

Graduate Texts in Physics

Carl S. Helrich

The Classical Theory of Fields

Electromagnetism

经典场论

Springer



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Carl S. Helrich

The Classical Theory of Fields

Electromagnetism

With 132 Figures



Springer

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by Carl S. Helrich

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Preface

The study of classical electromagnetic fields is an adventure. The theory is complete mathematically and we are able to present it as an example of classical Newtonian experimental and mathematical philosophy. There is a set of foundational experiments on which most of the theory is constructed. And then there is the bold theoretical proposal of a field-field interaction from James Clerk Maxwell, the validity of which was established in Heinrich Hertz' laboratory.

It is my intention here to present the theory of classical fields as a mathematical structure based solidly on laboratory experiments. I try to introduce the reader – the student – to the beauty of classical field theory as a gem of theoretical physics.

To keep the discussion fluid I placed the history in a beginning chapter and some of the mathematical proofs in the Appendices. Helmholtz' Theorem determines the form that will be taken by the field equations and the way in which we must understand each experiment. To obtain Maxwell's field equations is the goal. If the reader also learns to work through exercises that is good. But that is not the goal. The problems the reader will encounter as a practitioner will require thinking that must be based on a deep understanding of classical field theory.

And so I have tried to obtain Maxwell's Equations as soon as possible. I have not been completely successful because of my concerns about the reader's mathematical development. I felt compelled to include a rather extensive chapter on mathematical background for readers unfamiliar with some of the language. I have also included chapters on Green's Functions and Laplace's Equation between the static form of Maxwell's Equations and a discussion of Faraday's Experiment. These may be avoided by the reader already fluent in the mathematics.

The chapter on Einstein's relativity is an integral necessity to the text. This chapter is historically accurate and fairly complete for the level of the text. My treatment is based on original papers by Einstein, Hendrik A. Lorentz, and Hermann Minkowski, on the excellent historical analysis of Abraham Pais, and on some more modern treatments such as Wolfgang Pauli's and Wolfgang Rindler's. My goal is to demonstrate the covariance of Maxwell's Equations and to present the transformation theory, while not losing sight of the "step" that had been introduced. I do not suggest ignoring this chapter. It is good for the physicist's or engineer's

soul to know about this step. But it is not absolutely required for much of the use to which a practitioner will put field theory.

I have tried to be honest with the reader about our microscopic picture of matter. I avoid quantum mechanical descriptions, but not the fact that these lie behind our treatment of matter.

Our models of plasmas provide a good testing ground for electrodynamic theory that does not require quantum mechanics. I have used this at points in the text. This has been my guide in the chapter on particle motion and in my final chapter on waves in a dispersive medium.

My discussions of particle motion are based on Hamiltonian mechanics, which I outline. This results in a symmetry, as well as simplicity in the equations of motion. My treatment of magnetic mirrors relies on numerical solutions, which are simplified by the canonical equations. And I have based my discussion of coherent particle motion verbally on what is known of the dynamics of plasmas.

I have not intended this treatment to be exhaustive. The topics I have chosen reflect my interests as well as what I felt my own education lacked. I will, probably, readily agree with any criticism claiming that I have missed an indispensable topic. I do, however, believe that after finishing this text the reader should be able to encounter that topic with confidence.

I am grateful to generations of students who have helped in the development of my course in classical field theory. Their patience and enthusiasm has been an inspiration.

I am also grateful to my teachers and the directors of programs in which I have been involved. Among these I particularly want to acknowledge Leslie Foldy, David Mintzer, Marvin Lewis, and Günter Ecker. From each of these people I have learned to be thorough, unrelenting, and even confident. The first three of these people were inspiring teachers, Lewis was my doctoral mentor, and Ecker was my director in Jülich.

I have discussed modern plasma theory, of which I am no longer a part, extensively with my friend Wei-li Lee of the Princeton Plasma Physics Laboratory. Lee contributed directly to my discussions of gyrokinetic theory and its application to magnetically confined fusion plasmas.

I am grateful for the patience and understanding of my wife, Betty Jane, who has endured more than I could have expected as I wrote this, and who remained a constant source of encouragement.

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Contents

1	Origins and Concepts	1
1.1	Introduction	1
1.2	Magnetism	2
1.3	Gravitation	3
1.4	Faraday, Thomson, and Maxwell	3
1.5	Gravitation a Vector Field	4
1.6	Charges and Electric Fields	5
1.7	Priestly's Speculation	6
1.8	Voltaic Cell	7
1.9	Currents and Magnetic Fields	8
1.9.1	Oersted	8
1.9.2	Ampère	10
1.9.3	Electrical current	11
1.10	Induced Electric Field	12
1.11	The Mathematical Theory	13
1.11.1	The field equations	13
1.11.2	Maxwell	15
1.12	Experimental Evidence	20
1.12.1	Waves in the laboratory	20
1.12.2	Wave energy and momentum	23
1.13	Michelson and Morley Experiment	24
1.14	Relativity	27
1.15	Summary	29
	Questions	30
2	Mathematical Background	33
2.1	Introduction	33
2.2	Vectors	34
2.2.1	The Vector Space	34
2.2.2	Representation	36
2.2.3	Scalar Product	39
2.2.4	Vector Product	40

2.3	Multivariate Functions	41
2.3.1	Differentials	42
2.3.2	Cylindrical Coordinates	43
2.3.3	Spherical Coordinates	44
2.4	Analytic Functions	45
2.4.1	Taylor Series	45
2.4.2	Analyticity	46
2.5	Vector Calculus	46
2.5.1	Field Quantities	46
2.5.2	The Gradient	47
2.5.3	The Divergence	48
2.5.4	The Curl	54
2.5.5	The Laplacian Operator	61
2.6	Differential Equations	62
2.6.1	Helmholtz' Theorem	64
2.6.2	The Del Operator	65
2.6.3	Dirac Delta Function	66
2.7	Summary	71
	Exercises	72
3	Electrostatics	79
3.1	Introduction	79
3.2	Coulomb's Law	79
3.2.1	Coulomb's Experiment	79
3.2.2	Units	81
3.3	Superposition	81
3.4	Distributions of Charges	82
3.4.1	Distribution of Point Charges	83
3.4.2	Volume Charge Density	84
3.4.3	Surface Charge Density	85
3.5	The Field Concept	87
3.6	Divergence and Curl of $\mathbf{E}$	89
3.7	Integral Electrostatic Field Equations	91
3.7.1	Gauss' Theorem	92
3.7.2	Stokes' Theorem	93
3.8	Summary	93
	Exercises	94
4	The Scalar Potential	99
4.1	Introduction	99
4.2	Potential Energy	99
4.3	Potential Surfaces	101
4.4	Poisson's Equation	101
4.5	Multipole Expansion	108
4.6	Energy Storage	110
4.6.1	Electrostatic Energy Density	110
4.6.2	Energy of a Set of Conductors	111

4.7	Summary	116
	Exercises	117
5	Magnetostatics	123
5.1	Introduction	123
5.2	Current	124
5.2.1	Current Density	124
5.2.2	Charge Conservation	125
5.3	Oersted's Experiment	126
5.4	Ampère's Experiment	129
5.4.1	Direction of the Force	130
5.4.2	The Constant	131
5.5	Consequences of Ampère's Experiment	132
5.5.1	Force on a Charge	132
5.5.2	Field from a Straight Wire	133
5.5.3	Biot-Savart Law	134
5.6	Superposition	136
5.7	Multipole Expansion	138
5.8	Divergence and Curl of $\mathbf{B}$	141
5.9	Gauge Transformation	142
5.10	The Static Field Equations	143
5.11	Summary	145
	Exercises	145
6	Applications of Magnetostatics	149
6.1	Introduction	149
6.2	Biot-Savart Law	149
6.3	Vector Potential	151
6.4	Summary	159
	Exercises	159
7	Particle Motion	165
7.1	Introduction	165
7.2	Analytical Mechanics	165
7.2.1	Euler-Lagrange Formulation	165
7.2.2	Hamiltonian Formulation	166
7.3	Electrodynamics	167
7.3.1	The Lagrangian	167
7.3.2	The Hamiltonian	169
7.4	Particle Motion	169
7.4.1	Magnetic Fields	169
7.4.2	Electric and Magnetic Fields	176
7.5	Plasmas	180
7.6	Summary	182
	Exercises	182

8	Green's Functions	187
8.1	Introduction	187
8.2	General Formulation	188
8.3	Poisson's Equation	189
8.4	Green's Function in One Dimension	190
8.5	Vector Potential	200
8.6	Summary	200
	Exercises	201
9	Laplace's Equation	205
9.1	Introduction	205
9.2	Forms of Laplace's Equation	206
9.3	Rectangular Coordinates	207
9.3.1	Eigenvalue Problems	207
9.4	Cylindrical Coordinates	211
9.5	Spherical Coordinates	214
9.6	Summary	219
10	Time Dependence	221
10.1	Introduction	221
10.2	Faraday's Law	221
10.3	Displacement Current	224
10.4	Magnetostatic Energy	226
10.5	Maxwell's Equations	228
10.6	Summary	230
	Exercises	230
11	Electromagnetic Waves	239
11.1	Introduction	239
11.2	Wave Equations	240
11.3	Plane Waves	241
11.4	Plane Wave Structure	244
11.4.1	Polarization	246
11.5	General Wave Solutions	249
11.5.1	Spread of Waves	249
11.5.2	Representation in Plane Waves	250
11.5.3	Fourier Transform	250
11.6	Fourier Transformed Equations	254
11.7	Scalar and Vector Potentials	255
11.8	Summary	256
	Exercises	257
12	Energy and Momentum	261
12.1	Introduction	261
12.2	Transport Theorem	262
12.3	Electromagnetic Field Energy	263
12.4	Electromagnetic Field Momentum	266

12.5	Static Field Energies	269
12.6	Summary	270
	Exercises	270
13	Special Relativity	273
13.1	Introduction	273
13.2	The New Kinematics	274
13.2.1	Time	275
13.2.2	Space	278
13.2.3	Lorentz Transformation	280
13.3	Minkowski Space-Time	280
13.3.1	Four Dimensions	280
13.3.2	Four Vectors	282
13.3.3	The Minkowski Axiom	282
13.3.4	The Light Cone	283
13.4	Formal Lorentz Transform	284
13.5	Time and Space	285
13.5.1	Time Dilation	286
13.5.2	Space Contraction	287
13.5.3	Velocities	289
13.6	Tensors	291
13.7	Metric Space	293
13.8	Four-Velocity	294
13.9	Mass, Momentum, and Energy	295
13.9.1	Mass	295
13.9.2	Four-Momentum	296
13.9.3	Energy	297
13.10	Electrodynamics	300
13.10.1	Field Equations	300
13.10.2	Derivatives	300
13.10.3	Current and Potential Vectors	302
13.10.4	Electrodynamic Covariance	303
13.10.5	Field Strength Tensor	304
13.11	Moving Charges	306
13.12	Summary	311
	Exercises	312
14	Radiation	317
14.1	Introduction	317
14.2	Waves from Sources	317
14.3	Liénard–Wiechert Potentials	322
14.4	Plane Waves	326
14.5	Sources	327
14.5.1	Dipole Radiation	328
14.5.2	Charge in a Magnetic Field	330
14.6	Summary	331
	Exercises	332

15	Fields in Matter	335
15.1	Introduction	335
15.2	Experiments	336
15.2.1	Dielectrics in Capacitors	336
15.2.2	Solid Dielectrics	337
15.2.3	Magnetic Cores in Inductors	339
15.2.4	Magnetism in Solids	340
15.3	Potentials from Slowly Varying Fields	342
15.3.1	Atoms and Multipole Expansions	342
15.3.2	Polarization and Magnetization Densities	345
15.3.3	Polarization Charges and Magnetization Currents	347
15.4	Interaction of Fields	349
15.5	Maxwell's Equations in Matter	350
15.6	Constitutive Equations	351
15.6.1	Polarization	351
15.6.2	Magnetization	351
15.6.3	Permittivity and Permeability	352
15.7	Boundary Conditions on Fields	352
15.7.1	Electric Field	353
15.7.2	Magnetic Field	355
15.8	Ferromagnetism	357
15.8.1	Hysteresis	360
15.8.2	Modern Directions	361
15.9	Summary	361
	Exercises	362
16	Waves in Dispersive Media	373
16.1	Introduction	373
16.2	Waves in Matter	374
16.2.1	Representation of Waves	374
16.2.2	Dispersion Relation in Matter	375
16.2.3	Transverse and Longitudinal Waves	376
16.2.4	Wave Conductivity	377
16.2.5	Wave Energy	377
16.3	Nearly Monochromatic Waves	378
16.3.1	Dispersion of Monochromatic Waves	378
16.3.2	Time and Space Averages	379
16.3.3	Field Energy	381
16.3.4	Particle Energy	385
16.4	Note on Group Velocity	387
16.5	Application	389
16.6	Summary	391
	Exercises	392

Appendix A Vector Calculus	393
A.1 Differential Operators.....	393
A.2 Differential Operator Identities.....	394
Appendix B Dirac Delta Sequences	395
Appendix C Divergence and Curl of B	397
Appendix D Green's Theorem	399
Appendix E Laplace's Equation	401
Appendix F Poisson's Equation	407
Appendix G Helmholtz' Equation	409
Appendix H Legendre's Equation	413
Appendix I Jacobians	417
Appendix J Dispersion	423
Appendix K Answers to Selected Exercises	429
References	435
Index	439

Chapter 1

Origins and Concepts

Gravity must be caused by an Agent acting constantly according to certain Laws; but whether this agent be material or immaterial, I have left to the Consideration of my Readers.

Isaac Newton

The field concept is the product of a highly original mind, a mind which never got stuck on formulas.

Albert Einstein

1.1 Introduction

Classical field theory possesses a striking beauty in part because it comes to us as a complete theory in which we can tie almost each law and concept directly to a single experiment. Except for the qualifying “almost” we can present the subject as based on hard laboratory data and a very limited number of guiding ideas. But there is too much human thought that lies behind the word “almost” for us to drop it in our pursuit of understanding.

The history of science is a history of human thought. Because there is no simple understanding of the origin of ideas there can be no simple understanding of the development of any branch of science. And our understanding of a branch of science is linked to how well we comprehend the origin of the ideas on which it rests.

Classical field theory began as a logical extension of what we already knew from Isaac Newton’s experimental and mathematical philosophy [79, pp. 214–215] and the mechanics that resulted. Newton, however, knew that his law of universal gravitation begged an explanation that lay beyond the experimental data.

Michael Faraday (1791–1867), whom we acknowledge as the greatest among experimentalists, imagined that lines of force permeated space and were responsible for electric and magnetic phenomena. This was heresy. But it was believed by William Thomson (1824–1907) and James Clerk Maxwell (1831–1879). Maxwell’s

mathematical development of Faraday's ideas, and the conviction that light waves must emerge from the theory, will bring us to a point where theory presses experiment.

Heinrich Hertz (1857–1894), in the laboratory in Karlsruhe, carried out the experiments that identified propagating electromagnetic waves. But this alone did not reveal the full truth, as Hertz knew.

We must also encounter the crisis in scientific thought that marked the beginning of the twentieth century. The answers presented by Albert Einstein in 1905 required a revision of the bedrock of Newtonian thought: the concept of time and consequently of space.

Classical field theory will then bring us new ideas that we could not have anticipated. In this chapter we will trace the twisted historical path with the intention of seeing the origin of ideas and the consequences. To fully understand classical field theory as a product of human thought we must encounter these origins and consequences.

We will have many occasions to reference this chapter.

1.2 Magnetism

In 1849 Thomson first used the term *field of force*, or simply *field*, in reference to magnetic effects [18, p. 146]. He was providing new words for Faraday's idea of lines of force. He was also, at least in part, giving words to the same phenomenon that had such an impression on Einstein when he was four years of age. Einstein had marveled at the fact that a compass needle responded to a magnet although there was nothing between the needle and the magnet [65, p. 3].

Faraday was a mature scientist in 1849. His lines of force gave expression to a conviction that we should not simply accept a description in the language of action at a distance. In Faraday's mind the space surrounding a magnet or an electrical charge was not empty. It was penetrated by magnetic or electric lines of force. These lines of force are responsible for what we experience when we bring the like poles of two permanent magnets close to one another. But Faraday also believed that such lines of force were present around any mass and were the source of the gravitational force [18, p. 147].

In a lecture he gave at the Royal Institution in 1834, three years after his discovery that electric currents result from variations in magnetism, Faraday claimed that, "We cannot say that any one is the cause of the others, but only that they are connected and due to a common cause" [18, p. 145].

In 1845 Faraday found the effect of magnetism on the polarization of light. In the paper reporting this effect he stated his conviction that the various forms of the forces of matter have one common origin. This was a great scientist and a very careful experimentalist speaking out of his experience. Faraday saw a universe permeated by fields where the nineteenth century theoreticians saw Newtonian particles and action at a distance.