河海大学文天学院规划教材

运筹学引论（第二版）

Introduction to Operations Research

（2nd Edition）

葛久研◎主　编
（Ge Jiuyan）
姜忠鹤　徐晓迪　闫　杰◎副主编
（Jiang Zhonghe, Xu Xiaodi, Yan Jie）

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内容提要

运筹学是一门新兴的应用数学分支学科，解决问题的理论基础是最优化理论与技术。本书以优化理论基础为重点，主要涉及线性规划、图与网络规划、动态规划、对策论等，各部分内容着重阐明基本理论与基本方法。本书在阐述运筹学的基本理论和基础知识的基础上，更拓展了其在管理和商务活动中的运用。本书主要读者对象为大学商学院学生、经济管理专业研究生等。

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>> Foreword of the Second Edition

We have been deeply gratified with the response, which is from the students of Wentian College, Hohai University, to our first edition. To help to guide the students to our field is a heavy responsibility. It is enjoyable for us to meet the needs of new generations of students. Our goal for this edition is to help define the modern approach to teaching operations research effectively at an introductory level.

In this edition, the Integer Programming and Branch-and-Bound Technique (4.6) as well as the 0-1 Programming and Implicit Enumeration (4.7) are introduced. The Dual Simplex Method (5.5) and the Matrix Form of Simplex Method (5.6) are presented. An example of the Comprehensive Sensitivity Analysis (5.7) is given. The transportation problem with uncertainty demand (6.4.4), which has not been involved in the first edition, is supplemented. In addition, some examples are provided in Chapter 7 for interpreting the relationship between the dynamic programming and the linear programming.

We do hope that this edition would be helpful for students. If those students who would like to take part in the entrance exams for postgraduate school could have some inspiration, we would be very happy.

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Jiang Zhonghe, Xu Xiaodi, Yan Jie
Wentian College, Hohai University

2017-4-30

>> Preface

Since the emergence of management science/operations research (MS/OR) as a discipline in the 1940s, the application of these techniques has spread to encompass almost every facet of modern business management and decision making. The inevitable impact of this business education is that management science has become an integral part of business curricula, and a host of textbook dealing with the subject has been written.

We wrote this textbook because we felt that we could, in many respects, improve upon what has been written in the past. Three major pedagogical issues that have influenced the writting of this textbook are: (1) the role of the computer in teaching MS/OR, (2) the matching of presentation style to the audience of the text, and (3) the necessary content to accommodate a variety of instructor styles and syllabi.

Why are we spending time to learn about MS/OR when the computer can do all of this for us? How well the instructor fields this question may determine whether the class becomes a motivated learning group or a disinterested mass? The answer to this question lies in the fact that every decision problem involves three stages of effort: (1) problem formulation (or model development), (2) solution of the problem, and (3) interpretation and testing of the solution results. Current computer software and computer-based decision support systems are very useful for mathematical

computation and problem solving, few are able to provide much assistance in either the formulation of the problem or the interpretation of the result. We are not trying to downplay the importance of the computer, without which since most contemporary management science problems could not be solved without which. However, we do not think that the role of the trained decision-maker should be downplayed, since his or her role is just as crucial. An individual who is well-trained in management science techniques can very quickly adapt to user-friendly management-science computer software, while the best software available cannot help a decision-maker who does not understand these techniques.

There are two approaches that could be taken. The first is to use computer output liberally in the text and attempt to integrate the presentation of formulation and interpretation with computer codes for solving the problems. The problem here is that there is little standardization of computer hardware and software at colleges and universities. This makes this approach completely useful only at those schools that have hardware that can run the code used in the text. The other approach is to concentrate in the text on the MS/OR issues and defer the computer integration to the instructor, who can choose computer software that is compatible with available hardware and with syllabus design. We chose the latter approach.

The second issue of presentation style and audience is highly related to the first issue of the role of the computer. Since the computer will continue to be the prime method of problem solution, it is our belief that the vast majority of future managers and engineers will be involved in the formulation and interpretation

stages, with the solution stage left to the computer and individuals who have specialized in MS/OR.

The third issue is that of content. In an effort to make the textbook useful to as wide an audience as possible, we have included chapters on most topics normally considered to fall within the area of MS/OR. In order to keep the presentation simple and make the text readable, we have tried to keep the chapters short and make the presentation concise. Consequently, the text should be appropriate for a variety of different syllabi.

This textbook could not have been completed without the assistance of a great many people. Special acknowledgments should be made of the students who studied and/or are studying in Wentian College, Hohai University. They gave us a lot of good ideas and suggestions. We also would like to thank those students majoring in *International Economics and Trade* of grade 2008, Wentian College, Hohai University. Most of the record work was finished by them.

Any comments and criticisms the readers would like to make are welcome.

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2010-12-14

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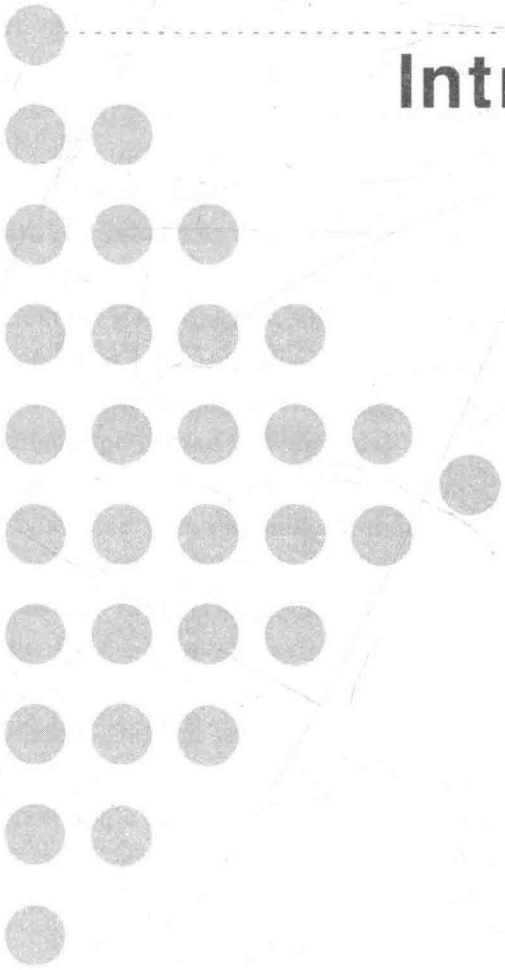
Chapter 1

Introduction



Chapter 1

Introduction

1.1 Origins of Operations Research

The roots of operations research (commonly referred to OR) can be traced back many decades ago, when early attempts were made to use a scientific approach in the management of organizations. However, the beginning of the activity called *operations research* has generally been attributed to the military services early in World War Ⅱ. Because of the war effect, there was an urgent need to allocate scarce resources to the various military operations and the activities within each operation in an effective manner. Therefore, the British and then the U.S military management called upon a large number of scientists to apply a scientific approach to deal with this and other strategic and tactical problems. In fact, they were asked to do *research* on (military) *operations*. These teams of scientists were the first OR teams. By developing effective methods of the tool of radar, these teams were instrumental in winning the Air Battle of Britain. Through their research on how to better manage convoy and antisubmarine operations, they also played a major role in winning the Battle of the North Atlantic. Similar efforts assisted the Island Campaign in the Pacific.

When the war was ended, the success of OR in the war effort spurred interest in applying OR outside the military. As the industrial boom following the war was running its course, the problems caused by increasing complexity and specialization in organizations were again coming to the forefront. It was becoming apparent to a growing number of people, including business consultants who had served on or with the OR teams during the war, that these were basically the same problems that had been faced by the military but in a different context. By the early 1950s, these individuals had introduced the use of OR to a variety of organizations in business, industry, and government. The rapid spread of OR soon followed.

At least two factors played a key role in the rapid growth of OR during this period. One was the substantial progress that was made early in improving techniques to OR. A prime example is the *simplex method* for solving linear programming problems, which was developed by George Dantzig in 1947. Many of the standard tools of OR, such as linear programming, dynamic programming, game theory, and inventory theory, were relatively well developed before the end of 1950s.

The other factor was the onslaught of the computer revolution, which gave great impetus to the growth of this field. A large amount of computation is usually required to deal most effectively with the complex problems typically considered by OR. Doing this by hand would often be out of the question. Today, a whole range of computers from mainframes to laptops now is being routinely used to solve OR problems.

1.2 Nature of Operations Research

Operations research means **research on operations**. Thus, it is applied to those problems that concern how to conduct and coordinate the operations (or the activities) within an organization. The nature of the organization is essentially immaterial, and in fact, OR has been applied extensively in such diverse areas as economics, management administration, manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, public services, and so on. Therefore, the breadth of its application is wide.

The *research* part of the name implies that OR uses a *scientific method* to study problems. The *scientific method* is adopted to investigate the problem concerned. In particular, the process begins by carefully observing and formulating the problem, including collecting all relevant data. The next step is to establish a scientific (typically mathematical) model that attempts to abstract the essence of the real problem. It is then assumed that the model is sufficiently precise to the essential features of the situation that conclusion (solution) obtained from the model is also valid to the real problem. Next, suitable experiments are taken to test this assumption (which is frequently referred to as

model validation). Thus, it can be said that *operations research* involves creative scientific research into the fundamental properties of operations. But, there is more to it than this. Specifically, OR is also concerned with the practical management of the organization. Therefore, to be successful, OR must also provide positive, understandable conclusions to the decision maker(s) when needed.

Another characteristic of OR is its broad viewpoint. As implied in proceeding, OR takes an organization point of views into account. It tries to resolve the conflicts of interest among the components of the organization in a way that is best (optimal) for the organization as a whole. This does not mean that the study of each problem must give explicit consideration to all aspects of the organization, rather, the objectives being sought must be consistent with those of the overall organization.

In addition, there is another characteristic which is that OR usually attempts to find *a* best solution to the problem under consideration (note, please, that we say *a* best instead of *the* best solution since there may be multiple solutions tied to be best). The goal is to identify a best possible course of action. Although, it must be interpreted carefully in terms of the practical needs of management, this **search for optimality** is really an important theme in OR.

It is evident that no single individual should be expected to be an expert on all the many aspects of OR work or the problems considered. This would require a group of individuals having diverse backgrounds and skills. When the study of a new problem is undertaken, it is necessary to use a *team approach*. Such an OR team typically needs to include those who collectively are highly educated in mathematics, statistics and probability theory, economics, business administration, computer science, engineering and physical science, the behavioral science, and other special techniques.

1.3 Overview of the Operations Research Modeling

This textbook is to devote the mathematical methods of operations research. This is quite appropriate because these quantitative techniques from what is

known about OR are very important. But this does not mean that the practical OR studies are purely mathematical exercises. In fact, the mathematical method often poses a small part of the total effort required only. The target of this section is to summarize the working procedures on OR, which has been successfully formed during resolving a great number of real problems considered. They are:

(1) **Defining the problem of interest** All the objectives, possible constraints, controlling variables, and the related parameters must be clear enough. All relevant data must be gathered.

(2) **Formulating a mathematical model** The relationship between and among the controlling variables, parameters, constraints and objectives must be expressed in a mathematical model which represents the real problem.

(3) **Deriving solution from the model** The solution could be obtained by means of any technique. The solution can be optimal, sub-optimal, or satisfied. It seems to imply that an OR study seeks to just find a solution, which may or may not be required to be optimal. In fact, this usually is not the case. An optimal solution for the original model may be far from ideal to the real problem, so additional analysis is needed. Therefore, **post-optimality analysis** (analysis done after an optimal solution is found) is a very important part of most OR studies.

(4) **Testing model and refine it as needed** The program used to resolve the OR model should be tested. A good way is to take a fresh look at the **overall model** to check for obvious errors or oversights. In addition, whether the solution derived from the model reflects the real problem or not must be examined. If necessary, the model should be refined.

(5) **Implementing the OR model** The OR model and its computer-based program would be applied by related sectors. This phase is critical because it is here, and only here, that the benefits of the study are reaped. Therefore, it is important for an OR team to participate in launching this phase, both to make sure that model solutions are accurately translated to an operating procedure and to rectify any flaws in the solutions that are then uncovered. The OR team must provide active guidance throughout the course of study to related sectors and state what may occur during the model application clear enough.

Surplus Comments In concluding this discussion of the major procedures of

an OR study, note that there usually are a lot of **exceptions**. Indeed, OR requires considerable ingenuity and innovation, so it is impossible to write down any **standard** procedure that is always followed. Rather, the proceeding description may be viewed as a model that roughly represents how successfully OR studies are conducted.