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AMERICAN MATHEMATICAL SOCIETY

An Introduction to Dynamical Systems

Continuous and Discrete Second Edition

连续和离散动力系统引论

第二版

R. Clark Robinson

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图字：01-2016-2521 号

连续和离散动力系统
引论 第二版
Lianxu he Lisan Dongli Xitong
Yinlun

图书在版编目 (CIP) 数据

连续和离散动力系统引论 = An Introduction to Dynamical Systems: Continuous and Discrete : 第二版 : 英文 / (美) R · 克拉克 · 罗宾逊 (R. Clark Robinson) 著 . — 影印本 . — 北京 : 高等教育出版社 , 2017.4
ISBN 978-7-04-047009-3
Ⅰ. ①连… Ⅱ. ①R… Ⅲ. ①连续动力系统—英文②离散动力系统—英文Ⅳ. ①O192 ②O193
中国版本图书馆 CIP 数据核字 (2016) 第 326742 号

策划编辑 李 鹏 责任编辑 李 鹏
封面设计 张申申 责任印制 毛斯璐

出版发行 高等教育出版社
社址 北京市西城区德外大街4号
邮政编码 100120
购书热线 010-58581118
咨询电话 400-810-0598
网址 http://www.hep.edu.cn
http://www.hep.com.cn
网上订购 http://www.hepmall.com.cn
http://www.hepmall.com
http://www.hepmall.cn
印刷:北京新华印刷有限公司

开本 787mm × 1092mm 1/16
印张 47.5
字数 1175 千字
版次 2017 年 4 月第 1 版
印次 2017 年 4 月第 1 次印刷
定价 199.00 元

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出版者的话

近年来，我国的科学技术取得了长足进步，特别是在数学等自然科学基础领域不断涌现出一流的研究成果。与此同时，国内的科研队伍与国外的交流合作也越来越密切，越来越多的科研工作者可以熟练地阅读英文文献，并在国际顶级期刊发表英文学术文章，在国外出版社出版英文学术著作。

然而，在国内阅读海外原版英文图书仍不是非常便捷。一方面，这些原版图书主要集中在科技、教育比较发达的大中城市的大型综合图书馆以及科研院所的资料室中，普通读者借阅不甚容易；另一方面，原版书价格昂贵，动辄上百美元，购买也很不方便。这极大地限制了科技工作者对于国外先进科学技术知识的获取，间接阻碍了我国科技的发展。

高等教育出版社本着植根教育、弘扬学术的宗旨服务我国广大科技和教育工作者，同美国数学会（American Mathematical Society）合作，在征求海内外众多专家学者意见的基础上，精选该学会近年出版的数十种专业著作，组织出版了“美国数学会经典影印系列”丛书。美国数学会创建于 1888 年，是国际上极具影响力的专业学术组织，目前拥有近 30000 会员和 580 余个机构成员，出版图书 3500 多种，冯·诺依曼、莱夫谢茨、陶哲轩等世界级数学大家都是其作者。本影印系列涵盖了代数、几何、分析、方程、拓扑、概率、动力系统等所有主要数学分支以及新近发展的数学主题。

我们希望这套书的出版，能够对国内的科研工作者、教育工作者以及青年学生起到重要的学术引领作用，也希望今后能有更多的海外优秀英文著作被介绍到中国。

高等教育出版社

2016 年 12 月

How many are your works, O Lord!
In wisdom you made them all;
the earth is full of your creatures.
–Psalm 104:24

Preface

Preface to the Second Edition

In the second edition of this book, much of the material has been rewritten to clarify the presentation. It also has provided the opportunity for correcting many minor typographical errors or mistakes. Also, the definition of a chaotic attractor has been changed to include the requirement that the chaotic attractor is transitive. This is the usual definition and it eliminates some attractors that should not be called chaotic. Several new applications are included for systems of differential equations in Part 1. I would encourage readers to email me with suggestions and further corrections that are needed.

R. Clark Robinson
March 2012

Preface to the First Edition

This book is intended for an advanced undergraduate course in dynamical systems or nonlinear ordinary differential equations. There are portions that could be beneficially used for introductory master level courses. The goal is a treatment that gives examples and methods of calculation, at the same time introducing the mathematical concepts involved. Depending on the selection of material covered, an instructor could teach a course from this book that is either strictly an introduction into the concepts, that covers both the concepts on applications, or that is a more theoretically mathematical introduction to dynamical systems. Further elaboration of the variety of uses is presented in the subsequent discussion of the organization of the book.

The assumption is that the student has taken courses on calculus covering both single variable and multivariables, a course on linear algebra, and an introductory course on differential equations. From the multivariable calculus, the material on partial derivatives is used extensively, and in a few places multiple integrals and surface integrals are used. (See Appendix A.1.) Eigenvalues and eigenvectors are the main concepts used from linear algebra, but further topics are listed in Appendix A.3. The material from the standard introductory course on differential equations is used only in Part 1; we assume that students can solve first-order equations by separation of variables, and that they know the form of solutions from second-order scalar equations. Students who have taken an introductory course on differential equations are usually familiar with linear systems with constant coefficients (at least the real-eigenvalue case), but this material is repeated in Chapter 2, where we also introduce the reader to the phase portrait. At Northwestern, some students have taken the course covering part one on differential equations without this introductory course on differential equations; they have been able to understand the new material when they have been willing to do the extra work in a few areas that is required to fill in the missing background. Finally, we have not assumed that the student has had a course on real analysis or advanced calculus. However, it is convenient to use some of the terminology from such a course, so we include an appendix with terminology on continuity and topology. (See Appendix A.)

Organization

This book presents an introduction to the concepts of dynamical systems. It is divided into two parts, which can be treated in either order: The first part treats various aspects of systems of nonlinear ordinary differential equations, and the second part treats those aspects dealing with iteration of a function. Each separate part can be used for a one-quarter course, a one-semester course, a two-quarter course, or possibly even a year course. At Northwestern University, we have courses that spend one quarter on the first part and two quarters on the second part. In a one-quarter course on differential equations, it is difficult to cover the material on chaotic attractors, even skipping many of the applications and proofs at the end of the chapters. A one-semester course on differential equations could also cover selected topics on iteration of functions from Chapters 9–11. In the course on discrete dynamical systems using Part 2, we cover most of the material on iteration of one-dimensional functions (Chapters 9–11) in one quarter. The material on iteration of functions in higher dimensions (Chapters 12–13) certainly depends on the one-dimensional material, but a one-semester course could mix in some of the higher dimensional examples with the treatment of Chapters 9–11. Finally, Chapter 14 on fractals could be treated after Chapter 12. Fractal dimensions could be integrated into the material on chaotic attractors at the end of a course on differential equations. The material on fractal dimensions or iterative function systems could be treated with a course on iteration of one-dimensional functions.

The main concepts are presented in the first sections of each chapter. These sections are followed by a section that presents some applications and then by a section that contains proofs of the more difficult results and more theoretical material. The division of material between these types of sections is somewhat arbitrary. The theorems proved at the end of the chapter are restated with their original theorem number. The material on competitive populations and predator–prey systems is contained in one of the beginning sections of the chapters in which these topics are covered, rather than in the applications at the end of the chapters, because these topics serve to develop the main techniques presented. Also, some proofs are contained in the main sections when they are more computational and serve to make the concepts clearer. Longer and more technical proofs and further theoretical discussion are presented separately at the end of the chapter.

A course that covers the material from the primary sections, without covering the sections at the end of the chapter on applications and more theoretical material, results in a course on the concepts of dynamical systems with some motivation from applications.

The applications provide motivation and illustrate the usefulness of the concepts. None of the material from the sections on applications is necessary for treating the main sections of later chapters. Treating more of this material would result in a more applied emphasis.

Separating the harder proofs allows the instructor to determine the level of theory of the course taught using this book as the text. A more theoretic course could consider most of the proofs at the end of the chapters.

Computer Programs

This book does not explicitly cover aspects of computer programming. However, a few selected problems require computer simulations to produce phase portraits of differential equations or to iterate functions. Sample Maple worksheets, which the students can modify to help with some of the more computational problems, will be available on the webpage:

http://www.math.northwestern.edu/~clark/dyn-sys.

(Other material on corrections and updates of the book will also be available at this website.) There are several books available that treat dynamical systems in the context of Maple or Mathematica: two such books are [**Kul02**] by M. Kulenović and [**Lyn01**] by S. Lynch. The book [**Pol04**] by J. Polking and D. Arnold discusses using Matlab to solve differential equations using packages available at **http://math.rice.edu/~dfield**. The book [**Nus98**] by H. Nusse and J. Yorke comes with its own specialized dynamical systems package.

Acknowledgments

I would like to acknowledge some of the other books I have used to teach this material, since they have influenced my understanding of the material, especially with regard to effective ways to present material. I will not attempt to list more advanced books which have also affected my understanding. For the material on differential equations, I have used the following books: F. Brauer and J. Nohel [**Bra69**], M. Hirsch and S. Smale [**Hir74**], M. Braun [**Bra73**], I. Percival and D. Richards [**Per82**], D.W. Jordan and P. Smith [**Jor87**], J. Hale and H. Koçak [**Hal91**], and S. Strogatz [**Str94**]. For the material on iteration of functions, I have used the following books: the two books by R. Devaney [**Dev89**] and [**Dev92**], D. Gulick [**Gul92**], and K. Alligood, T. Sauer, and J. Yorke [**All97**].

I would also like to thank three professors under whom I studied while a graduate student: Charles Pugh, Morris Hirsch, and Stephen Smale. These people introduced me to the subject of dynamical systems and taught me many of the ideas and methods that I have used throughout my career. Many of my colleagues at Northwestern have also deeply influenced me in different ways: these people include John Franks, Donald Saari, and Robert Williams.

I thank the following reviewers for their comments and useful suggestions for improvement of the manuscript:

John Alongi, Pomona College

Pau Atela, Smith College

Peter Bates, Brigham Young University

Philip Bayly, Washington University

Michael Brin, University of Maryland

Roman Grigoriev, Georgia Technological Institute

Palle Jorgensen, University of Iowa

Randall Pyke, Ryerson University

Joel Robbin, University of Wisconsin

Bjorn Sandstede, Ohio State University

Douglas Shafer, University of North Carolina at Charlotte

Milena Stanislavova, University of Kansas

Franz Tanner, Michigan Technological University

Howard Weiss, Pennsylvania State University

I also thank Miguel Lerma for help in solving various LaTeX and graphics problems, Marian Gidea for help with Adobe Illustrator, and Kamlesh Parwani for help with some Maple worksheets.

I gratefully acknowledge the photograph by Julio Ottino and Paul Swanson used on the cover of the first edition of the book depicting mixing of fluids. This photo had previously appeared in the article [**Ott89b**]. A brief discussion of his research is given in Section 11.5.4.

Most of all, I am grateful to my wife Peggie, who endured, with patience, understanding, and prayer, the ups and downs of writing this book from inception to conclusion.

R. Clark Robinson
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October 2003

Historical Prologue

The theory of differential equations has a long history, beginning with Isaac Newton. From the early Greeks through Copernicus, Kepler, and Galileo, the motions of planets had been described directly in terms of their properties or characteristics, for example, that they moved on approximately elliptical paths (or in combinations of circular motions of different periods and amplitudes). Instead of this approach, Newton described the laws that determine the motion in terms of the forces acting on the planets. The effect of these forces can be expressed by differential equations. The basic law he discovered was that the motion is determined by the gravitational attraction between the bodies, which is proportional to the product of the two masses of the bodies and one over the square of the distance between the bodies. The motion of one planet around a sun obeying these laws can then be shown to lie on an ellipse. The attraction of the other planets could then explain the deviation of the motion of the planet from the elliptic orbit. This program was continued by Euler, Lagrange, Laplace, Legendre, Poisson, Hamilton, Jacobi, Liouville, and others.

By the end of the nineteenth century, researchers realized that many nonlinear equations did not have explicit solutions. Even the case of three masses moving under the laws of Newtonian attraction could exhibit very complicated behavior and an explicit solution was not possible (e.g., the motion of the sun, earth, and moon cannot be given explicitly in terms of known functions). Short term solutions could be given by power series, but these were not useful in determining long-term behavior. Poincaré, working from 1880 to 1910, shifted the focus from finding explicit solutions to discovering geometric properties of solutions. He introduced many of the ideas in specific examples, which we now group together under the heading of chaotic dynamical systems. In particular, he realized that a deterministic system (in which the outside forces are not varying and are not random) can exhibit behavior that is apparently random (i.e., it is chaotic).

In 1898, Hadamard produced a specific example of geodesics on a surface of constant negative curvature which had this property of chaos. G. D. Birkhoff continued the work of Poincaré and found many different types of long-term limiting behavior, including the α- and ω-limit sets introduced in Sections 4.1 and 11.1. His work resulted in the book [**Bir27**] from which the term "dynamical systems" comes.

During the first half of the twentieth century, much work was carried out on nonlinear oscillators, that is, equations modeling a collection of springs (or other physical forces such as electrical forces) for which the restoring force depends nonlinearly on the displacement from equilibrium. The stability of fixed points was studied by several people including Lyapunov. (See Sections 4.5 and 5.3.) The existence of a periodic orbit for certain self-excited systems was discovered by Van der Pol. (See Section 6.3.) Andronov and Pontryagin showed that a system of differential equations was structurally stable near an attracting fixed point, [**And37**] (i.e., the solutions for a small perturbation of the differential equation could be matched with the solutions for the original equations). Other people carried out research on nonlinear differential equations, including Bendixson, Cartwright, Bogoliubov, Krylov, Littlewood, Levinson, and Lefschetz. The types of solutions that could be analyzed were the ones which settled down to either (1) an equilibrium state (no motion), (2) periodic motion (such as the first approximations of the motion of the planets), or (3) quasiperiodic solutions which are combinations of several periodic terms with incommensurate frequencies. See Section 2.2.4. By 1950, Cartwright, Littlewood, and Levinson showed that a certain forced nonlinear oscillator had infinitely many different periods; that is, there were infinitely many different initial conditions for the same system of equations, each of which resulted in periodic motion in which the period was a multiple of the forcing frequency, but different initial conditions had different periods. This example contained a type of complexity not previously seen.

In the 1960s, Stephen Smale returned to using the topological and geometric perspective initiated by Poincaré to understand the properties of differential equations. He wrote a very influential survey article [**Sma67**] in 1967. In particular, Smale's "horseshoe" put the results of Cartwright, Littlewood, and Levinson in a general framework and extended their results to show that they were what was later called chaotic. A group of mathematicians worked in the United States and Europe to flesh out his ideas. At the same time, there was a group of mathematicians in Moscow lead by Anosov and Sinai investigating similar ideas. (Anosov generalized the work of Hadamard to geodesics on negatively curved manifolds with variable curvature.) The word "chaos" itself was introduced by T.Y. Li and J. Yorke in 1975 to designate systems that have aperiodic behavior more complicated than equilibrium, periodic, or quasiperiodic motion. (See [**Li,75**].) A related concept introduced by Ruelle and Takens was a *strange attractor*. It emphasized more the complicated geometry or topology of the attractor in phase space, than the complicated nature of the motion itself. See [**Rue71**]. The theoretical work by these mathematicians supplied many of the ideas and approaches that were later used in more applied situations in physics, celestial mechanics, chemistry, biology, and other fields.

The application of these ideas to physical systems really never stopped. One of these applications, which has been studied since earliest times, is the description and determination of the motion of the planets and stars. The study of the mathematical model for such motion is called *celestial mechanics*, and involves a finite number of bodies moving under the effects of gravitational attraction given by the Newtonian laws. Birkhoff, Siegel, Kolmogorov, Arnold, Moser, Herman, and many others investigated the ideas of stability and found complicated behavior for systems arising in celestial mechanics and other such physical systems, which could be described by what are called *Hamiltonian differential equations.* (These equations preserve energy and can be expressed in terms of partial derivatives of the energy function.) K. Sitnikov in [**Sit60**] introduced a situation in which three masses interacting by Newtonian attraction can exhibit chaotic oscillations. Later, Alekseev showed that this could be understood in terms of a "Smale horseshoe", [**Ale68a**], [**Ale68b**], and [**Ale69**]. The book by Moser, [**Mos73**], made this result available to many researchers and did much to further the applications of horseshoes to other physical situations. In the 1971 paper [**Rue71**] introducing strange attractors, Ruelle and Takens indicated how the ideas in nonlinear dynamics could be used to explain how turbulence developed in fluid flow. Further connections were made to physics, including the periodic doubling route to chaos discovered by Feigenbaum, [**Fei78**], and independently by P. Coullet and C. Tresser, [**Cou78**].

Relating to a completely different physical situation, starting with the work of Belousov and Zhabotinsky in the 1950s, certain mathematical models of chemical reactions that exhibit chaotic behavior were discovered. They discovered some systems of differential equations that not only did not tend to an equilibrium, but also did not even exhibit predictable oscillations. Eventually, this bizarre situation was understood in terms of chaos and strange attractors.

In the early 1920s, A.J. Lotka and V. Volterra independently showed how differential equations could be used to model the interaction of two populations of species, [**Lot25**] and [**Vol31**]. In the early 1970s, May showed how chaotic outcomes could arise in population dynamics. In the monograph [**May75**], he showed how simple nonlinear models could provide "mathematical metaphors for broad classes of phenomena." Starting in the 1970s, applications of nonlinear dynamics to mathematical models in biology have become widespread. The undergraduate books by Murray [**Mur89**] and Taubes [**Tau01**] afford good introductions to biological situations in which both oscillatory and chaotic differential equations arise. The books by Kaplan and Glass [**Kap95**] and Strogatz [**Str94**] include a large number of other applications.

Another phenomenon that has had a great impact on the study of nonlinear differential equations is the use of computers to find numerical solutions. There has certainly been much work done on deriving the most efficient algorithms for carrying out this study. Although we do discuss some of the simplest of these, our focus is more on the use of computer simulations to find the properties of solutions. E. Lorenz made an important contribution in 1963 when he used a computer to study nonlinear equations motivated by the turbulence of motion of the atmosphere. He discovered that a small change in initial conditions leads to very different outcomes in a relatively short time; this property is called *sensitive* *dependence on initial conditions* or, in more common language, the *butterfly effect*. Lorenz used the latter term because he interpreted the phenomenon to mean that a butterfly flapping its wings in Australia today could affect the weather in the United States a month later. We describe more of his work in Chapter 7. It was not until the 1970s that Lorenz's work became known to the more theoretical mathematical community. Since that time, much effort has gone into showing that Lorenz's basic ideas about these equations were correct. Recently, Warwick Tucker has shown, using a computer-assisted proof, that this system not only has sensitive dependence on initial conditions, but also has what is called a "chaotic attractor". (See Chapter 7.) About the same time as Lorenz, Ueda discovered that a periodically forced Van der Pol system (or other nonlinear oscillator) has what is now called a chaotic attractor. Systems of this type are also discussed in Chapter 7. (For a later publication by Ueda, see also [**Ued92**].)

Starting about 1970 and still continuing, there have been many other numerical studies of nonlinear equations using computers. Some of these studies were introduced as simple examples of certain phenomena. (See the discussion of the Rössler Attractor given in Section 7.4.) Others were models for specific situations in science, engineering, or other fields in which nonlinear differential equations are used for modeling. The book [**Enn97**] by Enns and McGuire presents many computer programs for investigation of nonlinear functions and differential equations that arise in physics and other scientific disciplines.

In sum, the last 40 years of the twentieth century saw the growing importance of nonlinearity in describing physical situations. Many of the ideas initiated by Poincaré a century ago are now much better understood in terms of the mathematics involved and the way in which they can be applied. One of the main contributions of the modern theory of dynamical systems to these applied fields has been the idea that erratic and complicated behavior can result from simple situations. Just because the outcome is chaotic, the basic environment does not need to contain stochastic or random perturbations. The simple forces themselves can cause chaotic outcomes.

There are three books of a nontechnical nature that discuss the history of the development of "chaos theory": the best seller *Chaos: Making a New Science* by James Gleick [**Gle87**], *Does God Play Dice?, The Mathematics of Chaos* by Ian Stewart [**Ste89**], and *Celestial Encounters* by Florin Diacu and Philip Holmes [**Dia96**]. Stewart's book puts a greater emphasis on the role of mathematicians in the development of the subject, while Gleick's book stresses the work of researchers making the connections with applications. Thus, the perspective of Stewart's book is closer to the one of this book, but Gleick's book is accessible to a broader audience and is more popular. The book by Diacu and Holmes has a good treatment of Poincaré's contribution and the developments in celestial mechanics up to today.

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