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Third Edition

With a Foreword by Robert Fefferman

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Foreword

Robert A. Fefferman, University of Chicago

Surely, Antoni Zygmund's *Trigonometric Series* has been, and continues to be, one of the most influential books in the history of mathematical analysis. Therefore, the current printing, which ensures the future availability of this work to the mathematical public, is an event of major importance. Its tremendous longevity is a testimony to its depth and clarity. Generations of mathematicians from Hardy and Littlewood to recent classes of graduate students specializing in analysis have viewed *Trigonometric Series* with enormous admiration and have profited greatly from reading it. In light of the magnitude of Antoni Zygmund as a mathematician and of the impact of *Trigonometric Series*, it is only fitting that a brief discussion of his life and mathematics accompany the present volume, and this is what I have attempted to give here.[1] I can only hope that it provides at least a small glimpse into the story of this masterpiece and of the man who produced it.

Antoni Zygmund was born on December 26, 1900 in Warsaw, Poland. His parents had received relatively little education, and were of modest means, so that his background was far less privileged than that of the vast majority of his colleagues. Zygmund attended school through the middle of high school in Warsaw. When World War I broke out, his family was evacuated to Poltava in the Ukraine, where he continued his studies. When the war ended in 1918, his family returned to Warsaw, where he completed pre-collegiate work, and entered Warsaw University. Zygmund's main interest throughout his childhood was astronomy, but at Warsaw University at that time there were not sufficient courses offered in that subject to make it realistic as a specialization, and so Zygmund turned instead toward another of his interests, mathematics.

There were a number of excellent mathematicians and teachers who profoundly influenced Zygmund during this period. The greatest impact came from Aleksander Rajchman, and Stanislaw Saks. Rajchman was a junior faculty member who was an expert on Riemann's theory of trigonometric series, and Saks a fellow student who was a few years his senior. From Rajchman, he learned much of the Riemann theory, and his doctoral thesis in 1923 was on this subject. Zygmund became an active collaborator with both Rajchman and Saks, publishing a number of important articles with each of them. In

[1] I have been fortunate to have a number of excellent references to consult regarding the life of Antoni Zygmund. The reader interested in additional material may consult the references in the Bibliography.

addition, Saks and Zygmund produced *Analytic Functions*, one of the classic texts on complex analysis.

One year prior to his PhD, Zygmund was appointed to an instructorship at the Warsaw Polytchnical School, and, in 1926, he was appointed Dozent at the University of Warsaw. Awarded a Rockefeller fellowship, Zygmund used this to travel to England for the academic year of 1929–30 and visit for the first half of this year with G.H. Hardy at Oxford, and for the second half with J.E. Littlewood at Cambridge. This experience had a tremendous impact on the young Zygmund. Not only did he work with two of the greatest analysts of the time, but while in England, he also met another young mathematician, R.E.A.C. Paley, a student of Littlewood, with whom he had an extended and very fruitful collaboration. When he returned to Poland in 1930, Zygmund moved to Wilno where he took a chair in mathematics at the Stefan Batory University. It was here that Zygmund's talent and quiet charisma as a teacher of advanced mathematics began to have a very major impact on the field. In the fall of 1930, Zygmund met a new student at the University, Jozef Marcinkiewicz. Marcinkiewicz was recognized, even as a student of calculus, as being tremendously talented with the potential to become a serious mathematician. In the following year, which was only the second at Stefan Batory for both teacher and student, Zygmund decided to offer a course on trigonometric series preceded by lectures on Lebesgue integration. Marcinkiewicz attended this course, and thus began his association with Zygmund. It took just three years for Marcinkiewicz to obtain his Master's degree, with a thesis that contained the highly non-trivial result that it is possible for a continuous periodic function to have interpolating polynomials corresponding to equidistant nodal points diverging almost everywhere. This result was elaborated to form his PhD thesis in 1935, and in 1937, Marcinkiewicz became a Dozent in Wilno. In the period from 1935 to 1939, a collaboration between Marcinkiewicz and Zygmund developed that was incredibly successful. Though of relatively short duration, their work opened a number of new directions, and in a sense set the stage for the theory of singular integrals which would be Antoni Zygmund's greatest contribution.

The years in which Zygmund was a young professor in Wilno, though very productive mathematically, were not easy ones. This was due in large part to Zygmund's courageous opposition to the bigotry which was all too common around him, and which was supported by the higher authorities. An example of this was his strong dislike of anti-Semitic policies within his university. At one time, for instance, student organizations, somewhat analogous to modern day fraternities, were sufficiently influential to mandate that all Jewish students must sit on the left side of each classroom during lectures. For Zygmund, this was completely unacceptable, and in response, he decided to move his classes from the larger halls to small mathematics department seminar rooms where there were only long tables in a central arrangement, and hence no seats at the left or right of the room. Another illustration of Zygmund's sensitivity to issues of social justice had to do with his university's requirement that all student associations have faculty members as their academic sponsors. Zygmund regularly sponsored associations which were not in favor with the Polish government. These unpopular moves on Zygmund's part did not go unnoticed, and in the year 1931, as part of the political purges of the universities by the government, Zygmund was dismissed from his professorship. This immediately brought extremely strong reaction from some of the most distinguished mathematicians in Europe. From Lebesgue in France, and from Hardy and Littlewood in England came formal written protests which resulted in Zygmund's reinstatement as professor. It is therefore an important aspect of Zygmund's life that in a very real sense, he was a crusader for human rights well before this was fashionable.

Among the many remarkable contributions of the Wilno period is the production of the first version of this book, published in Warsaw under the title, *Trigonometric Series*. This was Zygmund's first book, and it was published as volume V of the series Monografie Matematyczne. This is the same series that the celebrated book, *Theorie des Operations Lineaires*, by S. Banach appears as volume I. The tremendous success of *Trigonometric Series* led to its expansion and revision into a second edition, published in 1959 by Cambridge University Press, and then to no fewer than six reprinted versions after that.

The time in Wilno which had featured the rapid achievement of success in so many different ways came to a sudden end in September of 1939 as World War 2 erupted. At that time, both Zygmund and Marcinkiewicz were mobilized as reserve officers in the Polish army, and, as a result of the temporary 'friendship' between Germany and Russia, Poland was partitioned. The Soviets were given control of much of the country, including the part containing Wilno, and they proceeded to round up and execute many of the Polish officer corps in the Katyn Forest massacre. Most likely this is how Marcinkiewicz perished. Almost by a miracle, Zygmund was able to return to his family and escape with them to the United States, but his loss was absolutely devastating. His principal collaborators up to that time besides Marcinkiewicz had been Saks, Rajchman and Paley. Both Saks and Rajchman were murdered by the Nazis, and Paley had died in a tragic accident in 1933. These losses were not just mathematical. Zygmund had been extremely close to each of them, and so the war period must surely have been one of the most difficult of his life.

By 1939, Zygmund had an international reputation, and many friends all over the mathematical world. Due to the efforts of some of these friends, such as Jacob Tamarkin, Jerzy Neyman, and Norbert Wiener, Zygmund was able to emigrate to the United States in 1940. During the time immediately prior to the entrance by the United States into the war, there were very few jobs available to mathematicians. Nevertheless, after teaching for a semester at MIT, Zygmund was offered and accepted a position at Mount Holyoke College in central Massachusetts. A few years later, other offers followed. In 1945, Zygmund became a professor at the University of Pennsylvania, and then, in 1947 he was offered a professorship at the University of Chicago.

The University of Chicago mathematics department, which had a tradition of great strength, experienced a period of decline prior to World War 2. During the war, the president of the university, Robert Maynard Hutchins, brought the Manhattan project to the campus, and with it came a number of outstanding scientists, such as Enrico Fermi. Hutchins decided to make it a priority to strengthen the mathematics department in order to match the high quality of physical science appointments that had been made. To this end, a new chairman, Marshall Stone, was brought to the University and asked to bring about this improvement. The result was something phenomenal. In the period just after the war, Stone was able to assemble one of the best mathematics departments in history. At this time, the faculty of mathematics included such members as A.A. Albert, S.S. Chern, L. Graves, S. MacLane, M. Stone, A. Weil, I. Kaplansky, A. Zygmund, P. Halmos, I. Segal, and E. Spanier. Together with this influx of great mathematicians there came a corresponding one of brilliant students.

The combination of such a strong mathematician and teacher as Zygmund with the unusually rich mathematical environment of the University of Chicago produced a golden period of creativity and of supervision of exceptional students for Zygmund that was the crowning achievement of his life's work. In several cases, the route of outstanding students to Chicago was not totally straightforward, and the most famous case was that of Alberto P. Calderon. The story of the means by which Calderon came to Chicago is legendary. The following, taken from the introduction to the book, *Essays in Honor of Alberto P. Calderon* [2] tells the story beautifully:

> In the years immediately after World War 2, the U.S. Department of State had a very active visitor's program that sent prominent scientists to Latin America. Thus, Adrian Albert, Marshall Stone, and George Birkhoff visited Buenos Aires, and Gonzalez Dominguez arranged through them the visit of Zygmund, whose work on Fourier Series he much admired. At the Institute of Mathematics, Zygmund gave a two-month seminar on topics in analysis, based on his book. This seminar was attended by Gonzalez Dominguez, Calderon, Mischa Cotlar, and three other young Argentine mathematicians. Each of the participants had to discuss a portion of the text. Calderon's assignment was to present the Marcel Riesz theorem on the continuity of the Hilbert transform in L^p. According to Cotlar's vivid recollection of the event, Calderon's exposition was entirely acceptable to the junior audience, but not to Zygmund, who appeared agitated and grimaced all the time. Finally, he interrupted Calderon abruptly to ask where had read the material he was presenting, and a bewildered Calderon answered that he had read it in Zygmund's book. Zygmund vehemently informed the audience that this was not the proof in his book, and after the lecture took Calderon aside and quizzed him about the new short and elegant proof. Calderon confessed that he had first tried to prove the theorem by himself, and then thinking he could not do it, had read the beginning of the proof in the book; but after the first couple of lines, instead of turning the page, had figured out

how the proof would finish. In fact, he had found himself an elegant new proof of the Riesz theorem! Zygmund immediately recognized Calderon's power and then and there decided to invite him to Chicago to study with him.

This anecdote illustrates one of Calderon's main characteristics ...

The anecdote above also illustrates one of Zygmund's main characteristics: his tremendous desire to work with people of the greatest mathematical ability, and his absolute devotion to them. Calderon came to the University of Chicago in 1949 on a Rockefeller fellowship, and only one year later received his PhD there under Zygmund's supervision. The thesis consisted of three research papers, each of which was a major work. In particular, among the results of the thesis was one of the greatest importance, concerning the boundary behavior of harmonic functions of several variables, which represented a crucial step in carrying out the real variable program of Zygmund which will be described below. The collaboration between Calderon and Zygmund which followed was certainly one of the greatest in the history of modern analysis, and created a theory, the so-called Calderon–Zygmund Theory of Singular Integrals, that not only allowed for the extension of much of classical Fourier analysis from one to several dimensions, but played a fundamental role in the development of the theories of partial differential equations and geometry as well.

More than simply creating a new powerful mathematical theory, at Chicago, Zygmund created a school, the Chicago School of Analysis, which was to have an enormous impact on the subject in the next five decades, and promises to continue to do so in the future. After Calderon, there came other students who worked with Zygmund and who individually made historical level contributions to mathematics. In 1955, Elias M.Stein received his doctorate under Zygmund, and, as is well known, by his brilliant research and teaching went on to establish a great school of his own at Princeton. A bit later, other remarkable students finished their thesis work with Zygmund, including Paul Cohen and Guido and Mary Weiss. Taking into account the generations of students whose mathematical ancestry is traceable back to Zygmund, it is almost hard to imagine what mathematical analysis would be like without their collective contribution.

At Chicago, Zygmund had a total of thirty-five students. His collected works include some 215 articles. Zygmund received many formal honors in his lifetime. He was a recipient of the Steele Prize of the American Mathematical Society, as well as the National Medal of Science, the highest award given by the United States government in recognition of scientific achievement. In addition, he was given membership in a number of academies, including the National Academy of Sciences and American Academy for Arts and Sciences (USA), the Polish Academy of Sciences, the Argentina Academy of Sciences, the Royal Academy of Sciences of Spain, and the Academy of Arts and Sciences of Palermo, Italy. Zygmund also held honorary degrees from Washington University, The University of Torun, Poland, The University of Paris, and the University of Uppsala, Sweden.

After a very long and productive life in which he published his last research article at the age of 79, he finally slowed considerably, and, following a long illness, died at the age of 91. Few mathematicians have provided such a striking and wonderful counterexample to G.H. Hardy's view on the rapidity of loss of creativity that mathematicians suffer with age.

Zygmund's life events and his mathematics, particularly that covered in the present volume, are heavily intertwined. In what follows, I would like to discuss this mathematics in the context of the historical perspective considered above.

That historical perspective on Zygmund's career begins with his interaction with Rajchman. One sees the influence of Rajchman clearly reflected in the beautiful treatment of Riemann's theory of trigonometric series (which are not necessarily Fourier series) in Chapter 9. Here the main theorems concern questions of uniqueness and localization for such series. It is far from trivial that if a trigonometric series converges to zero everywhere, then the series must be the trivial series whose terms are each zero. The proof of this result involves formally integrating the series term-by -term twice to produce an absolutely convergent series, and then investigating a certain appropriate notion of generalized second derivative applied to this twice integrated series. The twice integrated series, which was first considered by Riemann, is a key to understanding much of the theory of (non-Fourier) trigonometric series. Another fundamental result is that of localization: the behavior of a series at a given point depends only on the values of this function (gotten by twice term-by-term integrating the series) in an arbitrarily small interval around this point. The approach Zygmund takes to establish this localization is via the notion of formal multiplication of trigonometric series, which is due to Rajchman. Of course, knowing that a trigonometric series that converges to zero everywhere must be trivial leads to the question of which sets E have the following property: any trigonometric series which converges to zero outside E necessarily is the trivial series. Such sets are called sets of uniqueness, and the analysis of whether or not a given set is one such is highly non-trivial. Because these sets are all of Lebesgue measure zero, their analysis is particularly delicate involving such issues as the number-theoretic structure of the set. Zygmund made major contributions to this area as well. The important result that the countable union of closed sets of uniqueness is a set of uniqueness is referred to in this volume as a theorem of N. Bary: however the result is also associated with Zygmund (see [3] for the interesting history of this).

Next, the influence of Saks and particularly of Marcinkiewicz is also apparent in *Trigonometric Series*. One of the most important contributions of Zygmund was the realization that operators may behave differently according to the group of dilations under which they are invariant. One of the initial examples of this was discovered by Saks who had shown that the classical Lebesgue theorem on differentiation of the integral for integrable functions was not valid if instead of balls in n-dimensional space, one considered averaging integrable functions over rectangles with sides parallel to the axes. It was then shown by Zygmund that one did have the corresponding maximal function, the so-called Strong Maximal Function, bounded on L^p, for all $p > 1$ (and hence the strong differentiation of the integral for functions in such L^p). In a fundamental article published in 1935, the sharp results on these maximal operators were given by Jessen, Marcinkiewicz and Zygmund. The result from this paper and its consequences for Fourier series are reproduced in Chapter 17. It was Zygmund who fully realized the role of product dilations in the theory, and who considered other dilations, formulating conjectures some of which are presently still unsolved. The corresponding product theory of maximal functions and singular integrals has been thoroughly understood, and its generalizations to other dilation groups introduced by Zygmund have played a significant role in applications, such as to the analysis of weakly pseudo-convex domains in several complex variables (see for example Stein [5].

Marcinkiewicz's influence appears in several parts of this book, and there can be no doubt that it played an absolutely fundamental role in the theory of singular integrals in Euclidean space that was Zygmund's most important contribution. It is extremely difficult to imagine the program of singular integrals without the Marcinkiewicz Interpolation Theorem of Chapter 12, because, as is now well known, the study of singular integrals proceeds by examining their action on L^1 where they are not bounded, but merely of weak type. Furthermore, the important Marcinkiewicz Integral, which also plays a crucial role in the analysis of singular integrals on L^1, appears in Chapter 4, where it is part of a complete real variables approach to the L^p theory of the Hilbert transform, using F. Riesz's Rising Sun Lemma (replaced by the Calderon–Zygmund Decomposition in the Calderon–Zygmund higher-dimensional theory). The presentation of the Marcinkiewicz integral as applied here to the Hilbert transform gives the reader a beautiful preview of the Calderon–Zygmund Theory, and as such is a real highlight of this book. Finally, we should mention that the Marcinkiewicz Multiplier Theorem is included, in connection with applications of Littlewood–Paley Theory to Fourier series, and this theorem like the real variable approach to the Hilbert transform certainly has a flavor of more recent developments which came as a consequence of the program of Calderon and Zygmund.

As mentioned above, just prior to the Wilno period during which Zygmund met his student Marcinkiewicz, he traveled to England to work with Hardy and Littlewood, and, at the same time met Paley. It is probably not possible to overstate the importance of the impact of these two giants of analysis and their brilliant student on Zygmund. This is very clearly reflected in this volume. In the first place, the Hardy–Littlewood Maximal Theorem is quite properly given great emphasis, and introduced quite early in the book (page 29). Zygmund felt that the Maximal Theorem was of the greatest importance, and clearly communicated this in both his mathematics and his work with students. Although Hardy and Littlewood invented the idea, it is only fair to give Zygmund and his students such as Calderon and Stein much credit for realizing its pervasive role in analysis. The theory of Hardy Spaces is an early manifestation of the impact of maximal functions on singular integrals, a story told in entirety only several decades later with the work of Stein and Weiss and then of Charles Fefferman and Stein on the subject. Here, in this volume (Chapter 7) one finds the original approach to the subject by use of the theory of analytic functions of a complex variable. Making use of Blaschke products to reduce the study of H^p functions to the case where the function has no zeros, one can then raise these analytic functions to small positive powers, thereby passing from the case $p < 1$ to the case $p > 1$ and allowing the Hardy–Littlewood Maximal Operator to control things, since the Maximal operator is well behaved on the spaces L^p, $p > 1$. Another very major collection of theorems, comprising the Littlewood–Paley Theory is emphasized as well, in Chapters 14 and 15. Zygmund felt that this theory was, to a much greater extent than most of the material in his book, "ahead of its time" and history has proven him correct on this. The Littlewood–Paley Theory involves the study of a certain quadratic functional, the Littlewood–Paley function, associated with a given function on the circle, which in turn is defined via the gradient of the harmonic extension of this function. This associated Littlewood–Paley function has an L^p norm $(1 < p < \infty)$ which is comparable in size with the L^p norm of the function, and is therefore extremely useful in a number of important applications. In *Trigonometric Series*, one sees this functional applied to prove the Marcinkiewicz Multiplier Theorem, and to understand the theory of Hardy Spaces, to mention just two such applications. Treated via a combination of identities for harmonic and analytic functions, this theory was later seen to be very much related to the theory of martingales from probability theory and to the Calderon–Zygmund Theory of singular integrals. In fact one of the most elegant applications of the latter theory was to recapture a very simple and conceptual proof of the Littlewood–Paley Theorem by viewing this as a special case of the boundedness of singular integrals, if only one considers the case of Hilbert space valued kernels.

The Littlewood–Paley Theory and Marcinkiewicz Multiplier Theorem lead naturally to the scientific collaboration in Zygmund's life that was no doubt one of the greatest of the twentieth century, namely that with his student, Alberto Calderon. Although there is not a detailed account of the Calderon–Zygmund Theory in higher dimensions, one can see several highlights of the work of Calderon, and of Calderon–Zygmund in this volume. For example, this book includes Calderon's celebrated thesis result, that for harmonic functions in the upper half space, non-tangential boundedness and non-tangential convergence is almost everywhere equivalent (this is actually only presented in the context of harmonic functions in the disk, but as Zygmund points out the exact same proof extends to functions harmonic in the unit ball or upper half space in higher dimensions). And, as pointed out above, the Hilbert transform is analyzed by techniques which are the one variable predecessors of the

Calderon–Zygmund ones, such as the Rising Sun Lemma of F. Riesz instead of the Calderon–Zygmund Decomposition. There is no question that anyone who reads *Trigonometric Series* will not only gain an understanding of the classical one-dimensional theory of Fourier Analysis, but will get an excellent understanding for the background of the more modern methods in several variables, and an insightful preview of those methods. As far as a view of the development by Zygmund of the Chicago School of Analysis, this text is simply written too early to cover most of this period. In particular, it is too early to reflect the tremendous influence of Elias M. Stein who had just started his career shortly before the final major revision. Still, it should be noted that Zygmund includes in a prominent way the Stein theorem on interpolation of analytic families of operators, which has passed the test of time as one of the basic tools of modern harmonic analysis. It is also worth noting that Zygmund mentioned on a number of occasions his regret at not having included the Carleson–Hunt Theorem on almost everywhere convergence of the Fourier series of functions belonging to L^p, $p > 1$. At one time, he was considering a second major revision which would have included this result, but for some reason never produced this revision. That he must be forgiven this omission is completely clear, since all books must end somewhere.

In fact, what is surprising about the current volume is not what is missing. What is surprising is that a single person could write such an extraordinarily comprehensive and masterful presentation of such a vast field. This volume is a text of historic proportion, having influenced several generations of some of the greatest analysts of the twentieth century. It holds every promise to do the same in the twenty first.

Bibliography

[1] W. Beckner, A.P. Calderon, R. Fefferman and P. Jones, *Conference on Harmonic Analysis in Honor of Antoni Zygmund*, Wadsworth Math. Series, 1981.

[2] M. Christ, C. Kenig, and C. Sadosky, 'Harmonic analysis and partial differential equations', in *Essays in Honor of Alberto P. Calderon*, Chicago Lectures in Mathematics, The University of Chicago Press, 1999.

[3] C. Fefferman, J.-P. Kahane, and E.M. Stein, 'A. Zygmund's scientific work', in *Selected Papers of Antoni Zygmund*, Kluwer Academic Publishers, 1989.

[4] A. Hulanicki, P. Wojtaszczyk, and W. Zelazko, *Selected Papers of Antoni Zygmund*, Kluwer Academic Publishers, 1989.

[5] E.M. Stein, 'Calderon and Zygmund's theory of singular integrals in harmonic analysis and partial differential equations', in *Essays in Honor of Alberto P. Calderon*, Chicago Lectures in Mathematics, The University of Chicago Press, 1999.

[6] I. Wirszup, Antoni Zygmund, *The University of Chicago Record*, 1993.

TRIGONOMETRIC SERIES

VOLUMES I AND II

DEDICATED TO THE MEMORIES OF

A. RAJCHMAN AND J. MARCINKIEWICZ

MY TEACHER AND MY PUPIL

A. ZYGMUND

TRIGONOMETRIC SERIES

VOLUME I

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