Mathematical Modeling for the MCM/ICM Contests

Volume 3

MCM/ICM数学建模竞赛

第 3 卷

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Foreword by Sol Garfunkel

While it is hard for me to believe, the Mathematical Contest in Modeling (MCM) is fast approaching its 30th year. During this time we have grown from 90 US teams to over 5000 teams representing 25 countries from all across the globe. We have been especially buoyed by the enthusiasm shown by our international colleagues and the rapid growth in international participation. COMAP welcomes your involvement with open arms.

COMAP runs three contests in mathematical modeling; they are MCM, ICM (the Interdisciplinary Contest in Modeling), and HiMCM (the High School Mathematical Contest in Modeling). The purpose of all of these contests has never been simply to reward student efforts — as important as that is. Rather, our objective from the beginning has been to increase the presence of applied mathematics and modeling in education systems at all levels worldwide. Modeling is an attempt to learn how the world works and the use of mathematics can help us produce better models. This is not a job for one country, but for all. The COMAP modeling contests were conceived and evolved to be strong instruments to help achieve this much larger goal.

It is my supreme hope that through this excellent book series students will learn more about COMAP contests and more about the process of mathematical modeling. I hope that you will begin to work on the exciting and important problems you see here, and that you will join the MCM/ICM contests and the rewarding work of increasing the awareness of the importance of mathematical modeling.

Sol Garfunkel, PhD Executive Director COMAP November 2012

Foreword by Chris Arney

Undergraduate students who receive instruction and experiences in mathematical modeling become better and more creative problem solvers and graduate students. This book series is being published to prepare and educate students on the topics and concepts of mathematical modeling to help them establish a problem solving foundation for a successful career.

Mathematical modeling is both a process and a mindset or philosophy. As a process, students need instruction and experience in understanding and using the modeling process or framework. As part of their experience, they need to see various levels of sophistication and complexity, along with various types of mathematical structures (discrete, continuous, liner, nonlinear, deterministic, stochastic, geometric, and analytic). As a mindset, students need to see problems that are relevant, challenging, and interesting so they build a passion for the process and its utility in their lives. A major goal in modeling is for students to want to model problems and find their solutions. Recipes for structured or prescribed problem solving (canned algorithms and formulas) do exist in the real world, but mathematical modelers can do much more than execute recipes or formulas. Modelers are empowered to solve new, open, unsolved problems.

In order to build sufficient experience in modeling, student exposure must begin as early as possible — definitely by the early undergraduate years. Then the modeling process can be reinforced and used throughout their undergraduate program. Since modeling is interdisciplinary, students from all areas of undergraduate study benefit from this experience.

The articles and chapters in this series expose the readers to model construction, model analysis, and modeling as a research tool. All these areas are important and build the students' modeling skills. Modeling is a challenging and advanced skill, but one that is empowering and important in student development. In today's world, models are often complex and require sophisticated computation or simulation to provide solutions or insights into model behavior. Now is an exciting time to be a skilled modeler since methodology to provide visualization and find solutions are more prevalent and more powerful than ever before.

I wish the students well in their adventure into modeling and I likewise wish faculty well as they use the examples and techniques in this book series to teach the modeling process to their students. My advice to all levels of modelers is to build your confidence and skills and use your talents to solve society's most challenging and important problems. Good luck in modeling!

Chris Arney, PhD
United States Military Academy at West Point
Professor of Mathematics
Director of the Interdisciplinary Contest in Modeling
October, 2011

Foreword by Patrick J. Driscoll

Mathematical modeling is a challenging endeavor whether you are new to the task or have been professionally engaged in it for many years. Efficiencies are gained with experience, but never at the cost of assuming a new problem can be solved in the same way with exactly the same ingredients as a previous one. Time and circumstances affect modeling parameters and assumptions, amplifying some and negating others. Simply asking a different question about the exact same situation can introduce intractable elements into a previously well-defined problem.

These facts, among others, mark the participation of MCM teams as extraordinary, no matter what level of award is achieved. The mere fact that undergraduates are willing to commit a solid block of their own time to tackle an unfamiliar problem speaks highly of their motivation to learn and develop. Ah, but the satisfaction of experiencing a model function as intended, and yield significant results pertaining to a line of inquiry — that is what will bring someone back for more. Like the one beautiful golf shot on an otherwise unremarkable round.

A writer friend of mine once said that a novel is a bit like achieving mental telepathy in that an author conveys a scene, an idea, or a plot existing in her mind into the mind of a reader using text or illustrations. If successful, someone on the other side of the world could read the text and see the imagery that the author intended to convey. In many ways, teaching mathematics, and mathematical modeling in particular, is a similar endeavor.

As educators, we use various symbols, diagrams, text, and graphs to present our understanding, our imagery of challenging concepts to students in the hope that an insight gained from one or more of these approaches will become a foothold upon which students can construct their own logical understanding. When students attempt to do this, active learning occurs. When students attain this ownership, a level of understanding occurs that leads to innovation and creativity in mathematics, science, and engineering that is not possible to achieve through other means. Pattern matching and mimicry, while appearing to produce similar results, are

deceptively inadequate to the task.

Since the beginning of MCM, we've published a good deal of recommendations and critique concerning what judges consider to be elements of a proper engagement with mathematical modeling. This advice was primarily intended for teams and institutions new to mathematical modeling and its nuances, recognizing both the important role that a faculty advisor plays in preparing teams to compete and the limited time within which the tasks required by MCM must take place.

Teams participating from institutions with an existing strong commitment to mathematical modeling might use such advice as simply an editorial check prior to submitting their papers. Regardless, the advice was never intended to prescribe a lockstep adherence to a process as a recipe for success, but rather to provide a framework within which MCM teams might initiate their efforts and bring about a respectable level of completion.

There are some things in life that cannot be rushed, no expedients exist that can accelerate them, and while technologies and templates exist to augment and enhance processes associated with these activities, understanding demands time. Mathematical modeling is one of these. And this, we must remember, is what makes the accomplishments of every MCM team so special.

Patrick J. Driscoll, Ph.D United States Military Academy at West Point Professor of Operations Research Director of the Mathematical Contest in Modeling April, 2016

Editors

Dr. Amanda Beecher is Associate Professor of Mathematics at Ramapo College in Mahwah, New Jersey, USA. She received her PhD (2007), MS (2003), and BS (2001) degrees in Mathematics from the University at Albany, State University of New York. She has published research papers in commutative algebra. She taught at West Point for three years, where she began involvement with the MCM/ICM contests. She has been a judge for the ICM since 2008 and written commentaries since this service was offered in 2010. She has written problems for the ICM and been a final judge for an ICM problem since 2013. She has served as Head Judge for the environmental problem since its inception in 2015.

Dr. Jay Belanger is Professor of Mathematics at Truman State University in Kirksville, Missouri, USA. He received his BS degree in Mathematics from the University of Michigan at Ann Arbor and his PhD degree in Mathematics from Princeton University. He has published research papers in complex analysis, computational complexity theory, differential equations and the history of mathematics. He has judged for the MCM contest, including MCM 2016, and co-authored five books in the HEP series on MCM/ICM Contests Guides and Solutions. He has served as a member of the editorial board of the series since 2011.

Dr. Kate Coronges is Executive Director of the Network Science Institute at Northeastern University in Boston, Massachusetts, USA. She was Program Manager for a basic science research program in Social and Cognitive Networks at the Army Research Office, and Assistant Professor of Behavioral Sciences and Leadership at the US Military Academy. She received her PhD in Health Behavior (2009) and MS in Public Health (2005) from the University of Southern California. She was Managing Editor for Connections on International Network of Social Network Analysis for a decade. Her research has focused on social structures and dynamics of teams and communities and their impacts in communication patterns, behaviors

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Dr. Jessica Libertini is Assistant Professor of Applied Mathematics at the Virginia Military Institute in Lexington, Virginia, USA. She received her BS degree in Mechanical Engineering from Johns Hopkins University, her MS degree in Mechanical Engineering from Rensselaer Polytechic Institute, and her PhD degree in Applied Mathematics from Brown University. Prior to entering academia, she was an engineer at General Dynamics for nine years. Her research interests are in mathematical modeling for a wide variety of systems including medical imaging, satellite systems, and food policy. She has also published in mathematics education, including book chapters, articles, and national reports. She has been a judge for several mathematical modeling competitions since 2009, including the ICM. She has authored ICM problems and has been a final judge for the ICM since 2013. She had served as the Head Judge of Problem D for ICM 2016.

Dr. Rodney X. Sturdivant is Associate Professor of Clinical Public Health in the Biostatistics Division of the College of Public Health at the Ohio State University. He assumed this position in 2013 after retiring from the US Army. He was Professor of Applied Statistics at the United States Military Academy (West Point) during the final years of his military service. He holds MS degrees in Statistics and in Operations Research from Stanford and a PhD degree in Biostatistics from the University of Massachusetts Amherst. He has numerous publications, presentations, and workshops primarily in the areas of statistics education and applied

statistics and is co-author of a popular textbook Applied Logistic Regression, 3rd Edition (Wiley, 2013). He has served as a triage and final judge for the ICM for over ten years.

Dr. Jie Wang is Professor of Computer Science at the University of Massachusetts in Lowell. He chaired the Department of Computer Science from 2007 to 2016. He is COMAP's Director for China Partnerships. He has written problems for the MCM and served as a final judge, including MCM 2016 and ICM 2016. He received a BS degree in Computational Mathematics from Sun Yat-sen University, and PhD degree in Computer Science from Sun Yat-sen University, and PhD degree in Computer Science from Boston University. His research interests include big data modeling and technology transfers, text automation, algorithms and computational optimization, and network security. His research has been funded continuously by the National Science Foundation since 1991. IBM, Intel, Google, Wantology, Eola Solutions, Massachusetts Education International, Tianhan Technology, and the Natural Science Foundation of China also funded his research. He has published over 175 journal and conference papers, 12 books and 4 edited books. He is active in professional service, including chairing conference program committees, organizing workshops, and serving as journal editors.

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Preface

This book series is a collection and expositions of the ideas, background knowledge, and modeling methodologies for solving the problems for the Mathematical Contest in Modeling (MCM) and the Interdisciplinary Contest in Modeling (ICM). It is intended to promote, enrich, and advance mathematical modeling education for undergraduate students. It is also intended to provide guidance for students to participate in the MCM/ICM contests. It can be used not only as a reference book in mathematical modeling, but also as supplementary materials for teaching an undergraduate course on modeling. This series grew out of a similar book series on math modelings for the MCM/ICM contests, published in Chinese by the Higher Education Press since 2011.

For an introduction to the MCM/ICM contests and guidelines for preparation and participation of the contests, the reader is referred to the first two chapters of Volume 1 of the series.

This volume is organized as follows: Chapter 1 addresses a common wonder what makes a promising solution an Outstanding Winner. Chapters 2 to 7 comment on, respectively, different solutions to Problems A to F for the MCM/ICM 2016 contests. Problem A asks how to maintain the temperature in a bathtub at a comfortable degree, Problem B asks how to handle space debris caused by man-made objects, Problem C asks how to invest in higher eduction for improving academic performance of undergraduates, Problem D asks how to measure the evolution and influence in society's information networks, Problem E asks how to manage and deliver potable water to people and sustain their needs, and Problem F asks how to use mathematical models to help draw up policies for immigrations of refugees. Each chapter presents the official problem, an overview of the essential elements to fully addressing the underlying problem, the list of Outstanding Winners, an elaboration of different modeling solutions to the problem, comments from the judges, a set of exercises, and references.

Started from Volume 2, each volume includes a chapter discussing math mod-

eling in a cutting-edge research problem. This volume presents in Chapter 8 a current research problem in databases of modeling complex events and patterns in large volume of data streams with missing data.

Each chapter has a lead author, and the lead authors are listed as editors of this volume following the chapter order from Chapters 2 to 8. The complete list of authors for each chapter is listed in the chapter.

I am grateful to all the contributors for their high-quality contributions, which made the final editing feasible. Efforts were made to unify all chapters while retaining contributors' own writing styles. I thank Ying Liu of HEP and Sol Garfunkel of COMAP for their insights, support, and guidance to make this book series a success. The Chinese reader please note that you may check the website http://www.mcmbooks.net for additional information about the books published in Chinese. We welcome and appreciate feedback from our readers. Please email your comments and suggestions to the following address: mcmbooks@gmail.com.

Jie Wang, PhD Editor-in-Chief March 1, 2017