普通高等学校双语教学规划教材

Theoretical Mechanics

主编　马红玉

西安电子科技大学出版社
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内容简介

本书精选理论力学的基本内容，主要内容包括质点运动学、质点动力学、质点组力学、刚体力学、分析力学以及有心力的应用等。本书内容简练，公式推导详细，重点突出。

本书可作为高等学校物理类专业的教材，也可作为其他相关专业的参考书目，还可作为学习科技英语的参考书。

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前　言

理论力学既是物理专业的一门基础理论课，又与工程技术紧密连接，还是结构力学、弹性力学、流体力学、材料力学等学科的基础。它是高等院校物理专业与工程专业的一门重要的基础课，是学习其他理论物理学科的入门向导，也是近代工程技术的理论基础。

针对教学学时压缩，但常见的教材内容较多，篇幅较长，导致学生负担较重的现状，编者选取了基础内容，编写了这本适用于较少学时的教材。理论力学课程包括的公式较多，编者在推导过程中加入推导步骤，可帮助学生加深理解，获得更好的教学效果。在编写过程中，编者借鉴了国内外同类教材，结合当代大学生的实际情况，着重于基础理论，引导和启发学生理解理论力学的基本原理和概念。本书为英文版，学生在学习物理知识之余，还可学习相关普通英语语法和专业词汇。

由于编写时间仓促，作者水平有限，本书中难免存在不妥之处，恳请广大读者指正。

马红玉

2017 年 8 月于西安

Contents

Introduction

Theoretic mechanics studies the mechanics motion of objects. By mechanics motion we mean the object changes its relative positions. The mechanics motion is the basic motion and is constituted of the complex motion. Theoretic mechanics is the guider to learn the other theoretic physics and is the basic of the engineering technology.

The basic rule of macroscopic mechanical motion studied by theoretic mechanics can be used to solve the problem of multi-degree-of-freedom motion of the object. The object of this book is focused on the finite degree freedom of the object such as particle and rigid body. The main task of the theoretical mechanics is the basic rule of inductive followed by the mechanical movement and determine the movement of objects or the property of the force on them. The method is a series of reasoning process by the help of the use of rigorous mathematical tool. From the point of the order, we first study the mechanical movement kinematics. And then further study the rules of the dynamics of mechanical movement should follow. As for the statics of the balance problems can be as one part of the dynamic for science. But the statics is very important in engineering. So it can be an independent discipline.

Mechanics is one discipline which developed earlier. As early as in ancient times, people started to manufacture and use some simple production tools due to the need of agriculture. So people have had some knowledge and understanding for mechanical movement. With the development of production, people gradually deepen the knowledge of the mechanical movement. By the end of the 16th century, the understanding of the mechanics also produced a leap when Western Europe started the formation and development of capitalism. Newton published the famous three laws of motion on the basis of predecessors'research work and laid the foundation of classical mechanics. Since then, many scientists made in-depth research on the mechanics. And they constantly opened up new areas and revealed the new law. Especially calculus and the other mathematical tools are widely used which provides a powerful weapon for the development of mechanics and promoting the development of mechanics. By the 18th century, Lagrange wrote a book which is Analytical Mechanics. the mechanical problems can fully use the strict analysis method to deal with. As the further research and contributions of Hamiltonian and Jacobi's, the classical mechanics gradually developed into a system subject.

Theoretical mechanics is closely related with other branches of natural science (like mathematics, astronomy, meteorology, science and engineering technology and so on), especially more closely to mathematics. Theoretical mechanics widely used mathematical theory to deduction and reasoning of the theory system. Some problem on mechanics often promote the development of mathematics itself. There are a lot of examples on the history of science.

Due to the rapid development of advanced science and technology, like the satellite, the rockets and space, etc. , many new topics for mechanics is put forward. And it promote the development of the modern mechanics. Although theoretical mechanics is older subjects, it has a strong vitality. Theoretical mechanics still has an important role in life and in scientific experiments.

Chapter 1 Kinematics of Particles

1.1 Description of the particle motion

1. System of reference

The mechanical motion of an object or a system of objects is absolute. In order to clearly describe the motion, the space position of the subject in different time should be descried firstly. Since the position is always relative, it is meaningless to say the position of a single object without specifying its spatial relation to another object. So the motion is relative too. As the position is relative, we must have a reference body to define the position or motion of an object. The object is the system of reference. Although the reference object is limited in size, the system of reference should be considered as entire infinite space fixed with the object.

We can choose the system of reference freely when we describe the motion. The description of the motion is different in different reference. The standard of the reference selection is that the description of the motion is simple. Therefore, the system of reference is different in different condition. For example, we always choose the ground as the reference when we discuss the motion of the car or the plane. When we study the motion of the rocket, we can choose the earth as the reference system. We should choose the sun as the reference if we describe the motion of the planet.

2. Coordinate System

We have to build a coordinate system in the reference system for quantitative describing the position of the object. The most commonly used ones are rectangular, spherical or cylindrical coordinate system. All those are three dimensional. In two-dimensional case, we have plane polar coordinate system. The choice of coordinate system is rather arbitrary in kinematics. We prefer the system in which the motion looks simpler. For example, we choose a rectangular coordinate system for the motion on a straight line, or a polar coordinate system for a circular motion. The transformation from one coordinate system to another is a mathematical event, which will be discussed in the next part.

3. Particle and Position

1) Particle

In the practical problem, the shape and the size of the object has nothing with the problem. In such case, the object can be considered as a particle when we study the motion of it. The particle is a geometric point with mass. For example, we study the motion of the planet. Although the planet is very huge, it's radius is smaller than the radius of the orbit that the planet moves around the sun. The planet can be considered as a particle in such problem.

All objects can be considered as the assembler of particles in any situation. Therefore, the research of the mechanical problem usually start from the particle.

2) Description of the position of the particle

We can conveniently descript the spatial position when the reference system have been choose. One point O can be chosen as the reference point. The position of the particle in space P at moment t can be descript as a vector $\boldsymbol{r}$. The length of $\boldsymbol{r}$ is the distance between the P and the reference O. The direction of $\boldsymbol{r}$ is the orientation from the space P to the reference O.

The vector $\boldsymbol{r}$ can compactly descript the position of the particle. When we need to operate the physical quantity, the operation of the vector is difficult. Thus we build the coordinate system which origin is the point O. The vector $\boldsymbol{r}$ can be descript as one function or the coordinates.

In a rectangular coordinate system (Fig. 1. 1), the position of particle is denoted by a vector

$$\boldsymbol{r}=x\hat{\boldsymbol{i}}+y\hat{\boldsymbol{j}}+z\hat{\boldsymbol{k}} \tag{1.1.1}$$

Where $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$, $\hat{\boldsymbol{k}}$ are the basic vectors of the coordinate system. They have a unit length and are normally perpendicular to each other. x, y and z are the coordinates of the particle.

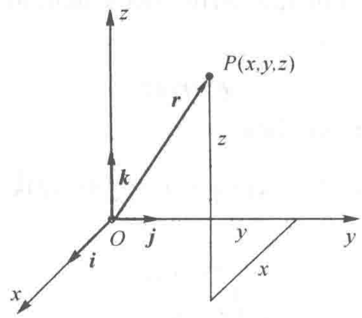

Fig. 1.1 Rectangular coordinate system

In a plane polar coordinate system, the two basic vectors are $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ (Fig. 1.2), which are both unit vectors in the direction of increasing the corresponding coordinate and perpendicular to each other. The position of the particle is denoted by

$$\boldsymbol{r}=r\,\hat{\boldsymbol{i}} \tag{1.1.2}$$

Where r is the length of the line OP and the θ is the polar angle. The basic vectors are variables. The length of the vectors are always equal to unite. But the orientation of the vectors are changing with the moving of the particle. The coordinates of the particle are r and θ. They are the time function

$$\begin{cases} r=r(t) \\ \theta=\theta(t) \end{cases} \tag{1.1.3}$$

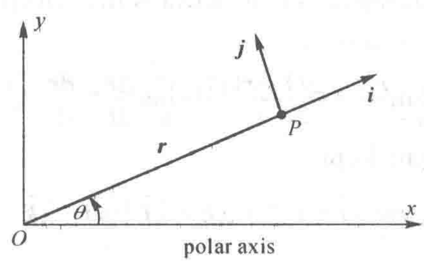

Fig. 1.2 Plane polar coordinate system

The nature coordinate system, cylindrical coordinate system and sphere coordinate system are also usually used to confirm the position of the particle.

3) Kinematical equation and trajectory

If the coordinates vary with time t, or x (and thus $\boldsymbol{r}$) are the function of time t, then we have a change in the position of the particle of a motion. The equation (1.1.4) is the kinematical equation

$$\boldsymbol{r}=\boldsymbol{r}(t) \tag{1.1.4}$$

In a rectangular coordinate system, then

$$\boldsymbol{r}(t)=x(t)\,\hat{\boldsymbol{i}}+y(t)\,\hat{\boldsymbol{j}}+z(t)\hat{\boldsymbol{k}} \tag{1.1.5}$$

The parameter equations are

$$\begin{cases} x=x(t) \\ y=y(t) \\ z=z(t) \end{cases} \tag{1.1.6}$$

In a rectangular coordinate system, the parameter equations are

$$\begin{cases} r=r(t) \\ \theta=\theta(t) \end{cases} \tag{1.1.7}$$

We get the trajectory equation when eliminating the parameter t of the parameter equations. All information about the motion is included in the trajectory or the function $\boldsymbol{r}(t)$. It is important to realize that all the parameter equations are continuous and differentiable as the space and time (and thus motion) are continuous by nature. The differential properties of $\boldsymbol{r}(t)$ represent the fundamental quantities of kinematics.

4. Velocity and Acceleration

1) Velocity

The most fundamental quantity in kinematics is the velocity $\boldsymbol{v}$, which is the first order derivation of $\boldsymbol{r}(t)$ with respect to time

$$\boldsymbol{v}=\lim_{\Delta t\to 0}\frac{\boldsymbol{r}(t+\Delta t)-\boldsymbol{r}(t)}{\Delta t}=\lim_{\Delta t\to 0}\frac{\Delta \boldsymbol{r}}{\Delta t}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\dot{\boldsymbol{r}}(t) \tag{1.1.8}$$

Or, if we write $\boldsymbol{v}$ in component form

$$\boldsymbol{v}=v_x\hat{\boldsymbol{i}}+v_y\hat{\boldsymbol{j}}+v_z\hat{\boldsymbol{k}}=\dot{x}\hat{\boldsymbol{i}}+\dot{y}\hat{\boldsymbol{j}}+\dot{z}\hat{\boldsymbol{k}} \tag{1.1.9}$$

Where

$$\begin{cases} v_x=\dot{x} \\ v_y=\dot{y} \\ v_z=\dot{z} \end{cases} \tag{1.1.10}$$

It is important to realize that the velocity $\boldsymbol{v}$ thus defined is belonging to an instant, not a time interval. For the latter we have only the average velocity, which is less powerful as it is not able to describe how the velocity varies in the time interval.

Obviously, velocity is a vector, as can be seen from Fig. 1. 3. The direction of $\boldsymbol{v}$ is always along the tangent of the trajectory at each point. The magnitude of $\boldsymbol{v}$ is called the speed, which is calculated by the general rule for the norm of a vector

$$v=\sqrt{v_x^2+v_y^2+v_z^2}=\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2} \tag{1.1.11}$$

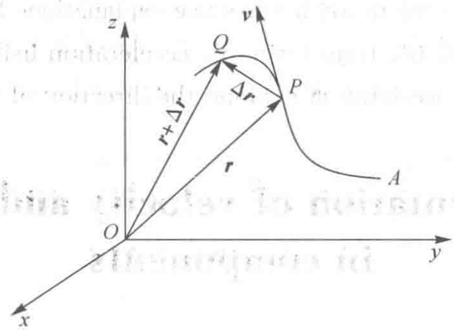

Fig. 1. 3 The definition of velocity

2) Acceleration

As the velocity $\boldsymbol{v}(t)$ is also a function of time t(Fig. 1. 4), its time rate can be defined as the acceleration $\boldsymbol{a}$

$$\boldsymbol{a}=\lim_{\Delta t\to 0}\frac{\Delta \boldsymbol{v}}{\Delta t}=\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}=\dot{\boldsymbol{v}}=\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}=\ddot{\boldsymbol{r}}(t) \tag{1.1.12}$$

which is the second order derivative of the trajectory $\boldsymbol{r}(t)$.

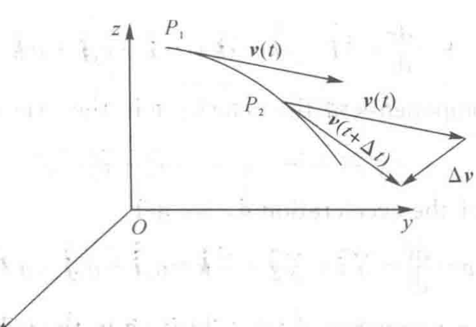

Fig. 1. 4 The definition of acceleration

The magnitude of an acceleration can also be calculated by the general rule for the norm of a vector, i.e., we have

$$a=\sqrt{a_x^2+a_y^2+a_z^2}=\sqrt{\dot{v}_x^2+\dot{v}_y^2+\dot{v}_z^2}=\sqrt{\ddot{x}+\ddot{y}+\ddot{z}} \tag{1.1.13}$$

Where

$$\begin{cases} a_x=\dot{v}_x=\ddot{x} \\ a_y=\dot{v}_y=\ddot{y} \\ a_z=\dot{v}_z=\ddot{z} \end{cases} \tag{1.1.14}$$

However, the direction of acceleration needs some explanation. Since the velocity is always in the tangential direction of the trajectory, the acceleration usually isn't in the tangential direction. The direction of acceleration is along the direction of the vector d$\boldsymbol{v}$.

1.2 Representation of velocity and acceleration in components

1. In the rectangular coordinate system

In a rectangular coordinate system, the basic vectors are $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$, $\hat{\boldsymbol{k}}$ which are invariant. If in the time t, the particle locate in the point P. The origin point of the rectangular coordinate is O. The vector $\overrightarrow{OP}$ define the location of the particle. It's coordinate is (x, y, z), then

$$\boldsymbol{r}=x\hat{\boldsymbol{i}}+y\hat{\boldsymbol{j}}+z\hat{\boldsymbol{k}} \tag{1.2.1}$$

The velocity is

$$\boldsymbol{v}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\dot{x}\hat{\boldsymbol{i}}+\dot{y}\hat{\boldsymbol{j}}+\dot{z}\hat{\boldsymbol{k}}=v_x\hat{\boldsymbol{i}}+v_y\hat{\boldsymbol{j}}+v_z\hat{\boldsymbol{k}} \tag{1.2.2}$$

Where $\dot{x}$, $\dot{y}$, $\dot{z}$ are the components of the velocity $\boldsymbol{v}$ in the axle x, y, z. The speed is

$$v=\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}=\sqrt{v_x^2+v_y^2+v_z^2} \tag{1.2.3}$$

From the definition of the acceleration $\boldsymbol{a}$, we get

$$\boldsymbol{a}=\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}=\ddot{x}\hat{\boldsymbol{i}}+\ddot{y}\hat{\boldsymbol{j}}+\ddot{z}\hat{\boldsymbol{k}}=a_x\hat{\boldsymbol{i}}+a_y\hat{\boldsymbol{j}}+a_z\hat{\boldsymbol{k}} \tag{1.2.4}$$

Where $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ are the components of the velocity $\boldsymbol{a}$ in the axle x, y, z. The magnitude of $\boldsymbol{a}$ is

$$a=\sqrt{a_x^2+a_y^2+a_z^2}=\sqrt{\dot{v}_x^2+\dot{v}_y^2+\dot{v}_z^2}=\sqrt{\ddot{x}^2+\ddot{y}^2+\ddot{z}^2} \tag{1.2.5}$$

The vector $\boldsymbol{r}$, the velocity $\boldsymbol{v}$ and the acceleration $\boldsymbol{a}$ are the function of time. The components of them must be the function of time. From the equations (1.2.1), (1.2.2) and (1.2.4), we can find out of the other two if we know one of them. For example, we want to determine the velocity and the acceleration of the particle. We can choose one proper coordinate system. Then we confirm the location coordinate. Calculating the first and second order derivation of $\boldsymbol{r}(t)$ with respect to time t, we get the velocity $\boldsymbol{v}$ and the acceleration $\boldsymbol{a}$. If we know the velocity $\boldsymbol{v}$, we have to integral to calculate the vector $\boldsymbol{r}$.

2. In the plane polar coordinate system

In a plane polar coordinate system, the two basic vectors are $\hat{\boldsymbol{i}}'$ and $\hat{\boldsymbol{j}}'$ (Fig. 1.5), which are both unit vectors in the direction of the increasing the corresponding coordinate and perpendicular to each other. They are related to the basic vectors of Cartesian coordinate system y, we have

$$\begin{cases}\hat{\boldsymbol{i}}'=\hat{\boldsymbol{i}}\cos\theta+\hat{\boldsymbol{j}}\sin\theta \\ \hat{\boldsymbol{j}}'=-\hat{\boldsymbol{i}}\sin\theta+\hat{\boldsymbol{j}}\cos\theta\end{cases} \tag{1.2.6}$$

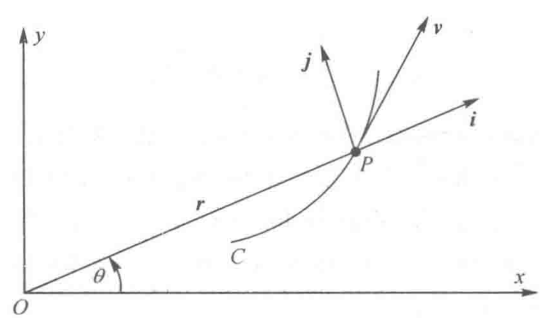

Fig. 1.5 The plane polar coordinate system

The general vector $\boldsymbol{r}$ is written as

$$\boldsymbol{r}=r\hat{\boldsymbol{i}}' \tag{1.2.7}$$

Where $\hat{\boldsymbol{i}}'$ is the unit vector in the radial direction. The derivative of $\boldsymbol{r}$ with respect to time t is

$$\boldsymbol{v}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}t}\hat{\boldsymbol{i}}'+r\frac{\mathrm{d}\hat{\boldsymbol{i}}'}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}t}\hat{\boldsymbol{i}}'+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{\boldsymbol{j}}' \tag{1.2.8}$$

Where in the last step, the derivative of $\hat{i}'$ is calculated from (1. 2. 6) as following

$$\frac{\mathrm{d}\hat{i}'}{\mathrm{d}t}=-\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{i}+\cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{j}=\frac{\mathrm{d}\theta}{\mathrm{d}t}(-\sin\theta\hat{i}+\cos\theta\hat{j})=\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{j}' \tag{1. 2. 9}$$

The two terms of (1. 2. 7) represent the radial v_r and angular v_θ velocities, respectively. We have

$$\begin{cases} v_r=\dot{r} \\ v_\theta=r\dot{\theta} \end{cases} \tag{1. 2. 10}$$

The acceleration is the derivative of $\boldsymbol{v}$. It can be calculated from (1. 2. 8) as following

$$\begin{aligned} \boldsymbol{a}&=\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}=\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\hat{i}'+\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{j}'+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\hat{j}'+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\mathrm{d}\hat{j}'}{\mathrm{d}t} \\ &=\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\hat{i}'+2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{j}'+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\hat{j}'+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\left(-\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\hat{i}' \end{aligned} \tag{1. 2. 11}$$

Or

$$\boldsymbol{a}=\left[\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}-r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right]\hat{i}'+\left(2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t}+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right)\hat{j}' \tag{1. 2. 12}$$

Where the first term in radial component is the radial acceleration, the second term is the angular acceleration.

$$\begin{cases} a_r=\ddot{r}-r\dot{\theta}^2 \\ a_\theta=r\ddot{\theta}+2\dot{r}\dot{\theta}=\dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta}) \end{cases} \tag{1. 2. 13}$$

In the polar coordinate system, the direction of the radial velocity and the angular velocity are changing with time. The radial acceleration usually don't equal to the $\ddot{r}$ although the radial velocity is the first order derivation of $\boldsymbol{r}$. In plane polar coordinate system, the component of the acceleration is complex. It just can be solved with the problem of the plane curve motion.

3. In the cylindrical coordinate system

The cylindrical coordinate system can be thought as the plane polar coordinate plus the vertical axial z. In cylindrical coordinate system, coordinates r, θ and z are related to Cartesian coordinate by

$$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases} \tag{1. 2. 14}$$

Similarly, the three basic vectors are (Fig. 1. 6)

$$\hat{e}_r=\cos\theta\hat{\boldsymbol{i}}+\sin\theta\hat{\boldsymbol{j}}$$

$$\hat{e}_\theta=-\sin\theta\hat{\boldsymbol{i}}+\cos\theta\hat{\boldsymbol{j}} \qquad (1.2.15)$$

$$\hat{e}_z=\hat{\boldsymbol{k}}$$

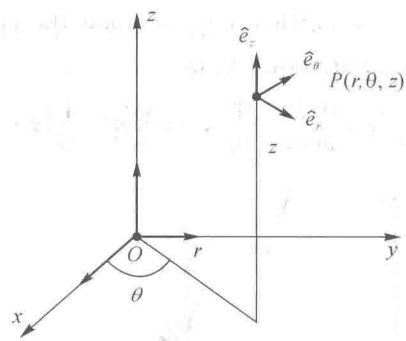

Fig. 1. 6 The cylindrical coordinate system

They are in the direction of increasing the corresponding coordinates and perpendicular to each other. The general vector $\overrightarrow{OP}$ is represented in this coordinate system by

$$\overrightarrow{OP}=r\,\hat{e}_r+z\,\hat{e}_z \qquad (1.2.16)$$

To find out the expression for velocity and acceleration, we first perform differentiation on (1. 2. 15) and obtain the following differential relations between basic vectors

$$\frac{\mathrm{d}\hat{e}_r}{\mathrm{d}t}=(-\sin\theta\hat{i}+\cos\theta\hat{j})\frac{\mathrm{d}\theta}{\mathrm{d}t}=\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{e}_\theta$$

$$\frac{\mathrm{d}\hat{e}_\theta}{\mathrm{d}t}=-(\cos\theta\hat{i}+\sin\theta\hat{j})\frac{\mathrm{d}\theta}{\mathrm{d}t}=-\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{e}_r \qquad (1.2.17)$$

Then we differentiate (1. 2. 16) with respect to time t. By using (1. 2. 17) we calculate the expression for the velocity to be

$$\boldsymbol{v}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}t}\hat{e}_r+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{e}_\theta+\frac{\mathrm{d}z}{\mathrm{d}t}\hat{e}_z \qquad (1.2.18)$$

And the expression for the acceleration to be

$$\boldsymbol{a}=\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}=\left[\frac{\mathrm{d}^2r}{\mathrm{d}t^2}-r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right]\hat{e}_r+\left(2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t}+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right)\hat{e}_\theta+\frac{\mathrm{d}^2z}{\mathrm{d}t^2}\hat{e}_z \qquad (1.2.19)$$

Then various terms have similar explanations as in the case of plane polar coordinate system.