方面进行太多的正式讲述。本书的目的不在于从细节上研究随机微积分或者鞅定价，因为它们不是理解书中内容的先决条件。虽然在某些情况下有些理论是不可避免的，但我将尽可能给出足够的解释，以保证读者在不需要深入了解其背后理论或派生理论的前提下可以继续阅读。

本书由两部分组成。第一部分描述了各种衍生品合约定价的方法技术和各种模型及其过程的估计。在第二部分中，我们着重于模型校准、校准步骤、滤波和参数估计等方面。

第 1 章回顾了一些基本概念，主要涉及随机过程的特征函数。这一章展示了如何应用特征函数生成结果分布的矩以及如何派生出不同过程的特征函数。同时，书中还回顾了各类标准分布的特征函数。在这一章中我提供了一个独立的列表，其中包括一些从业者在衍生品定价模型中最常用的随机过程，但这还不是一份最全面的列表，它并不能覆盖在实际

中使用到的每一个随机过程。在描述这些过程时，我们尽可能提供详细的数学描述，包括每一个分布的特征函数、存在的封闭形式以及存在封闭式的随机微分方程。最后，回顾了风险中性定价及测度变换。与标的资产的随机模型相结合后，这些理论构成了金融衍生品定价算法的基础。

第 2~6 章涵盖了多种衍生品合约定价的计算，包括 (a) 转换技术，(b) 有限差分法求解偏微分方程和部分可积微分方程和 (c) 蒙特卡罗模拟。第 2 章讲述了一系列变换技术，其中包括快速傅里叶变换技术，分形快速傅里叶变换，Fourier-cosine(COS) 方法和鞍点法。讨论了每种方法的利弊，并提供大量的交叉比较。第 3 章介绍了如何使用有限差分方法对偏微分方程进行数值求解，其重点是用几种最常用的有限差分技术求解偏微分方程，即显式的、隐式的、Crank-Nicolson 和多步法等几种方法，并讨论了这些方法的稳定性以及偏微分方程离散化后生成的刚性矩阵的不同结构，并且提供了解决线性方程组的方法。同时，本章提供了一个通过有限差分进行导数逼近的通用方法。第 4 章利用第 3 章中介绍的有限差分法对 Vanilla 及奇异衍生品进行定价。其中，这类衍生品的价格是可以被诸如 Black-Scholes 模型、一维局部波动率模型、二维 Heston 随机波动率模型等偏微分方程模型表示的。讲述如何实现边界条件和运动边界，建立非均匀网格点和协调转换以及如何处理带跳的条件。第 5 章涵盖了通过有限差分技术对部分可积微分方程进行数值求解来对各类衍生品进行定价。介绍了出现在纯跳框架下的 PIDE，例如方差伽马 (VG) 模型和 CGMY 模型。

对于闭合式、无特征函数、同时到期结算结构相当复杂的衍生品合约，例如非 Markov 过程，或者高维度的过程或模型，则利用蒙特卡罗模拟对其进行定价和估值。第 6 章讲述了蒙特卡罗模拟方法。讨论了不同的抽样方法和不同分布的抽样方法，同时也会涉及一部分蒙特卡罗积分和随机微分方程数值积分的内容。由于模拟时产生的方差会对结果的精确性造成影响，这一章也讲述了一些方差缩减技术来解决这一主要缺点，同时还深入研究了对一些纯跳过程的模拟。

本书的第二部分着重分析了实际市场中衍生品定价和估值的基本步骤。第 7 章讨论了如何通过调整模型参数使模型价格符合市场价格。关于局部波动率以及在股票、外汇、利率模型中使用的扩散或者带跳模型的校准也在本章进行了讨论。这一章还详细讲述了在校准过程中的两个重要步骤，即目标函数的构建以及优化方法，同时还讨论了模型风险的表示法。本书的最后一章，即第 8 章，涵盖了各种滤波技术及其在时间序列上的实现，同时阐述了模型中的最佳参数集，并提供了滤波技术示例以及各类模型、过程的参数估计。

本书提供了大量的习题和研究案例来帮助读者和学生测试他们在学习定价、估值、场景分析、校准、优化和参数估计时的掌握程度。

我想对那些直接或间接帮助我完成此书的人表达我的感激之情。我欠我的博士生导师以及我的合作者 Dilip B. Madan 一个大人情。特别感谢我的合作者 Peter Carr、Georges Courtadon 和 Massoud Heidari，在与他们的各种讨论和合作中我获得了极大的帮助。感谢 Alireza Javaheri、Michael Johannes 和 Nicholas G.Polson，在滤波和参数估计方面，我通过与他们的共同工作得到了巨大收获。从我的博士导师 Howard C. Elman，Ricardo H.Nochetto，R. Bruce (Royal) Kellogg 和 Jeffrey Cooper 那里，我学到了许多数值分析和科学计算方法的知识，没有他们的培养和指导我将无法达到今天的学识水平。

Preface

"In order to make any progress, it is necessary to think of approximate techniques, and above all, numerical algorithms ... Once again, what became a major endeavor of mine, the computational solution of complex functional equations, was entered into quite diffidently. I had never been interested in numerical analysis up to that point. Like most mathematicians of my generation, I had been brought up to scorn this utilitarian activity. Numerical solution was considered the last resort of an incompetent mathematician. The opposite, of course, is true. Once working in this area, it is very quickly realized that far more ability and sophistication is required to obtain a numerical solution than to establish the usual existence and uniqueness theorems. It is far more difficult to obtain an effective algorithm than one that stops with a demonstration of validity. A final goal of any scientific theory must be the derivation of numbers." This is an excerpt[1] from *Eye of the Hurricane* [30] on page 185 by Richard Bellman. It seems appropriate to start the preface with this quote considering advances in quantitative finance would have been impossible without utilizing computational/numerical techniques and their impact on the evolution of the field in recent years.

In most applications and physical phenomena, we are in search of a solution that happens to be an approximation of the true solution. As a result, some sort of a computational method/technique or a numerical procedure is a must. In quantitative finance, aside from a few cases with an analytical or a semi-analytical solution, we typically wind up with an approximation as well. As today's financial products have become more complex, quantitative analysts, financial engineers, and others in the financial industry now require robust techniques for numerical solutions. Computational finance has been a field that has been growing tremendously and intricacy of products and markets suggests there will be an even higher demand in the field.

This book is based on lecture notes I have used in my courses at Columbia University and my course at the Courant Institute of New York University. The selection of topics has been influenced by students and market requirements throughout my teaching over the years. Rama Cont, my colleague and friend, suggested to incorporate these notes into a textbook and referred me to the publisher.

My goal has been to write a textbook on computational methods in finance bringing together a full-spectrum of methods and schemes for pricing of derivatives contracts and related products, simulation, model calibration and parameter estimation with many practical examples. This book is intended for first/second year graduate students in the financial engineering or mathematics of finance field as well as practitioners, quants, researchers, technologists implementing models, and those who are interested in the field. My intention has been to keep the book self-contained and stand-alone.

Overall I have been pretty informal about theory.[2] The aim has not been to get into detail on stochastic calculus or martingales pricing as they are not prerequisites for understanding

[1]This quote was brought up to my attention by Michael Johannes, a colleague and friend, of Columbia Business School.

[2]An example of this is the Itô lemma for semi-martingales without defining semi-martingales or the Girsanov theorem without stating the theorem.

the procedures in the book. Yet in some cases it has been unavoidable, and I try to give sufficient explanation so that the reader can proceed without any need to delve into the derivation or the theory behind it.

This book is composed of two parts. The first part of the book describes various methods and techniques for the pricing of derivative contracts and the valuation of a variety of models and processes. In the second part, the book focuses on model calibration, calibration procedure, filtering, and parameter estimation.

Chapter 1 reviews some basic concepts, principally relating to the construction of the characteristic function of stochastic processes. It then shows how the characteristic function can be used to generate the moments of the resulting distribution and some methods used in our derivations of the characteristic functions of different processes. In addition, it reviews various characteristic functions of standard distributions. I then provide a self-contained list of some of the most commonly used stochastic processes that practitioners employ to model assets for derivative pricing applications. However, this list is by no means comprehensive and will certainly not cover every stochastic process used in practice. In describing these processes, I provide as detailed a mathematical description of each process as possible, including the characteristic function for every process, in closed form where available, as well as the stochastic differential equation where a closed form exists. Finally, the chapter contains a basic review of risk-neutral pricing and change of measure. When combined with a model of the stochastic evolution of the underlying asset, this forms the basis for all the derivative pricing algorithms in this book.

Chapters 2–6 cover many computational approaches for pricing derivatives contracts, including (a) transform techniques, (b) the finite difference method for solving partial differential equations and partial-integro differential equations, and (c) Monte Carlo simulation. Chapter 2 presents a range of transform techniques that comprise the fast Fourier transform, fractional fast Fourier transform, the Fourier-cosine (COS) method, and the saddlepoint method. I discuss the pros and cons of each approach and provide plenty of cross comparison. Chapter 3 introduces the finite difference method used for numerically solving partial differential equations. This chapter focuses on the most commonly used finite difference techniques utilized to solve partial differential equations, namely, explicit, implicit, Crank–Nicolson, and multi-step schemes. I discuss stability analysis of those schemes and different structure for the stiffness matrix arising from the discretization of partial differential equations and provide routines for solving the linear equations. A generic approach to derivative approximation by finite differences is also provided. Chapter 4 utilizes finite differences introduced in Chapter 3 to price vanilla and exotic derivatives under models for which a partial differential equation describing derivative prices can be formulated such as the Black–Scholes model and the local volatility models in the one-dimensional case and the Heston stochastic volatility model in the two-dimensional case. I discuss how to implement boundary conditions and exercise boundaries, setting up non-uniform grid points and coordinate transformation as well as dealing with jump conditions. Chapter 5 covers numerical solutions of partial-integro differential equations via finite differences for pricing various different derivative contracts. I look at PIDEs which arise in the pure jump framework, for instance, variance gamma (VG) and CGMY processes.

Not having the characteristic function in closed form, having a fairly complex payoff structure for the derivative contract under consideration, having a non-Markov process, or a high dimensional process or model, we have to utilize Monte Carlo simulation for pricing and valuation as the method of last resort. Chapter 6 covers Monte Carlo simulation. I discuss different sampling methods and sampling from various different distributions. I also go over Monte Carlo integration and numerical integration of stochastic differential equations. The output from simulation is associated with a variance that limits the accuracy of the simulation results. It is the major drawback to simulation and, naturally, various reduction techniques are studied and examined in this chapter. I also delve into simulation of some pure jump processes.

In the second part, the book focuses on essential steps in real-world derivatives pricing and estimation. In Chapter 7, I discuss how to calibrate model parameters so that model prices will be compatible with market prices. Construction of the local volatility surface and calibration of various different models in diffusion or pure-jump framework used for equity, foreign exchange, or interest rate modeling are discussed. The two essential steps in the calibration procedure, namely, the objective function and the optimization methodology are addressed in detail. I also discuss the notation of model risk. Chapter 8, the last chapter of the book covers various filtering techniques and their implementations used on the time series of data to unravel the best parameter set for the model under consideration and provide examples in filtering and parameter estimation of various different models and processes.

The book provides plenty of problems and case studies to help readers and students test their level of understanding in pricing, valuation, scenario analysis, calibration, optimization, and parameter estimation.

I would like to express my gratitude to several people who have influenced me directly or indirectly on this book. I owe a particular debt to my PhD advisor and co-author Dilip B. Madan. Special thanks to my co-authors Peter Carr, Georges Courtadon, and Massoud Heidari. I gained enormously from our many discussions and working together on a variety of different topics. I am thankful to Alireza Javaheri, Michael Johannes, and Nicholas G. Polson; I benefited tremendously on joint work with them regarding filtering and parameter estimation. I learned a great deal from my PhD advisor Howard C. Elman, Ricardo H. Nochetto, R. Bruce (Royal) Kellogg, and Jeffrey Cooper on numerical analysis and scientific computing; without their teaching and guidance I would not have been able to reach the level I am today.

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List of Symbols and Acronyms

B_t money market account at time t starting with \$1 at time 0 and rolling at the instantaneous short rate
i complex number $\sqrt{-1}$
$\mathbb{E}(x)$ expectation of x under some measure
$\mathbb{E}_t(x)$ expectation of x under some measure conditional on knowing all information up to t
$f(\mathrm{x})$ probability distribution function
$F(\mathrm{x})$ cumulative distribution function
$f(t,T)$ instantaneous forward rate at calendar time t with maturity T
$F(t,T,S)$ simply compounded forward rate for $[T,S]$ prevailing at t
$L(t,T)$ LIBOR rate at calendar time t with maturity T
$\mathbb{N}$ set of natural numbers
$\mathbb{P}$ real-world (physical) measure
$\mathbb{P}^T$ forward measure
$\mathbb{P}^{n+1,N}$ swap measure
$p(t,T)$ price of a zero-coupon at time t maturing at T
$P_{k|k-1}$ error covariance matrix at time k given observations up to and including time $k-1$
$P_{k|k}$ error covariance matrix at time k given observations up to and including time k
$\mathbb{Q}$ risk-neutral measure
ρ correlation
$\mathbb{R}$ set of real numbers
$\mathbb{R}^+$ set of positive real numbers
r_t instantaneous short rate at calendar time t
$R(t,T)$ continuously compounded spot rate with maturity T prevailing at t
$R(t,T,S)$ continuously compounded forward rate for $[T,S]$ prevailing at t
σ instantaneous volatility
$s(t,T)$ swap rate at calendar time t with maturity T
S_t underlying price value at time t
$Var(x)$ variance of random variable x
$\hat{x}_{k|k-1}$ estimate of the state at time k given observations up to and including time $k-1$
$\hat{x}_{k|k}$ estimate of the state at time k given observations up to and including time k
$\mathbb{Z}$ set of integers
ACT/360 day count convention — actual days assuming 360 days in a year
BVP boundary value problem
CDF cumulative distribution function
CDO collateralized debt obligation
CGF cumulant generating function
CGMY Carr–Geman–Madan–Yor
COS Fourier-cosine
DOC down-and-out call
DOP down-and-out put
FRA forward rate agreement
FFT fast Fourier transform
FrFFT fractional fast Fourier transform
GBM geometric Brownian motion
GBMSA geometric Brownian motion with stochastic arrival — Heston
NIG normal inverse Gaussian
PDE partial differential equation
PDF probability distribution function
PIDE partial-integro differential equation
SAE sum of the absolute errors
SDE stochastic differential equation
SRE sum of relative errors
SSAE sum of the squares of absolute errors
SSRE sum of the squares of relative errors
VG variance gamma
VGSA variance gamma with stochastic arrival
UOC up-and-out call
UOP up-and-out put

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